DEVELOPMENT OF WEIGHTED OPTIMALITY CRITERIA AND GENETIC ALGORITHM FOR DESIGN SELECTION OF RESPONSE SURFACE DESIGNS

BY

MR. APISAK CHAIROJWATTANA

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
(STATISTICS INTERNATIONAL PROGRAM)
DEPARTMENT OF MATHEMATICS AND STATISTICS
FACULTY OF SCIENCE AND TECHNOLOGY
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MR. APISAK CHAIROJWATTANA

ENTITLED

DEVELOPMENT OF WEIGHTED OPTIMALITY CRITERIA AND GENETIC ALGORITHM FOR DESIGN SELECTION OF RESPONSE SURFACE DESIGNS

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ABSTRACT

The purposes of the research are to study the weighted D and G optimality criteria (D_w and G_w) for the response surface designs and develop genetic algorithm (GA) for generating the designs that optimize the weighted D and G optimality criteria.

In this study, several cases are considered which are 2, 3, and 4 input variables, 6, 7, . . ., 29 design points depend on the design, and 16 combinations of prior probabilities assigned to the model.

For all cases, the designs are generated and obtained the reduced model using weak heredity (WH) and then calculate the D and G optimality criteria for find D_w and G_w. D_w and G_w are studied and compared.

The results of this study are as follows. In case of the 2 and 4-dimensional hypercube and hypersphere, GA designs are the best designs considering of D_w and G_w value. For the 3-dimensional hypercube, GA designs are the best designs in term of D_w, however, in term of G_w, the CCDs are the best designs. In case of 3-dimensional hypersphere, the GA designs are the best designs in term of D_w and G_w.

In addition, the catalogues of the best design are also created based on each weighted optimality criterion across a range of design sizes N and number of input variables k are also created.

Finally, this research shows that the GA design is the best design for response surface design because it's highest efficient in the weighted D and G optimality criteria.

Keywords: Computer-generated design, The Experimental Design, Optimality Criteria, Weak heredity.

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Mr. Apisak Chairojwattana

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CHAPTER 1

INTRODUCTION

1.1 Background

Running an experimental design is very important in many research situations. For example, the experimentation is a part of the scientific processes and is a way to learn about how a system or process works. In the engineering world, the experimental design is an important tool for improving the performance of a manufacturing process. It also has extensive applications in the development of new processes. The application of experimental design is adopted for reducing variability, reducing development time, reducing overall cost, and evaluating the process performance.

Response surface designs are one class of experimental designs and its methodology is defined as a collection of statistical and mathematical techniques useful for developing, improving and optimizing processes (Myers and Montgomery, 2002) [28]. The response surface methodology is helpful in solving many types of problems which can be classified in three general categories. First, the problem of fitting a response surface model over a particular region of interest. For example, Drain, Borror, Anderson-Cook, and Montgomery (2005) [11] applied the response surface design for correlated noise variables. Gallina, Martowicz, and Uhl (2010) [16] used the response surface techniques applied to investigate the effect of uncertainly on the eigenvalues and eigenvectors dispersion of the thickness of windscreen layer. Secondly, the problem of determining the optimization of the response. For instance, Del Castillo, Montgomery, and

McCarville (1996) [13] presented modified desirability functions for a multiple response optimization. Eren and Kaymak-Ertekin (2007) [15] used response surface methodology to determine the optimum processing conditions that yielded maximum water loss, weight reduction, minimum solid gain and water activity during osmotic dehydration of potatoes. Finally, the problem is selection of operating conditions to achieve certain specifications or customer requirements. In most response surface problems, there are several responses that must be considered. The major goal is to maintain an optimal or near-optimal value for the response function, while simultaneously satisfying specifications for other responses that are imposed by the customer.

In response surface methodology, we often uses a first-order, interaction, or second-order model as the model to approximate the response function. The second-order model is very useful in response surface methodology because it is very flexible and easy to estimate the parameters by the least squares method. In addition, it works well as an approximating model for real data sets in response surface problems. When fitting the model, the use of coded variables in place of the actual process variables facilitates in the construction of the experimental design. The advantages of using coded variable are computational ease, increased accuracy in estimating the model coefficients, and enhanced interpret ability of the coefficient estimates in the model (Khuri and Cornell, 1996) [23].

After considering practical designs, design optimality criteria are often used to evaluate a proposed experimental design. The common design optimality criteria are D, G, A, and IV criteria. Based on these criteria, D, G, A efficiencies and the scaled average prediction variance (IV) criterion can be calculated and can be used to compare designs. For example, Borkowski and Valeroso (2001) [5] presented the comparison of design optimality criteria of reduced models for response surface design in the hypercube. Chomtee (2003) [10] presented the comparison of design optimality criteria of reduced models for response surface design in the spherical design region. Dette, Wong, and Zhu (2005) [14] applied the D-optimality criteria for the placebo-treatment problem. Anderson-Cook, Borror, and Montgomery (2009) [1] used the optimality criteria for evaluated and compare the response surface designs. The selection of a design via optimality criteria is dependent on the choice of the estimating response surface model. That is, for different models, the efficiency values will also differ. After the model is fitted, many parameters may be deemed insignificant, and a reduced model retaining only significant terms is adopted for practical applications. Therefore, a design should be robust over the set of potential reduced models, that is, the design's optimality criteria should be left high over a wide classification of potential model particularly for those models held to be most likely a priori. Most statistical review focuses only on comparisons associated with the initially proposed model. Although graphical methods, such as variance dispersion graphs, fraction of design space plots, and quantile dispersion graphs allow designs to be compared based on their prediction variance properties throughout the experimental design region, these methods also assume that one a priori model is the correct approximating response surface model. See Borkowski (2006) [4] for a review of these graphical methods.

Li and Nachtsheim (2000) [24] proposes the topic of design robustness by developing a class of factorial designs that can be used for efficient estimation of main effects and any combination of interactions. Other researchers have pointed to the design selection problem when the proposed estimating model is an under-parameterized estimation of the true response surface. The general case is the use of a low order polynomial when higher order polynomial is a better approximating function.

This dissertation address model misspecification for which the second-order model is overparameterized. Based on weighted D and G optimality criteria measures, the robustness properties of several family of response surface designs over a collection of reduced model are evaluated. Li and Nachtsheim (2000) [24] considered two weighted criterias that were based, respectively, on a design's estimation capacity and the D criterion that was equally-weighted across models. Atkinson, Donev, and Tobias (2007) [2] discussed composite design optimality criteria in their book.

Chipman (1996) [8] presented the two classes of reduce model based on heredity, which are weak heredity (WH) and strong heredity (SH). This research will lead to the development and application of only WH to calculate a weighted average of the efficiency values across all models given the assignment of prior probabilities to model effects.

Although the weighted optimality criteria can be used evaluate and compare designs, the designs that optimize these weighted optimality criteria are the most desirable. Currently, the optimal designs based on weighted optimality criteria are not known. A genetic algorithm (GA), however, is potential to generate these optimal designs.

GAs are an evolutionary search strategy based on simplified rules of biological population genetics of evolution. The foundations of GAs were developed by John Holland (1960) and provided solutions for complex problems in optimization, machine learning, programming, and job scheduling (See Holland (1975) [22], Michalewicz (1996) [26], Haupt and Haupt (2004) [19], and Gen, Cheng, and Lin (2008) [17]). The GAs were adopted in many disciplines that include the response surface designs. For example, Borkowski (2003) [3] used a GA to generate small exact response surface designs. Heredia-Langner, Carlyle, Montgomery, Borror, and Runger (2003) [21] developed GAs for the construction of D-optimal designs. Heredia-Langner, Carlyle, Montgomery, and Borror (2004) [20] approached GAs for model-robust optimal designs. Drain, Carlyle, Montgomery, Borror, and Anderson-Cook (2004) [12] presented a genetic algorithm hybrid for constructing optimal response surface designs. Goldfarb, Borror, Montgomery, and Anderson-Cook (2005) [18] used GAs to generate mixture-process experimental designs involving control and noise variables.

Historically, classical response surface designs (such as Central Composite Designs (CCDs) and Box-Behnken Designs (BBDs)) have been applied widely in response surface design. Computer-generated designs (such as designs generated by the Optex Procedure in *SAS* statistical software) have also been used widely but they require specifying a single model. For both the classical designs and the computer-generated designs, very little research has been done to study their model robustness properties. Because the application of GAs to statistical design problems is relatively new, very little research has been performed on the use of GAs for generating designs. Part of this research will include the introduction of the use of GAs to study the model robustness properties of classical CCDs and BBDs by using the D and G weighted optimality criteria.

1.2 Research Objectives

The objectives of the study are as follows:

1. To find the weighted D and G-efficiencies for commonly-used response surface designs. These include Central Composite Designs (CCDs), Box-Behnken Designs (BBDs), and computer-generated designs.
2. To develop genetic algorithms (GAs) for generating designs that optimize the weighted D and G optimality criteria for a variety of response surface design situations.

1.3 Research Scope

The scope of the research includes the following:

1. To study the robustness properties of classical response surface designs and computer-generated designs for a number of input variables, the following designs will be considered:
   - (a) CCDs having $k = 2$, 3, and 4 input variables.
   - (b) BBDs having $k = 3$ and 4 input variables.
   - (c) Computer-generated designs having $k = 2$, 3, and 4 input variables.
2. For these designs, D and G-criteria will be calculated for each of the reduced models assuming weak heredity (WH) for the following sets of prior probabilities assigned to model terms:

No	Weak heredity p_l	p_1	p_2	p_q
1	0.50	0.10	0.35	0.35
2	0.50	0.10	0.35	0.95
3	0.50	0.10	0.95	0.35
4	0.50	0.10	0.95	0.95
5	0.50	0.70	0.35	0.35
6	0.50	0.70	0.35	0.95
7	0.50	0.70	0.95	0.35
8	0.50	0.70	0.95	0.95

No	Weak heredity p_l	p_1	p_2	p_q
9	0.90	0.10	0.35	0.35
10	0.90	0.10	0.35	0.95
11	0.90	0.10	0.95	0.35
12	0.90	0.10	0.95	0.95
13	0.90	0.70	0.35	0.35
14	0.90	0.70	0.35	0.95
15	0.90	0.70	0.95	0.35
16	0.90	0.70	0.95	0.95

3. Designs will be generated by GAs under the following conditions:

   (a) For $k = 2$, 3, and 4 design variables.

   (b) In two different design regions: the k-dimensional hypercube and the k-dimensional hypersphere of radius $\sqrt{k}$.

   (c) For the following number of design points N will be considered:

k	CCDs	BBDs	Computer-generated
2	$N = 9, 10, \ldots, 14$		$N = 6, 7, \ldots, 10$
3	$N = 15, 16, \ldots, 19$	$N = 13, 14, \ldots, 17$	$N = 10, 11, \ldots, 14$
4	$N = 25, 26, \ldots, 29$	$N = 25, 26, \ldots, 29$	$N = 16, 17, \ldots, 20$

   (d) For weighted D and G-criteria.

   (e) For weak heredity (WH).

1.4 Benefits of the Research

The benefits of this research include:

1. Generating the best designs for the conditions defined in the Research Scope. These designs will form a catalog of designs that will be very useful in industry. These designs can be used to optimize a manufacturing process. It also can reduce production costs as well.

2. For production planning, the best model can also be used in planning how to set the levels of the input variables for an industrial process. By controlling the level and interaction of input variables, the experimenter can achieve optimum process yield.

CHAPTER 2

REVIEW OF LITERATURE

2.1 Response Surface Methodology

Response Surface Methodology (RSM) is a group of statistical techniques that are useful for the solving and analysis of problems in which a response variable is influenced by several input variables. The objective of RSM is to optimize this response. RSM was defined in many definition. For example, Myers and Montogomery (2002 page 1) [28] defined

> "Response surface methodology(RSM) is a collection of statistical and mathematical techniques useful for developing, improving, and optimizing processes. It also has important applications in the design, development, and formulation of new products, as well as in the improvement of existing product designs."

and that the objectives of RSM are the mapping of a response surface over a particular region of interest, the optimization of the response of interest, or the selection of operating conditions to achieve specific conditions for the response (such as a specified target value for the response). Khuri and Cornell (1996 page 3) [23] defined response surface methodology as a set of techniques that encompasses:

> "(1) Setting up a series of experiments (designing a set of experiments) that will yield adequate and reliable measurements of the response of interest.
>
> (2) Determining a mathematical model that best fits the data collected from the design chosen in (1), by conducting appropriate tests

of hypotheses concerning the model's parameters.

(3) Determining the optimal setting of the experimental factors that produce the maximum (or minimum) value of the response."

In RSM, the response variable (y) is typically assumed to be related to the input variables $(\xi_1, \xi_2, \ldots, \xi_k)$ and the relationship is

$$y = f(\xi_1, \xi_2, \ldots, \xi_k) + \varepsilon \tag{2.1}$$

where $f(\xi_1, \xi_2, \ldots, \xi_k)$ is the unknown true response function, and ε is a term of the other sources of variability not accounted for in f. Thus, the measurement error on the response will be included in ε. We will assume ε to have a normal distribution with mean zero and constant variance σ^2. Therefore,

$$\begin{aligned} E(y) &= E[f(\xi_1, \xi_2, \ldots, \xi_k)] + E(\varepsilon) \\ &= f(\xi_1, \xi_2, \ldots, \xi_k). \end{aligned} \tag{2.2}$$

The input variables $\xi_1, \xi_2, \ldots, \xi_k$ are called the *natural variables* and are transformed into *coded variables* $(x_1, x_2, \ldots, x_k)$. The coded variables are defined to be dimensionless with mean zero and have the same standard deviation. The coded variables can be written as

$$x_i = \frac{2\xi_i - (L_i + H_i)}{H_i - L_i} \qquad i = 1, 2, \ldots, k \tag{2.3}$$

Where H_i and L_i are the experimental high and low levels of ξ_i, respectively. For example, consider the chemical reaction process just discussed where the input variables are temperature ξ_1, time ξ_2, and pressure ξ_3. The minimum and maximum levels of the input variables are 250°F and 450°F for temperature, 5 and 10 seconds for time, and 75 psi and 125 psi for pressure. The coded variables of ξ_1, ξ_2, and ξ_3 are $x_1 = \frac{2\xi_1-(450+250)}{450-250}$, $x_2 = \frac{2\xi_2-(5+10)}{10-5}$, and $x_3 = \frac{2\xi_3-(75+125)}{125-75}$. Thus, when an input variable has only two levels. The Equation (2.3) produces the familiar ± 1 notation for the level of the coded variables associated with the two-level factorial arrangements. Therefore, $x_1 = -1$ when $\xi_1 = 250$°F, $x_1 = 1$ when $\xi_1 = 450$°F, $x_2 = -1$ when $\xi_2 = 5$, $x_2 = 1$ when $\xi_2 = 10$, $x_3 = -1$ when $\xi_3 = 75$, and $x_3 = 1$

when $\xi_3 = 125$. Without loss of generality, the true response function (2.2) can be written in term of the coded variables as

$$\eta = f(x_1, x_2, \ldots, x_k) \tag{2.4}$$

Because the true response function f is unknown, we usually use a low-order polynomial (such as a first-order or second-order polynomial) to approximate it. The first-order model in terms of the code variables is

$$\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k \tag{2.5}$$

Equation (2.5) is often called a *main effects model.* The model can be extended to an interaction effects model by adding the interaction terms to the model. The interaction effects model is

$$\eta = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum\sum_{i<j} \beta_{ij} x_i x_j. \tag{2.6}$$

In many cases when the response surface has strong curvature, the second-order model is usually adequate and useful as an approximation the true response surface f. The second-order model is

$$\eta = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum\sum_{i<j} \beta_{ij} x_i x_j \tag{2.7}$$

The second-order models work well in solving real response surface problems since it can take on a wide variety of functional form. In addition, parameters in the second-order model are easy to estimate. Thus, the second-order model is very widely used in application of response surface methodology.

2.2 Response Surface Designs

In RSM, the full and fractional factorial designs are an important class of designs. The two-level full and fractional factorial designs are used for variable screening which is accomplished with a design for fitting a first-order model. However, when the goal is optimization, the experimental design should allow the experimenter to fit the second-order model that contains $1 + 2k + k(k-1)/2$ parameters. Important designs for fitting the second-order model are the central composite designs (CCDs) and the Box-Behnken designs (BBDs). CCDs are designs that consist of points from a full or fractional factorial design, center runs at the middle level of each design variable, and axial points located on the axes for each design variable. The factorial points contribute to the estimation of linear terms and two-factor interactions in the model. The axial points and center runs contribute to the estimation of quadratic terms in the model. BBDs are formed by combining two-level full or fractional factorial designs with incomplete block designs. The detail of these designs are described in next sections.

2.2.1 Full and Fractional Factorial Designs

The full factorial designs are a class of designs applied to the study of the effects of two or more factors on a response of interest. In general, factorial designs are the most efficient design for these experimental goals. By definition, a factorial design is, the design includes all possible combinations of the levels of the factors.

A very important special case of the factorial design is the 2^k factorial design for k factors of interest and each factor has only two levels. The model for a 2^k design will include k main effects, $\binom{k}{2}$ two-factor interactions, $\binom{k}{3}$ three-factor interactions, ..., and one k-factor interaction. That is, for a 2^k design the complete model will contain $2^k - 1$ effects. By definition, the *main effect* of a factor is defined to be the change in response produced by a change in the level of the factor, and a *two-factor interaction* occurs when the difference in response between the levels of one factor is not the same at all levels of the other factor. In RSM, the full factorial designs are the most powerful for the first-order model or the model that includes the two-factor interactions.

For example, the model of 2^2 design have 2 main effects, and 1 two-factor interaction and the complete model will contain 3 effects. The model of 2^3 design have 3 main effects, 3 two-factor interactions, 1 three-factor interaction.

The complete model of 2^3 design will contain 7 effects. It is also assumed that all models wil contain an intercept term in addition to these effects. The geometries of 2^2 and 2^3 design are shown in Figure 2.1 and Figure 2.2 , respectively.

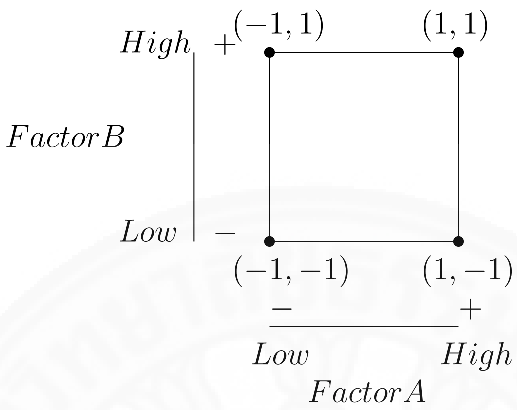
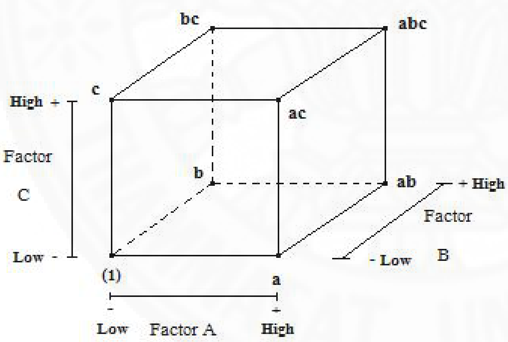


Figure 2.1: The 2^2 design.



(a) Geometric view

Run	Factor		
	A	B	C
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+

(b) The design matrix

Figure 2.2: The 2^3 factorial design.

When the number of factors in a 2^k factorial design increases, the number of all possible combinations of the levels of the factors rapidly increases. For example, the 2^7 design requires 128 runs and the model can include 7 main effects, 21 two-factor interactions, and 99 three-factor and higher-order interactions.

If the experimenter can reasonably assume that certain high-order interaction are negligible, then information on the main effects and low-order interactions may be obtained by running only a fraction of the complete factorial experiment to save time and money resources. The fractional factorial designs are the most widely used for this situation.

The fractional factorial designs require fewer runs than the full factorial designs to study main effects and low-order interactions. The 2^{k-p} fractional factorial designs containing 2^{k-p} runs is called $1/2^p$ fraction of the 2^k designs. In these design, the experimenter must select p $(1 \leq p \leq k)$ independent design generators. The defining relation for the design consists of the p generators initially chosen and their $2^p - p - 1$ generalized interactions that are used to form the alias structure. The construction and analysis of 2^{k-p} fractional factorial design is described in Myers and Montgomery (2002) [28].

2.2.2 Central Composite Designs

The central composite designs (CCDs) were introduced by Box and Wilson (1951). CCDs are an alternative class of three-level factorial designs for fitting the second-order response surface model. A central composite design consists of three sets of points:

1. The factorial portion of the design is a 2^k full factorial design or a 2^{k-p} fractional factorial design, where the factor levels are coded as ± 1 values. The design points in this portion are called the factorial points that represent a variance-optimal design for a first-order model or a first-order plus two-factor interaction model. The number of factorial points is $n_f = 2^k$ or $n_f = 2^{k-p}$ for a full or fractional factorial design, respectively.

2. The center points or the central runs provide information about the existence of curvature in the response surface. If curvature is found, the addition of axial points allow for efficient estimation of the pure quadratic terms. Let n_c be the number of center points.

3. The axial portion of the design contain the two axial points on the axis of each input variable at a distance of α from the center point. The number of

axial points is $n_a = 2k$.

Therefore, the total number of design points is $N = n_f + n_c + n_a$. The choice of the number of center points n_c will have an influence on the distribution of $NVar[\hat{y}(x)]/\sigma^2$ in the region of interest. The region of interest for a CCD depends on the selection of the axial point distance α. In general, the value of α varies from 1.0 to $\sqrt{k}$. When the axial distance equals to 1.0 ($\alpha = 1.0$), the design is called a *face-centered cube design* and if $\alpha = \sqrt{k}$, the design is called *spherical design*. For example, the design matrice for CCDs for $k = 3$ are shown in Table 2.1 and the geometric of design is shown in Figures 2.3 - 2.4. For more information on CCDs, see Myers and Montgomery (2002) [28], Atkinson, Donev, and Tobias (2007) [2], and Khuri and Cornell (1996) [23].

Table 2.1: The design matrix of the 15-Point Central Composite Design (CCD) for $k = 3$

	Cube design			Spherical design		
No.	x_1	x_2	x_3	x_1	x_2	x_3
1	1	1	1	1	1	1
2	1	1	-1	1	1	-1
3	1	-1	1	1	-1	1
4	1	-1	-1	1	-1	-1
5	-1	1	1	-1	1	1
6	-1	1	-1	-1	1	-1
7	-1	-1	1	-1	-1	1
8	-1	-1	-1	-1	-1	-1
9	1	0	0	$\sqrt{3}$	0	0
10	-1	0	0	$-\sqrt{3}$	0	0
11	0	1	0	0	$\sqrt{3}$	0
12	0	-1	0	0	$-\sqrt{3}$	0
13	0	0	1	0	0	$\sqrt{3}$
14	0	0	-1	0	0	$-\sqrt{3}$
15	0	0	0	0	0	0

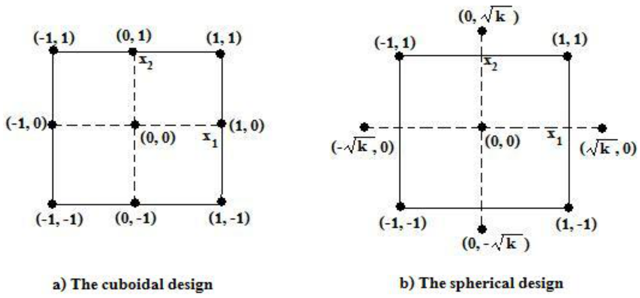


Figure 2.3: Central composite design for $k = 2$.

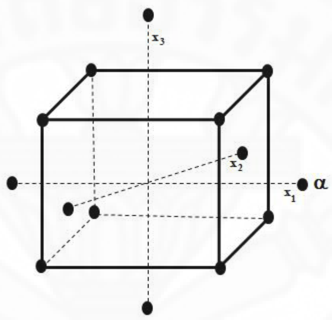


Figure 2.4: Central composite design for $k = 3$ and $\alpha = \sqrt{k}$.

2.2.3 Box-Behnken Designs

Box and Behnken (1960) introduced a class of three-level designs for estimation of the parameters in the second-order response surface model. The Box-Behnken designs (BBDs) are generated by combining two-level full or fractional factorial designs with balanced incomplete block designs. The general BBDs are the following parameters:

k = the number of design variables.

b = the number of blocks in the BIBD.

t = the number of design variables per block.

r = the number of blocks in which a design variable appears.

λ = the number of times that each pair of design variables appears in the same block. It must hold that $\lambda = \frac{r(t-1)}{k-1}$.

To generate a BBD, the t design variables appearing in each block in the BIBD are replaced with the t columns defining a 2^t full-factorial design or 2^{t-p} fractional-factorial design with levels ± 1. The remaining $k-t$ columns are set at mid-level 0 and n_c center points are include in the design. The total number of design points is $N = n_f b + n_c$ where $n_f = 2^t$ or $n_f = 2^{t-p}$ depending on if a full or fractional-factorial design was used. The levels are coded so that they lie on a sphere of radius $\alpha = \sqrt{k}$. For example, if we consider a balanced incomplete block design with three treatments and three blocks, the BIBD construction is given as Figure 2.5.

Block	Treatment		
	1	2	3
Block 1	×	×	
Block 2	×		×
Block 3		×	×

Figure 2.5: Construction of BIBD design for three treatments and three blocks

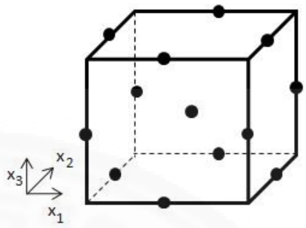
In the response surface setting, variables x_1 and x_2 in the first block are paired together in a 2^2 factorial (scaling ± 1) while x_3 remains fixed at the center ($x_3 = 0$). The same applies for blocks 2 and blocks 3, with a 2^2 factorial being represented by each pair of treatments while the third factor remains fixed at 0. As a result, the $k = 3$ Box-Behnken design is given by Figure 2.6. For more information on BBDs, see Myers and Montgomery (2002) [2] and Khuri and Cornell (1996) [23].

2.3 Design Optimality Criteria

Keifer and Wolfowitz (1959) introduced optimal design theory for evaluating and comparing RSM designs. Their work is presented in a measure theoretic approach in which an experimental design is interpreted as a design measure. The best design is dependent on sample size, model, range of variable levels, and other

$$D = \begin{bmatrix} -1 & -1 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & -1 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \\ 0 & -1 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

(a) The design matrix



(b) The Geometric view

Figure 2.6: The $k = 3$ BBD with a center point.

constraints. In practice, design selection and optimality criteria are highly dependent on the approximating response surface model. Therefore, different models lead to different design optimality criteria values. Design optimality criteria are characterized by letters of the alphabet and are often called *alphabetic optimality criteria*. In this dissertation four optimality criteria (D, A, G, and IV optimality) are considered. These optimality criteria are functions of ($\mathrm{X}'\mathrm{X}$) where X is the expanded design matrix.

Suppose x is any point in the design region $\mathcal{X}$. N is the design size, and $\mathrm{f}(\mathrm{x}) = [\mathrm{f}_1(\mathrm{x}), \mathrm{f}_2(\mathrm{x}), \ldots, \mathrm{f}_p(\mathrm{x})]$ is a vector of p real-valued functions based on the p model terms. For the second-order response surface model,

$$\mathrm{f}(\mathrm{x}) = [1, x_1, \ldots, x_k, x_1x_2, \ldots, x_{k-1}x_k, x_1^2, \ldots, x_k^2].$$

In this dissertation, f(x) will be a vector containing a subset of terms of the full model vector given in the second-order model. A review will now be given for the four design optimality criteria.

1. D-optimality

   D-optimality is the best-known and most often used criterion. This criterion is based on the idea that the experimental design should be chosen to optimize certain properties of the moment matrix $\mathrm{M} = \frac{\mathrm{X'X}}{N}$, where X′X is the information matrix. *D*-optimality requires the maximization of $|\mathrm{X'X}|$ or equivalently, the minimization of $|(\mathrm{X'X})^{-1}|$. Because $|\mathrm{X'X}|$ is inversely proportional to the square of the volume of the confidence region on the coefficients in the response surface model, a larger $|\mathrm{X'X}|$ implies better estimation of model parameters. A *D*-optimal design is one in which $\mathrm{M} = \frac{\mathrm{X'X}}{N}$ is maximized. It implies that the maximum is taken over all possible designs. As a result, it is natural to define the *D*-efficiency of an *N*-point design as

$$D_{\text{eff}} = 100\frac{|\mathrm{X'X}|^{1/p}}{N} \tag{2.8}$$

where p is the number of parameters in the model. Comparison of *D*-efficiencies allows for designs that have different sample sizes to be compared.

2. *A*-optimality

   A-optimality deals with the individual variances of the coefficients of the response surface model. The goal of *A*-optimality is to minimize the sum of variances of the coefficients (weighted by N). Because the variances of the coefficients are equal to σ^2 times the the diagonal elements of $(\mathrm{X'X})^{-1}$, *A*-optimality is defined as minimizing the trace of $[(\mathrm{X'X})^{-1}]$. By the definition, the *A*-efficiency of a design is defined as

$$A_{\text{eff}} = 100\frac{p}{\text{trace}[N(\mathrm{X'X})^{-1}]} \tag{2.9}$$

3. *G*-optimality

   G-optimality is based on the scaled prediction variance function $v(\mathrm{x}) = N\mathrm{Var}[\hat{y}(\mathrm{x})]/\sigma^2$ which is an important measure of design performance. One disadvantage is that $v(\mathrm{x})$ is not single number but rather depends on the location x at which one is predicting. In fact,

$$v(\mathrm{x}) = N\mathrm{Var}[\hat{y}(\mathrm{x})]/\sigma^2 = N\mathrm{f}'(\mathrm{x})(\mathrm{X'X})^{-1}\mathrm{f}(\mathrm{x})$$

   where vector f(x) reflects location of x in the design space as well as the nature of the model. *G*-optimality and the corresponding *G*-efficiency emphasize choosing a design for which the maximum $v(\mathrm{x})$ in the region of the design is not too large. A G-optimal design is a design ζ that satisfies the following minimax criterion

$$\min_{\zeta}\left[\max_{\mathrm{x}\in\mathcal{X}} v(\mathrm{x})\right]$$

which is equivalent to

$$\min_{\zeta}\left[\max_{\mathrm{x}\in\mathcal{X}} \mathrm{f}'(\mathrm{x})(\mathrm{X}'\mathrm{X})^{-1}\mathrm{f}(\mathrm{x})\right]$$

where $\mathcal{X}$ is the design space, and ζ is the space of all possible designs. The G-efficiency of design is defined as

$$G_{\text{eff}} = 100\frac{p/N}{\hat{\sigma}^2_{max}} \tag{2.10}$$

4. IV-optimality

Another important design optimality criterion that addresses the prediction variance is IV-optimality. The goal of IV-optimality is the minimization of the average of the scaled prediction variance in the design space. An IV-optimal design ζ satisfies

$$\min_{\zeta} \quad [\text{average}(v(\mathrm{x})) \ \text{ over } \mathrm{x}\in\mathcal{X}]$$

The IV-criterion of design is defined as

$$IV_{\text{crit}} = N\sigma^2_{ave} \tag{2.11}$$

When comparing designs, larger-is-better for D, A, and G efficiencies while smaller-is-better for the IV-criterion. For more information about design optimality criteria and the application of these criteria for the comparison of experimental designs, see Myers and Montgomery (2002) [28] and Atkinson, Donev, and Tobias (2007) [2].

2.4 Weighted Optimality Design Criteria

Design selection based on optimality criteria is highly dependent on the estimating response surface model that is proposed prior to data collection. Hence, for different models, the design optimality criteria values will be different. However, after data are collected and the model parameters are fitted, many parameters are judged insignificant. A reduced model retaining only significant terms is then adopted for use. Therefore, a design should be robust over the set of potential reduced models.

Chipman (1996) [8] introduced the heredity theory for the two classes of reduced models which are weak and strong heredity. The reduced models are formed by removing terms based on hierarchy. Specifically:

Weak heredity (WH) requires that

1. If a model contains an x_i^2 term, then it must contain the corresponding x_i term and
2. If a model contains an $x_i x_j$ term, then it must contain either the x_i or x_j term or both.

Strong heredity (SH) requires that

1. If a model contains an x_i^2 term, then it must contain the corresponding x_i term and
2. If a model contains an $x_i x_j$ term, then it must contain both of the x_i or x_j terms.

Both x_i and x_j are called parents of $x_i x_j$, and x_i is the single parent of x_i^2. Weak and strong inheritance possess the immediate inheritance principle. A term inherits immediately from another term if it inherits from that term and that term is of the next lowest order (Chipman 1996). For instance, if a model had an $x_i^2 x_j$ term, it would inherit immediately from x_i^2 and $x_i x_j$, and it inherits, but not immediately, from x_i and x_j. For WH, there are

$$\sum_{i=0}^{k}\binom{k}{i}(2^{ki-i(i-1)/2})$$

reduced models of the model in Equation (2.7). Thus, for $k = 3, 4, 5$ design variables, there are respectively, 185, 3905, and 160929 WH models. For SH there are

$$\sum_{i=0}^{k}\binom{k}{i}(2^{i(i+1)/2})$$

reduced models of the model in Equation (2.7). Thus, for $k = 3, 4, 5$ design variables, there are respectively, 95, 1337, and 38619 SH models. As k increases, the number of reduced models becomes very large (especially for $k = 5$ variables). However, for many designs, any permutation of the input variables will yield the same X′X matrix. Any design that has this property will be called *symmetric*. By exploiting the symmetry of a design, the number of models that actually need to be considered is much smaller and can be easily handled. For example, consider the 3-factor CCD design. Any permutation of the labels x_1, x_2, and x_3 will yield the same X′X matrix:

$$\mathrm{X'X_{CCD}} = \left[\begin{array}{c|ccc|ccc|ccc}
15 & 0 & 0 & 0 & 0 & 0 & 0 & 14 & 14 & 14 \\
\hline
0 & 14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 14 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\
\hline
14 & 0 & 0 & 0 & 0 & 0 & 0 & 26 & 8 & 8 \\
14 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 26 & 8 \\
14 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 8 & 26
\end{array}\right]$$

Suppose SH holds and the reduced model is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \epsilon$$

Then X′X associated with this reduced model would be formed from rows and columns 1, 2, 3, 5, and 8 of $\mathrm{X'X_{CCD}}$. Because of the symmetry of the CCD design, this model is equivalent to 6 other models for D, A, G, and IV. For example, it is equivalent to the following models:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{22} x_2^2 + \epsilon$$

$$y = \beta_0 + \beta_1 x_1 + \beta_3 x_3 + \beta_{13} x_1 x_3 + \beta_{11} x_1^2 + \epsilon$$

$$y = \beta_0 + \beta_1 x_1 + \beta_3 x_3 + \beta_{13} x_1 x_3 + \beta_{33} x_3^2 + \epsilon$$

$$y = \beta_0 + \beta_2 x_2 + \beta_3 x_3 + \beta_{23} x_2 x_3 + \beta_{22} x_2^2 + \epsilon$$

$$y = \beta_0 + \beta_2 x_2 + \beta_3 x_3 + \beta_{23} x_1 x_3 + \beta_{33} x_3^2 + \epsilon$$

By exploiting the symmetry of designs such as the Box-Draper designs, the small composite designs (SCDs), the Plackett-Burman composite designs (PBCDs), the Hoke designs, and the Notz designs, the 185, 3905, 160929 WH models for $k = 3, 4, 5$ can be reduced to 41, 138, and 406 nonequivalent WH models, and the 95, 1337, 38619 SH models for $k = 3, 4, 5$ can be reduced to 25, 60, and 126 nonequivalent SH models.

After the experimenters have reduced the number of models, the next step is to apply experimenter-assigned weights to each of the reduced models, i.e., a prior specification of probabilities to the WH or SH model space. For a given model, the set of effects present in the model can be represented by a δ vector

$$\delta = (\delta_1, \delta_2, \ldots, \delta_k, \delta_{12}, \delta_{13}, \ldots, \delta_{(k-1)k}, \delta_{11}, \delta_{22}, \ldots, \delta_{kk})$$

such that an effect is active if its associated element of δ is equal to 1. Otherwise, it is inactive effect if the associated element of δ is equal to 0. Then,

for $k = 2$, $\quad \delta = (\delta_1, \delta_2, \delta_{12}, \delta_{11}, \delta_{22})$

for $k = 3$, $\quad \delta = (\delta_1, \delta_2, \delta_3, \delta_{12}, \delta_{13}, \delta_{23}, \delta_{11}, \delta_{22}, \delta_{33})$, and

for $k = 4$, $\quad \delta = (\delta_1, \delta_2, \delta_3, \delta_4, \delta_{12}, \delta_{13}, \delta_{14}, \delta_{23}, \delta_{24}, \delta_{34}, \delta_{11}, \delta_{22}, \delta_{33}, \delta_{44})$

are the corresponding δ-vectors. For instance, the vector $\delta = (1, 1, 0, 1, 0, 0, 1, 0, 0)$ when $k = 3$ corresponds to the model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \epsilon$$

In this dissertation, independence of effects is assumed (as in Chipman 1996). That is, we assume

1. The linear effect δ_i's are independent.

2. The interaction δ_{ij}'s are independent of each other and only depend on the parent δ_i's.

3. The second-order (quadratic) δ_{ii}'s are independent of each other and only depend on the parent δ_i.

The joint probability density function of δ can be written as:

$$\Pr(\delta) = \left(\prod_{i=1}^{k} \Pr(\delta_i)\right)\left(\prod_{i<j}^{k} \Pr(\delta_{ij}|\delta_i, \delta_j)\right)\left(\prod_{i=1}^{k} \Pr(\delta_{ii}|\delta_i)\right) \tag{2.12}$$

where $\Pr(\delta_i)$, $\Pr(\delta_{ij}|\delta_i, \delta_j)$, and $\Pr(\delta_{ii}|\delta_i)$ are experimenter-assigned prior probabilities that the x_i, x_{ij}, and x_i^2 terms, respectively, are in the model.

If the experimenter assumes the input variables are equally important so that prior probabilities do not depend on particular variables, but only on the type of model term, then prior probabilities are equal for linear effects. Similarly, prior probabilities are equal for interaction effects and are equal for second-order effects. That is,

$$\begin{aligned}
\Pr(\delta_i = 1) &= p_l \quad \text{for all } i, && (2.13)\\
\Pr(\delta_{ij} = 1|\delta_i, \delta_j) &= p_0 \quad \text{if } (\delta_i, \delta_j) = (0, 0), \\
&= p_1 \quad \text{if } (\delta_i, \delta_j) = (0, 1) \text{ or } (1, 0), \\
&= p_2 \quad \text{if } (\delta_i, \delta_j) = (1, 1) && (2.14)\\
\Pr(\delta_{ii} = 1|\delta_i) &= p_q \quad \text{if } \delta_i = 1, \\
&= 0 \quad \text{if } \delta_i = 0 && (2.15)
\end{aligned}$$

For WH and SH, $p_0 = 0$ and for SH, $p_1 = 0$. Thus, the experimenter will need to set the values of p_l, p_1, p_2, and p_q for WH or set values p_l, p_2, and p_q for SH when designing the experiment.

A *weighted design optimality criteria* are the weighted averages of the efficiency values across all models based on the D, A, G, and IV criteria. The weighted D-optimality criterion (D_w) will be defined as

$$D_w = \sum_{i=1}^{K} D(i)\Pr(\Delta_i) \tag{2.16}$$

where K is the number of reduced models, Δ_i is δ-vector for model i, $D(i)$ is D-efficiency for model i, and $\Pr(\Delta_i)$ is the probability model $\Pr(\delta)$ in Equation (2.12) evaluated for Δ_i.

For symmetric designs, Equation (2.16) can be simplified if we further assume each design variables is equally important. Specifically, D_w can be written as

$$D_w = \sum_{i=1}^{K^*} m(i)D(i)\Pr(\Delta_i^*) \tag{2.17}$$

where K^* is the number of reduced nonequivalent models, $m(i)$ is the number of models equivalent to model i, Δ_i^* is δ-vector for model i, $D(i)$ is D-efficiency for model i, and $\Pr(\Delta_i^*)$ is probability model $\Pr(\delta)$ in Equation (2.12) evaluated for Δ_i^*.

When $k = 3$, 4, and 5, there are $K = 185$, 3905, 160929 WH models, and $K = 95$, 1337, and 38619 SH models. But, for symmetric designs, the number of nonequivalent models is reduced to $K^* = 41$, 138, 406 WH models and $K^* =$ 25, 60, and 126 SH models when $k = 3$, 4, and 5,

The weighted A, G, and IV-optimality criteria denoted A_w, G_w, and IV_w are defined by replacing $D(i)$ with $A(i)$, $G(i)$, and $IV(i)$ in Equation (2.17).

Chipman (1996) [8] presented Bayesian variable selection with related predictors. The application of the methods is illustrated via the stochastic search variable selection algorithm which is modified to utilize the new priors. Shirakura and Tong (1996) [29] applied the weighted A-optimality for fractional 2^m factorial designs of resolution V. Butler (2000) [7] applied weighted design optimality criteria for polynomial response surface and is shown that there are some particularly simple parameterizations of the response surface for which weighted optimality criteria are equivalent to A-optimality or weighted A- optimality. This is useful in practice as A-optimality search algorithms are readily available. Borkowski and Valeroso (2001) [5] used WH in their comparison of design efficiencies for several commonly-used response surface designs. Chomtee (2003) [10] presented a comparison of design optimality criteria for reduced models for response surface designs in a spherical design region. Borkowski, Turk, and Chomtee (2011) [6] used WH and SH to evaluate weighted design optimality criteria and compare several types of response surface designs in a cuboidal design region. They also provided an algorithm for estimating weighted design optimality criteria based on monte-carlo methods.

2.5 Genetic Algorithms

A genetic algorithm (GA) is an optimization and search technique based on the principles of genetics and natural selection. This algorithm was developed by John Holland in the 1960s. A GA maintains a population of candidate solutions for a problem to evolve under specified selection rules to a state that maximizes an objective function. After the selection process in a GA, the most fit candidate solutions are changed by a reproduction process that have operators that will produce improved solutions for the next generation. The process continues with each generation developing more fit solutions until an acceptable solution has evolved.

Some of the advantages of a GA include that it can be used to optimize with either continuous or discrete variables and it does not require an objective function that has derivatives. A GA can handle a large number of variables, and it can provide a list of optimum or near-optimum variable values (not just a single solution), and it works with numerically-generated data, experimental data, or analytical functions. These advantages can produce stunning results when traditional optimization approaches fail miserably.

A GA differs from conventional search techniques because it starts with an initial set of candidate solutions called a *population*, and each member of the population satisfies all imposed system constraints. Each potential solution to the problem in the population is called a *chromosome*. A chromosome is a string of symbols usually that is represented by a string of *genes* that are either binary-encoded or real-number encoded.

In this dissertation, the real number representation will be used because it works effectively on mathematical optimization problems. It also allows for the use of numerical reproduction operators. A chromosome C will be an $N \times k$ matrix with the N rows representing the N design points in k factors. When C is expanded to include columns for all the p terms in the model, it yields the $N \times p$ expanded design matrix X. A gene is any of the $N \times k$ real-number x_{ij} entries (rows) in the chromosome C. That is, x_{ij} is the coded setting of the i^{th} factor in the j^{th} experimental run. A gene's value is dictated by the design region $\mathcal{X}$. For example, it the experimental design region is cuboidal, then each $x_{ij} \in [-1, 1]$.

The *objective function* F measures a chromosome's fitness as a solution and it is the function we wish to optimize. F takes a chromosome as input and outputs of an objective function value. In this dissertation, the designs will be evaluated using the D, A, G, and IV criteria. Thus, four objective functions will be used for both weak heredity (WH) and strong heredity (SH). For WH, the goal of the research is to evolve a design that maximizes an objective function F_i where

$$F_D = D_w(\mathrm{X}) \qquad F_A = A_w(\mathrm{X}) \qquad F_G = G_w(\mathrm{X}) \qquad F_{IV} = IV_w(\mathrm{X}).$$

For SH, the goal of the research is to evolve a design that maximizes an objective function F_i where

$$F_D = D_s(\mathrm{X}) \qquad F_A = A_s(\mathrm{X}) \qquad F_G = G_s(\mathrm{X}) \qquad F_{IV} = IV_s(\mathrm{X}).$$

For each case, F_i measures C's fitness as a solution in the search for an optimal design relative to criterion i.

In the selection process of a GA, chromosomes in the current population are selected to reproduce offspring which are then evaluated using the objective function. The objective function values then determine which parent or offspring chromosomes are fit enough to survive and form the next generation. One simple method for selecting chromosomes for the reproduction process is the following:

1. In each generation, the population consists of M chromosomes (designs). One or two chromosomes that provide the largest objective function values are chosen. The number of chromosomes depends on the number in the population. The two best or *elite* chromosomes are chosen when M is an even number and one elite chromosome is chosen when M is an odd number. The remaining chromosomes are randomly partitioned into $(M-2)/2$ or $(M-1)/2$ pairs of *parent chromosomes*.

2. Each of these parental pairs reproduces. After the reproduction process is completed there is an *offspring chromosome* associated with each parent. Thus, there are the 1 or 2 elite, $(M-1)$ or $(M-2)$ parents, and $(M-1)$ or $(M-2)$ offspring chromosomes for total of $2M-1$ or $2M-2$ chromosomes at the end of a reproduction cycle.

3. The objective function F_i values are calculated for all of the elite, parent, and offspring chromosomes. For each parent-offspring pair, the chromosome with the higher F_i value survives to be a future parent and the other chromosome is removed from the population. Thus, at the end of every generation, future parent chromosomes and the current elite chromosomes survive to the next generation. These M chromosomes are then sorted and the one or two with be the best F_i values become the elite chromosomes for the next generation.

In the GA, the selection and reproduction processes will be iterated until an acceptable solution has evolved. The elite chromosomes will converge to the best solutions as the number of generations increases. It may be possible for the elite chromosome to converge quickly to either the best design in the entire space of designs (global optimum) or to the best design in a subspace of the entire space of designs (local optimum). This will depend on the nature of the objective function. For example, some objective functions have many local minima. One goal is to create a GA that avoids the tendency to converge quickly to a local rather than a global maximum. To avoid this problem, the GA should explore other regions of the design space by randomly introducing changes, or mutations, in some of the variables during the *reproduction process*.

The reproduction process will operate on the genes to produce offspring chromosomes. A reproduction operator is applied to chromosomes if a probability test is passed. A probability test is based on a Bernoulli trial with probability of success α. Specifically, suppose u be a random deviate from a uniform distribution with 0 and 1, $U(0,1)$. If $0 \leq u \leq \alpha$ then the probability test is passed and the reproduction operator is applied. Otherwise, the probability test is not passed and the chromosome is left unchanged.

Several possible reproduction operators are blending, zero genes, extreme genes, sign change, creep, and elitism. All of these operators except elitism requires a probability test defined by Bernoulli parameters α_b, α_z, α_e, α_s, and α_c, respectively. The following is a description of each reproduction operator:

The blending operator: The blending operator will combine row A_a of A with a random row of B, say B_b to form two new rows: A_a^* and B_b^* via the linear combinations:

$$A_a^* = \beta A_a + (1-\beta)B_b \qquad \text{and} \qquad B_b^* = \beta B_b + (1-\beta)A_a$$

where A and B are two parent chromosomes paired in the reproduction process and β be a random $[0,1]$ uniform variate. A total of N probability tests are applied to A, one for each row.

The zero gene operator: If a probability test is passed for gene x_{ij}, then the gene is set to 0.

The extreme gene operator : If a probability test is passed for gene x_{ij}, then the gene is randomly set to either ± 1.

The sign change operator : If a probability test is is passed for gene x_{ij}, then the sign of the gene is changed. That is, we replace x_{ij} with $-x_{ij}$.

The creep operator : If a probability test is passed for gene x_{ij}, then a random variate ϵ from a normal $N(0,\sigma^2)$ distribution is added to x_{ij} to form a new gene $x_{ij}^* = x_{ij} + \epsilon$. If the creep operator takes $x_{ij}^* > 1$ or $x_{ij}^* < -1$ it will be reset to the boundary value $x_{ij}^* = \pm 1$. The variance σ^2 is set by the researcher.

Elitism operator : Elite chromosomes from the present generation are retained to the next generation which guarantees that the most fit chromosomes that have evolved will survive to the next generation. The 1 or 2 best designs are retained but are not allowed to participate in the reproduction process until some future chromosomes with better fitness evolve to take their place.

The crossover operator : The crossover operator will change gene of A with a random gene of B. If a probability test is passed for gene x_{ij}, then the cutpoint of gene is randomly decimal point. The cutpoint of gene A and B are applied to A and B by $A(r, c) = cutA + B - cutB$ and $B(r, c) = cutB + A - cutA$ respectively.

For zero gene, extreme gene, sign change, and creep operators, a probability test is passed is applied to every gene x_{ij} of each parent chromosome in the reproduction process. These four operators sweep through all of the chromosome's genes. Thus, a subset of the $N \times k$ entries in a design C will be modified during a generation of the GA. These operators allow exploration of the design space and reduce the chance of being trapped at a local optimum. Moreover, they also let evolution to a superior design proceed rapidly.

For this dissertation, probability tests are implemented in the following order: blending, zero gene, sign change, crossover, creeping, and extreme gene. These operators will be applied multiple times per generation when more than one probability test is pass. This will ensure a diversity of changes during the reproduction process, especially, when the creep operator is last.

For more information of GA in Michalewicz (1994) [26], Haupt and Haupt (2004) [19], Gen, Cheng, and Lin (2008) [17]. GA is interested from many researcher. For example, Heredia-Langner, Carlyle, Montgomery, Borror, and Runger (2003) [21] applied the GA to construct the D-optimal design. Borkowski (2003) [3] used a GA to generate small exact response surface designs. This research provided a catalog of designs given for 1, 2, and 3 design factors and calculated the efficiencies for classical response surface designs relative to exact optimal designs of the same design size. Drain, Carlyle, Montgomery, Borror, and Anderson-Cook (2004) [12] presented GA hybrid for constructing optimal response surface designs. Heredia-Langner, Montgomery, Carlyle, and Borror (2004) [20] present a technique to construct model-robust alphabetically-optimal designs using GA. This technique is useful in situations where computer-generated designs are most likely to be employed, particularly experiments with mixtures and response surface experiments in constrained regions. Goldfarb, Borror, Montgomery, and Anderson-Cook (2005) [18] used GA to generate mixture-process experimental designs involving control and noise variables.

The primary goal of the dissertation is to develop a GA that will generate an exact N-point k-variable response surface design for various choices of N, k, and weighted optimality criterion. This is equivalent is to finding an $N \times k$ matrix that optimizes a weighted design optimality criterion.

CHAPTER 3

RESEARCH METHODOLOGY

The research goals are to propose the weighted optimality as criteria for generating model-robust designs and to develop algorithms for generating these designs that optimize these weighted optimality criteria. The designs will be generated for a variety of response surface design situations by using MATLAB version 7.6.0.324(R2008a) and OPTEX procedure in *SAS*. The research will be presented in two parts. The first part is the evaluation of weighted D and G-efficiencies for classical and computer-generated designs. The second part is developing the GA for generating designs that optimize these weighted optimality criteria.

3.1 Methodology for Finding the Weighted Optimality Criteria

For the first part of the proposed research, the methodology used to calculate the weighted optimality criteria values for all possible reduced models of the most widely-used designs for fitting the quadratic response surface designs is described as follows:

1. Generate the design matrix for CCD, BBD, or computer-generated design with the number of input variables (k) and the number of design points (N) outlined in the research scope (Chapter 1).

2. Reduce the full model using the principles of weak heredity (WH).

3. Calculate the D and G efficiencies from each reduced model following the equation of (2.8) and (2.10), respectively.

4. Determine the model probability for each reduced model. In each reduced model, we have the Δ-vector that consists of the set of linear (δ_i) effects, the set of interaction (δ_{ij}) effects which depend on the δ_i and δ_j values, and the set of second-order (δ_{ii}) effects which depend on the δ_i values. If these effects are assumed to be independent (as in Chipman 1996), then the joint probability density function of Δ is

$$\Pr(\Delta) = \left(\prod_{i=1}^{k} \Pr(\delta_i)\right)\left(\prod_{i<j}^{k} \Pr(\delta_{ij}|\delta_i, \delta_j)\right)\left(\prod_{i=1}^{k} \Pr(\delta_{ii}|\delta_i)\right).$$

where Δ is the effect model (δ-vector) in the reduced model, each element of Δ is either 0 or 1 indicating whether or not an effect is inactive or active when δ_i represent the indicator function value of linear effect of the x_i term in the model, δ_{ij} represent the indicator function value of two-factor interaction effect given by factor i and factor j of the $x_i x_j$ term in the model, and δ_{ii} represent the indicator function value of second-order effect given by factor i of the x_i^2 term in the model. The prior probabilities are defined by

$$\begin{aligned}
\Pr(\delta_i = 1) &= p_l \quad \text{for all } i, \\
\Pr(\delta_{ij} = 1|\delta_i, \delta_j) &= p_0 \quad \text{if } (\delta_i, \delta_j) = (0,0), \\
&= p_1 \quad \text{if } (\delta_i, \delta_j) = (0,1) \text{ or } (1,0), \\
&= p_2 \quad \text{if } (\delta_i, \delta_j) = (1,1), \\
\Pr(\delta_{ii} = 1|\delta_i) &= p_q \quad \text{if } \delta_i = 1, \\
&= 0 \quad \text{if } \delta_i = 0
\end{aligned}$$

By definition, for WH, $p_0 = 0$. Therefore, the researcher must assign values p_l, p_1, p_2, and p_q to be able to calculate $\Pr(\Delta)$. To study a wide variety of probability assignments, the values of these probabilities are defined in research scope in Chapter 1.

5. Calculate the weighted D-optimality criteria (D_w) by

$$D_w = \sum_{i=1}^{K^*} m(i) D(i) \Pr(\Delta_i^*)$$

where K^* is the number of reduced nonequivalent model, when $k = 2$, 3, and 4, K^* is 17, 41, 138 WH models, $m(i)$ is the number of models equivalent to model i, Δ_i^* is δ-vector for model i, $D(i)$ is D-efficiency for model i, and $\Pr(\Delta_i^*)$ is probability model evaluated for Δ_i^*.

6. Continue calculating the weighted G-optimality criteria, denoted G_w is defined by replacing $D(i)$ with $G(i)$ in the equation of D_w.

3.2 Methodology for Generating Designs by the Genetic Algorithm

The methodology for generating designs that optimize the different weighted optimality criteria by a GA is outlined as follows:

1. Specify $k = 2$, $N = 6$, and the number of chromosomes (designs) in GA population M equal to 9.

2. Generate M design matrices of the chromosomes for the hypercube design or hypersphere design in the population given the design properties from Step 1.

3. Calculate the objective function of the GA for each criterion which is the D weighted optimality criteria for each chromosome. In other words, we use these weighted optimality criteria as the objective functions of GA.

4. Choose a elite chromosome that produce the highest of the objective function. The remaining chromosomes randomly partitioned into $(M-1)/2$ pairs of parent chromosomes.

5. Apply the reproduction process to each of these parental pairs. The reproduction process will operate on the genes to produce offspring chromosomes. The six operations will be applied in the following order: blending, zero genes, extreme genes, sign chance, rotation, crossover, and creep by setting the rate of these operators as 0.10, 0.15, 0.15, 0.05, 0.05, 0.10 and 0.05, respectively. These rates can be tuned to improve the speed of finding a solution using the GA. After the reproduction process, there are the 1 elite,

$(M-1)$ parents, and $(M-1)$ offspring chromosomes. About of detail for reproduction process, you can see in Chapter 2.

6. Calculate the objective function for each of the parent chromosomes and each of the offspring chromosomes.

7. Compare the objective function values for each parent-offspring pair. The chromosome with the better objective function value survives to be a future parent and the other is removed from the population. Hence, at the end of each generation, M chromosomes (elite and future parents) exist to the next generation.

8. Sort the objective function values and select the one with the best objective function values become the elite for the next generation.

9. Iterate step 5 to step 8 for a large number of generations. In this dissertation, 2,500 generations are used to generate design for 2, 3, and 4 factors. If needed, the number of generations can be increased if convergence does not occur.

10. Show the best design for GA and compare the result with other criterion.

Remark: for weighted G-optimality criteria is defined by changing the objective function, that is, by replacing D optimality criteria by G optimality criteria. In other condition will be changed follow as the research scope in Chapter 1.

CHAPTER 4

RESULT AND DISCUSSION

The objectives of this study are to find the weighted D and G-efficiencies for commonly-used response surface designs and to develop genetic algorithms (GAs) for generating designs that optimize the weighted D and G-optimality criteria for a variety of response surface design situations.

The results are contained in 3 sections :

1. The results of the weighted D and G-optimality criteria values for CCDs.
2. The results of the weighted D and G-optimality criteria values for BBDs.
3. The results of the weighted D and G-optimality criteria values for computer generated designs.

4.1 The results of the weighted D and G-optimality criteria values for CCDs

4.1.1 for $k = 2$ factors

The cuboidal CCDs

The boxplots of weighted D-optimality criterion values for the cuboidal CCDs slightly decrease when N increases (Figure 4.1). The presence of near parallel lines in Figure 4.5 and 4.6 indicates no interaction between pairs of p_i probabilities on D_w values. Similar patterns occur in all plots as N increases, and weighted D-optimality criterion values decrease. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The decrease in D_w when increasing p_l is larger than increasing p_1 and p_2. This is shown in Figure 4.5 (a), (e), (f) and Figure 4.6 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in D_w when increasing p_l is similar when p_2 is at its low level ($p_2 = 0.35$) and when it is at its high level ($p_2 = 0.95$). This is reflected in the near parallelism in the Figure 4.5 (b), (c) and Figure 4.6 (b), (c). There is only a small decrease in D_w when increasing p_1 or p_2. The decreases are similar which is reflected by the clear parallelism in Figure 4.5 (d) and Figure 4.6 (d). Thus D_w is quite robust to changes in p_1 or p_2.

The boxplots of weighted G-optimality criterion values for the cuboidal CCDs decrease when N increases (Figure 4.2). The presence of near parallel lines in Figure 4.7 and 4.8 indicates no interaction between pairs of p_1, p_2 and p_q probabilities on G_w values. Similar patterns occur in all plots as N increases, and weighted G-optimality criterion values decrease. Increasing p_l, p_1, and p_2 lowers G_w values, but there is the increase in G_w when increasing p_q. This is shown in Figure 4.7 (a), (e), (f) and Figure 4.8 (a), (e), (f). However, the decrease in G_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in G_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.7 (b), (c) and Figure 4.8 (b), (c). The values of G_w decrease when p_1 or p_2 increased, but with a larger decrease in G_w when increasing p_2. The decreases are similar when increasing p_1 for both p_2 levels which is reflected by the parallelism in Figure 4.7 (d) and Figure 4.8 (d). Thus G_w is more robust to changes in p_1 than to changes in p_2.

From the results for the cuboidal CCD design, the D_w values are robust to change in p_1 and p_2 and slightly less robust as N increases. However, the D_w values are sensitive to change in p_l and p_q. The G_w values are slightly sensitive (weakly robust) to change in p_1 and p_2. However, the G_w values are less robust to change in p_l and very sensitive to change in p_q.

The spherical CCDs

The boxplots of weighted D-optimality criterion values for the spherical CCDs decreases slowly when N increases (Figure 4.3). The presence of parallel lines in Figure 4.9, 4.10 and 4.11 indicates no interaction between pairs of p_1, p_2 and p_q probabilities on D_w values. Similar patterns occur in all plots as N increases, and weighted D-optimality criterion values decrease. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The rate of change in D_w decreases when N increases. The decrease in D_w when increasing p_l is larger than increasing p_1 and p_2. This is shown in Figure 4.9 (a), (e), (f), Figure 4.10 (a), (e), (f) and Figure 4.11 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in D_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.9 (b), (c), Figure 4.10 (b), (c) and Figure 4.11 (b), (c). There is a small decrease in D_w when increasing p_1 or p_2. The decreases are similar which is reflected by the clear parallelism in Figure 4.9 (d), 4.10 (d) and Figure 4.11 (d). Thus D_w is quite robust to changes in p_1 or p_2.

The boxplots of weighted G-optimality criterion values for the spherical CCDs increase from $N = 9$ to 10, but the G_w values decrease after $N = 10$ (Figure 4.4). The effect of changing p_l or p_q on G_w depends on the size of the CCD. When the number of center points increases from 2 to 6 ($N = 10$ to $N = 14$), the positive effect of increasing p_l on G_w at the high level of p_q becomes smaller until there is almost no effect on G_w. However, there is an increasing negative effect on G_w when increasing p_l at the low level of p_q as the number of center points increases from 2 to 6. An example can be seen in the comparison of the plots in Figure 4.13 (a) and Figure 4.14 (a). An exception occurs when changing from 1 to 2 center points ($N = 9, 10$). If on additional center point is added to the CCD ($N = 9, 10$), the interaction effect between p_l and p_q on G_w is reversed. This can be seen in Figures 4.12 (a) and 4.13 (a).

The rate of change in G_w when increasing p_l is smaller when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the rate of change in G_w when increasing p_l is larger when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). For $N = 9$, G_w values decrease, and when $N > 9$, G_w values increase and then gradually decreases as N increases. This is reflected in the lack-of-parallelism in the Figure 4.12 (b), (c), Figure 4.13 (b), (c) and Figure 4.14 (b), (c).

The G_w values increase as p_1, p_2, and N increase. For $N = 9$, G_w values decrease when p_q increases. For $N > 9$, G_w values increase when p_q increases for all levels of p_1 and p_2. In the case of the pair p_1 and p_q, values of G_w have changed in a parallel manner. That is, the parallel lines indicate no interaction between pairs of p_1 and p_q on G_w values. Also, the increase in G_w when increasing p_q is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.12 (e), (f), Figure 4.13 (e), (f) and Figure 4.14 (e), (f). There is a small increase in G_w when increasing p_1 or p_2. The increases are similar which is reflected by the clear parallelism in Figure 4.12 (d) to Figure 4.14 (d). Thus G_w is quite robust to changes in p_1 or p_2.

From the results for the spherical CCD design, the D_w values are robust to change in p_1 and p_2 and slightly less robust as N increases. However, the D_w values are sensitive to change in p_l and p_q and robust to change in p_q as N increases. The G_w values are slightly sensitive (weakly robust) to change in p_1 and p_2. However, the G_w values are very sensitive to change in p_l and p_q. In addition, the G_w values are more robust to change in p_q as N increase.

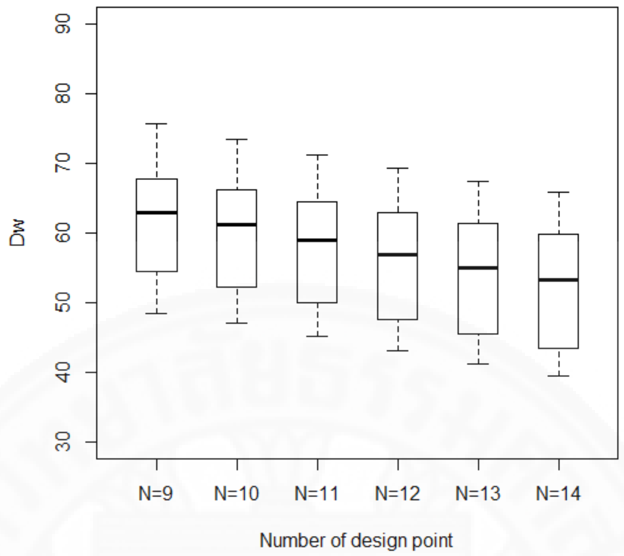


Figure 4.1: Boxplots for the weighted D-optimality criterion values in a cuboidal region for CCDs, $k = 2$

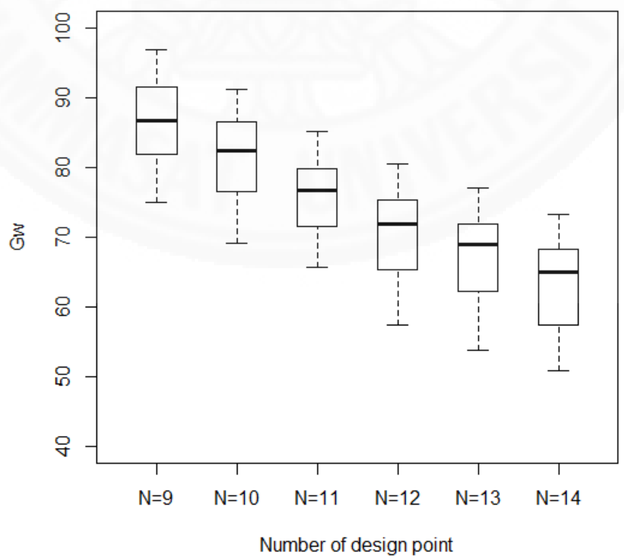


Figure 4.2: Boxplots for the weighted G-optimality criterion values in a cuboidal region for CCDs, $k = 2$

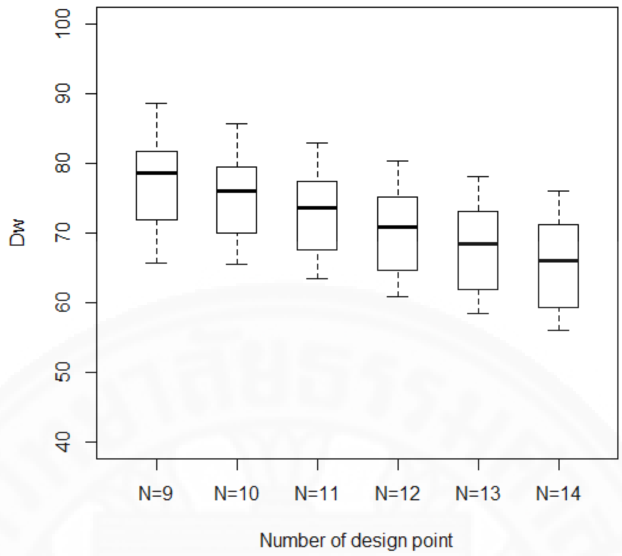


Figure 4.3: Boxplots for the weighted D-optimality criterion values in a spherical region for CCDs, $k = 2$

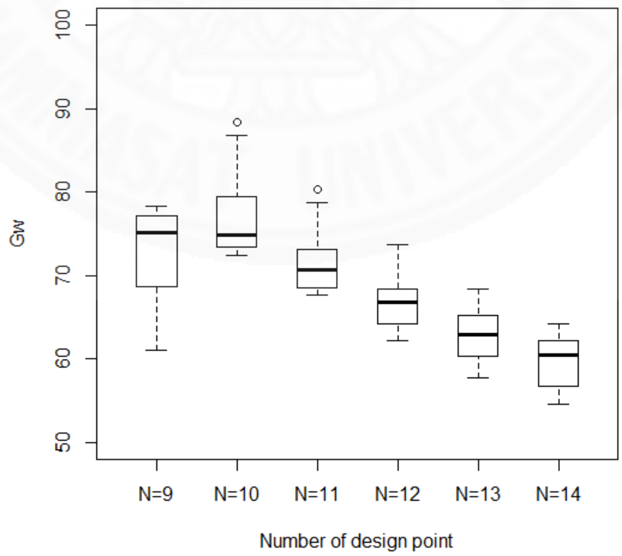


Figure 4.4: Boxplots for the weighted G-optimality criterion values in a spherical region for CCDs, $k = 2$

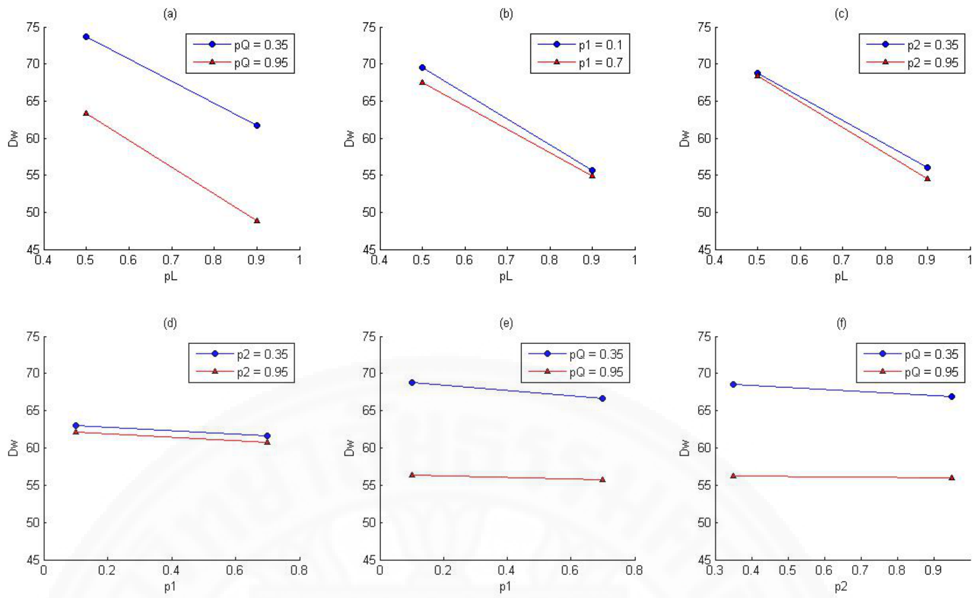


Figure 4.5: The weighted D-optimality criterion values in a cuboidal region for CCDs, $k = 2$ and $N = 9$

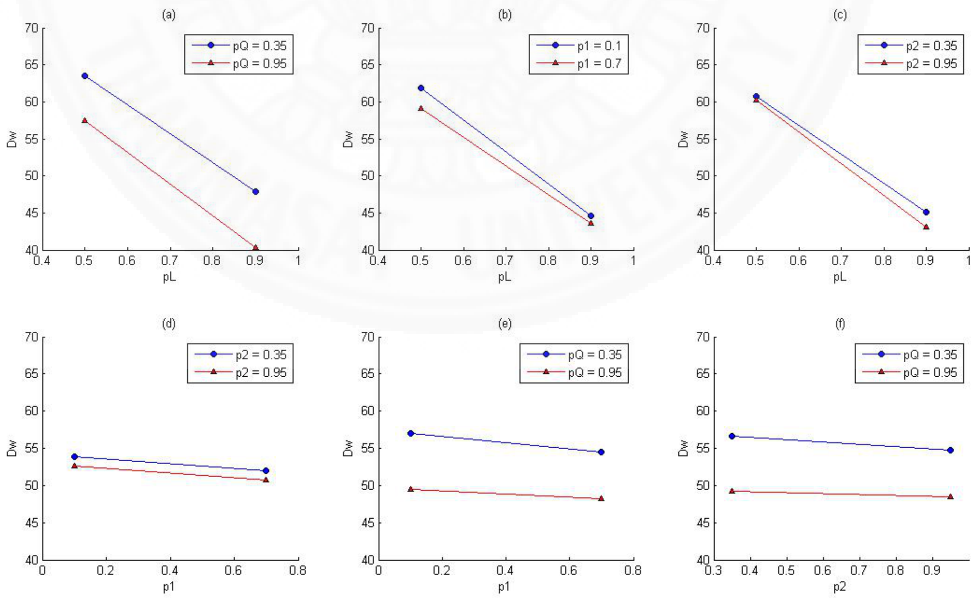


Figure 4.6: The weighted D-optimality criterion values in a cuboidal region for CCDs, $k = 2$ and $N = 14$

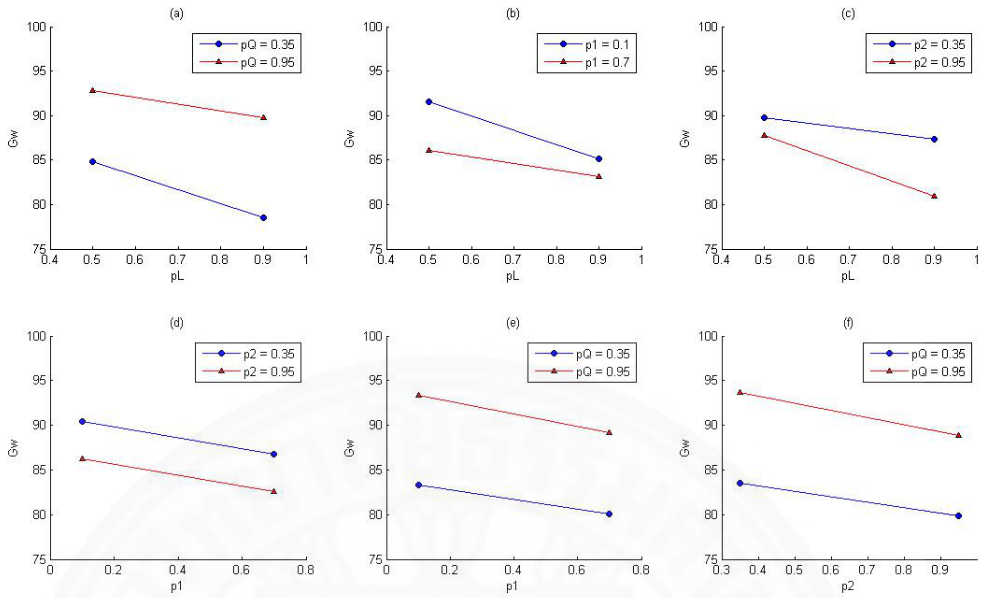


Figure 4.7: The weighted G-optimality criterion values in a cuboidal region for CCDs, $k = 2$ and $N = 9$

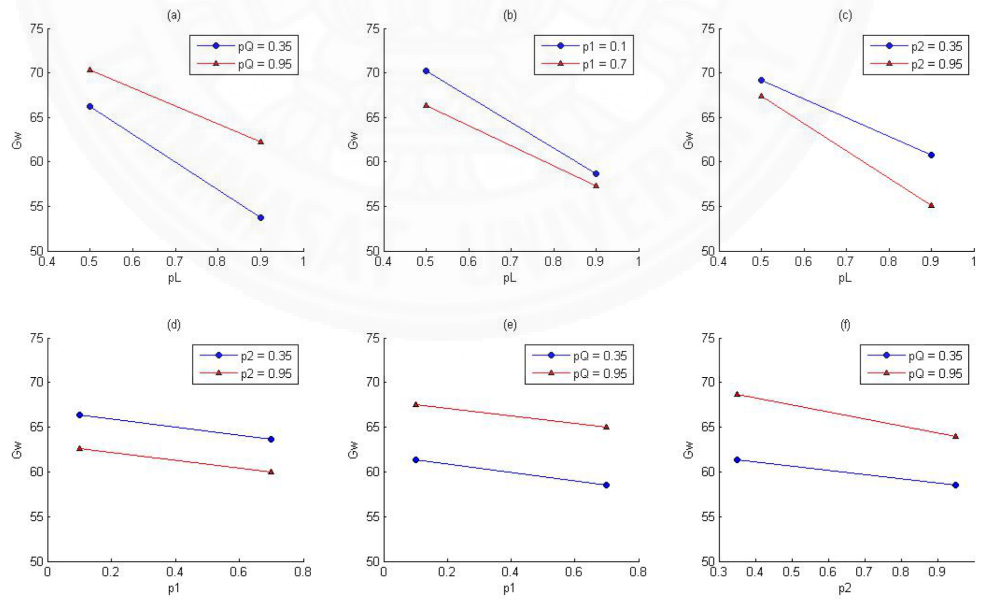


Figure 4.8: The weighted G-optimality criterion values in a cuboidal region for CCDs, $k = 2$ and $N = 14$

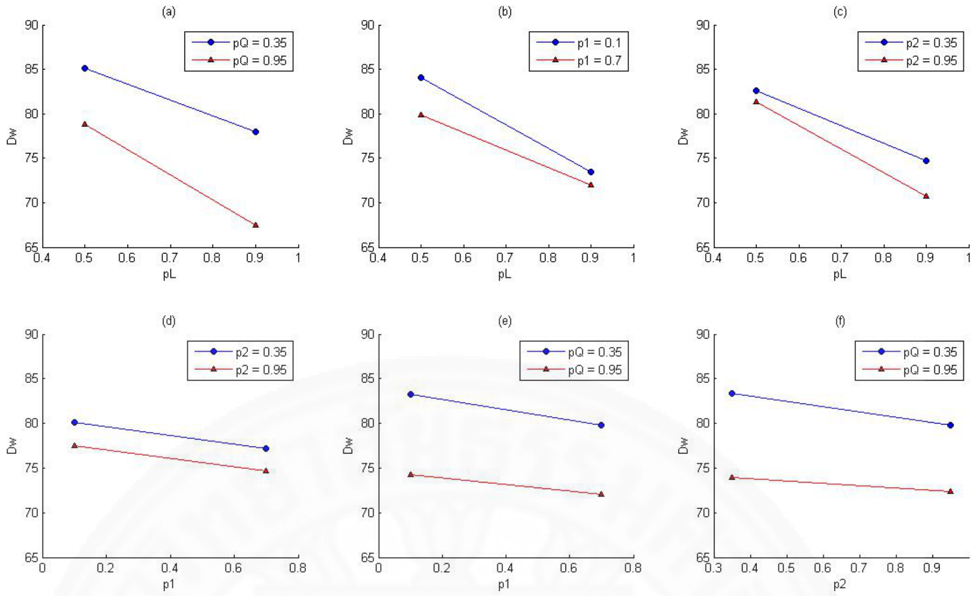


Figure 4.9: The weighted D-optimality criterion values in a spherical region for CCDs, $k = 2$ and $N = 9$

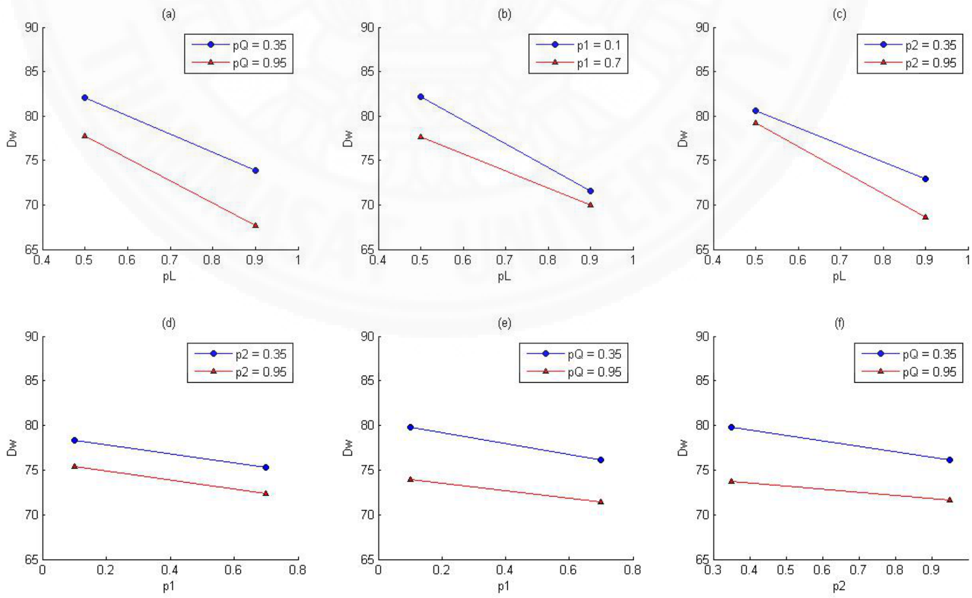


Figure 4.10: The weighted D-optimality criterion values in a spherical region for CCDs, $k = 2$ and $N = 10$

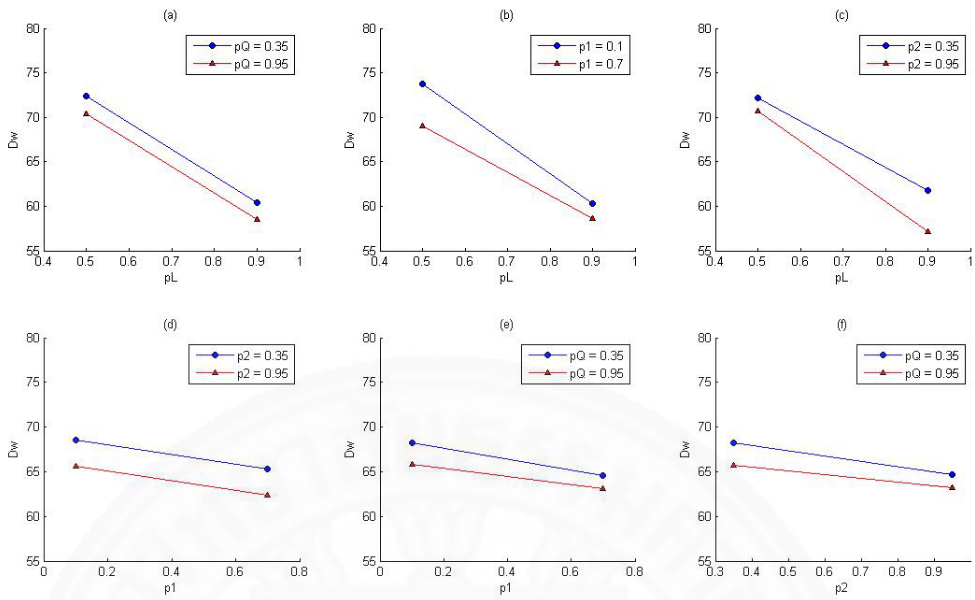


Figure 4.11: The weighted D-optimality criterion values in a spherical region for CCDs, $k = 2$ and $N = 14$

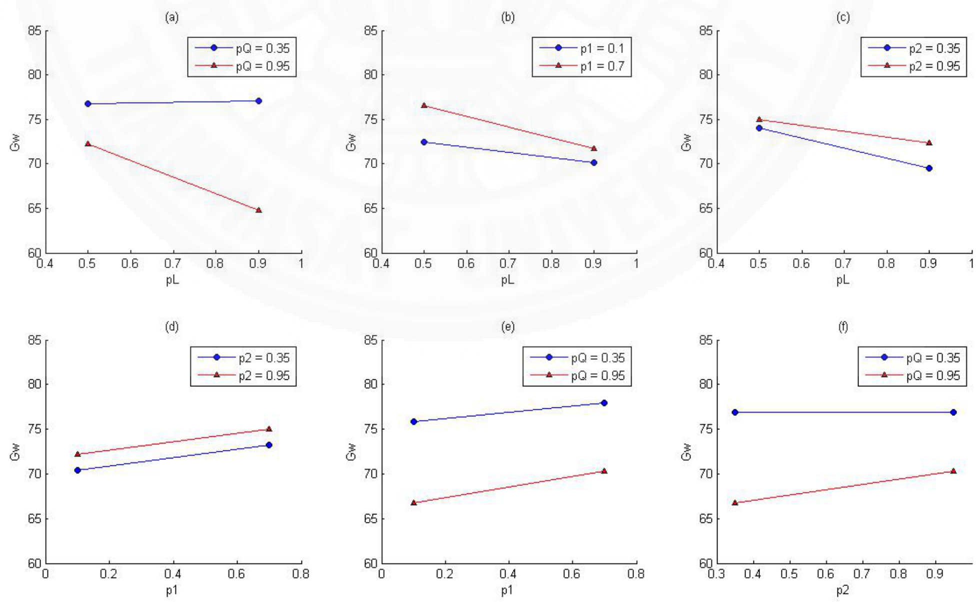


Figure 4.12: The weighted G-optimality criterion values in a spherical region for CCDs, $k = 2$ and $N = 9$

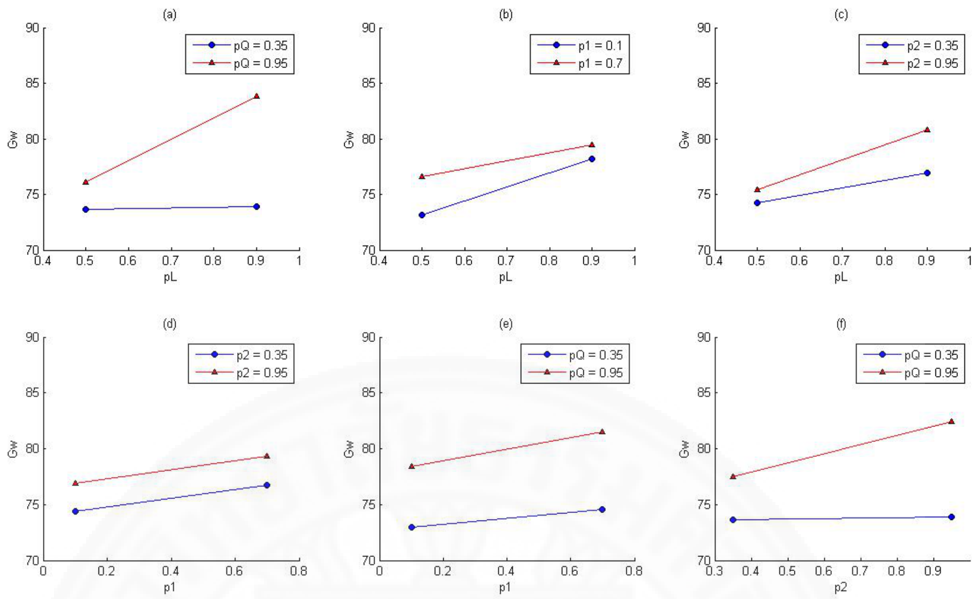


Figure 4.13: The weighted G-optimality criterion values in a spherical region for CCDs, $k = 2$ and $N = 10$

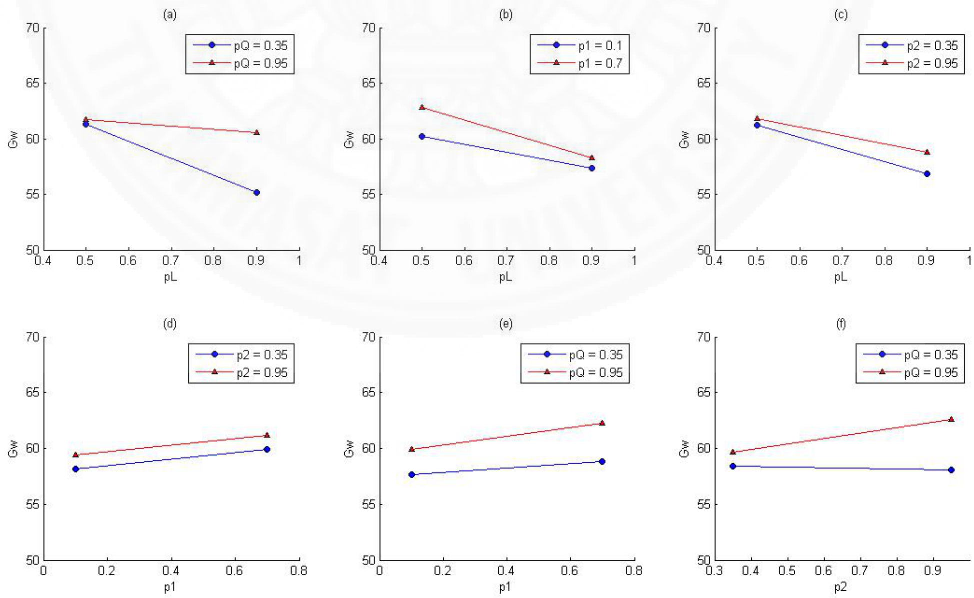


Figure 4.14: The weighted G-optimality criterion values in a spherical region for CCDs, $k = 2$ and $N = 14$

4.1.2 for $k = 3$ factors

The cuboidal CCDs

The boxplots of weighted D-optimality criterion values for the cuboidal CCDs decreases slowly when N increases (Figure 4.15). The presence of parallel lines in Figure 4.19 and 4.20 indicates no interaction between pairs of p_i probabilities on D_w values. Similar patterns in all plots occur as N increases, and weighted D-optimality criterion values decrease. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The decrease in D_w when increasing p_l is larger than increasing p_1 and p_2. This is shown in Figure 4.19 (a), (e), (f) and Figure 4.20 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger than increasing p_1 or p_2. This is reflected by the nearly parallel in the Figure 4.19 (b), (c) and Figure 4.20 (b), (c). There are small decreases in D_w when increasing p_1 or p_2. The decreases are similar which is reflected by the clear horizontal parallelism in Figure 4.19 (d), (e), (f) and Figure 4.20 (d), (e), (f). Thus D_w is quite robust to changes in p_1 or p_2.

The boxplots of weighted G-optimality criterion values for the cuboidal CCDs decrease when N increases (Figure 4.16). The presence of parallel lines in Figure 4.21 and 4.22 indicates no interaction between pairs of (p_l, p_q), (p_1, p_2), and (p_1, p_q) probabilities on G_w values. Similar patterns occur in all plots as N increases, and weighted G-optimality criterion values decrease. The G_w values slightly decrease when increasing p_l, p_1, and p_2, but there is increase in G_w values when p_q increases. This is shown in Figure 4.21 (a), (d), (e) and Figure 4.22 (a), (d), (e). However, when increasing p_l and p_1, at the low level of p_l ($p_l = 0.5$), the rate of decrease in G_w values is larger than when it is at its high level ($p_l = 0.9$). The rate of decrease in G_w values when increasing p_l and p_2 is at its low level ($p_l = 0.5$) is smaller than the rate of increase in G_w when increasing p_l and p_2 is at its high level ($p_l = 0.9$). This is reflected in the moderate lack-of-parallelism in the Figure 4.21 (b), (c) and Figure 4.22 (b), (c). The values of G_w increase when p_2 increases and p_q is at its high level ($p_q = 0.95$), but it slightly decreases when p_2 increases and p_q is at its low level ($p_q = 0.35$). These patterns are similar in all plots as N increases, which is reflected by the lack-of-parallelism in Figure 4.21 (f) and Figure 4.22 (f).

From the results for the cuboidal CCD design, the D_w values are robust to change in p_1 and p_2 and slightly less robust as N increase. However, the D_w values are sensitive to change in p_l and p_q. The G_w values are slightly sensitive (weakly robust) to change in p_1 and p_2. However, the G_w values are robust to change in p_l and very sensitive to change in p_q.

The spherical CCDs

The boxplots of weighted D-optimality criterion values for spherical CCDs decrease when N increases (Figure 4.17). The presence of parallel lines in Figure 4.23, 4.24 and 4.25 indicates no interaction between pairs of p_1, p_2 and p_q probabilities on D_w values. Similar patterns occur in all plots as N increases, and weighted D-optimality criterion values decrease. The D_w values decrease across each combination of p_l, p_1, p_2 and p_q. When N is increasing, the D_w values when p_q is at its high level are higher than when p_q is at its low level. The decreases are similar which is reflected by the parallelism in Figure 4.23 (a), (e), (f), Figure 4.24 (a), (e), (f) and Figure 4.25 (a), (e), (f). However, the decrease in D_w values when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). But then decrease in D_w value when increasing p_l is larger when p_2 is at its high level ($p_2 = 0.95$) than when it is at its low level ($p_2 = 0.35$). These interactions are reflected in the lack-of-parallelism in the Figure 4.23 (b), (c), Figure 4.24 (b), (c) and Figure 4.25 (b), (c). The clear parallelism in Figure 4.23 (d), Figure 4.24 (d) and Figure 4.25 (d) indicate no interaction between p_1 and p_2.

The boxplots of weighted G-optimality criterion values for the spherical CCDs tend to be larger in the tails of the distribution for $N = 15$ compared to $N = 16$, but have similar medians. The G_w values decrease after $N = 16$ (Figure 4.18). The effect of changing p_l or p_q on G_w depends on the size of the CCD. When the number of center points increase from 2 to 6 ($N = 16$ to $N = 20$), the increase in G_w when increasing p_l is larger when p_q is at its high level ($p_q = 0.95$) than when it is at its low level ($p_q = 0.35$) as the number of center points increases from 2 to 6. An example can be seen in the comparison of the plots in Figure 4.27 (a) and Figure 4.28 (a). An exception occurs when changing from 1 to 2 center points ($N = 15, 16$). If one additional center point is added to the CCD ($N = 15, 16$), the interaction effect between p_l and p_q on G_w is reversed. This can be seen in Figure 4.26 (a) and Figure 4.27 (a).

The rate of change in G_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the rate of change in G_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.26 (b), (c), Figure 4.27 (b), (c) and Figure 4.28 (b), (c).

There is a small increase in G_w when increasing p_1 or p_2. The increases are similar which is reflected by the clear parallelism in Figure 4.26 (d), Figure 4.27 (d) and Figure 4.28 (d).

The G_w values increase as p_1 and p_2 are increasing. For $N = 15$, G_w values increase when p_q increases. While $N > 15$, G_w value increase when p_q increase for all level of p_1 and p_2. In this case, values of G_w have changed in a parallel manner in Figure 4.26 (e), (f), Figure 4.27 (e), (f) and Figure 4.28 (e), (f).

From the results for the spherical CCD design, the D_w values are weakly robust to change in p_l, p_1, and p_2 and slightly less robust as N increase. However, the D_w values are robust to change in p_q. The G_w values are slightly sensitive (weakly robust) to change in p_l, p_1 and p_2. However, the G_w values are robust to change in p_q.

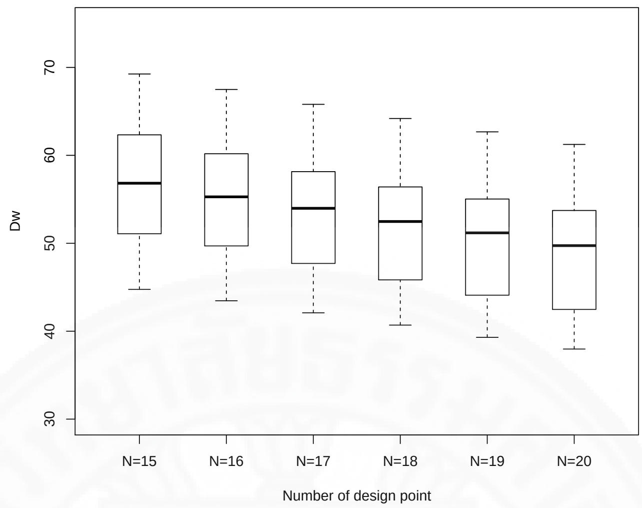


Figure 4.15: Boxplots for the weighted D-optimality criterion values in a cuboidal region for CCDs, $k = 3$

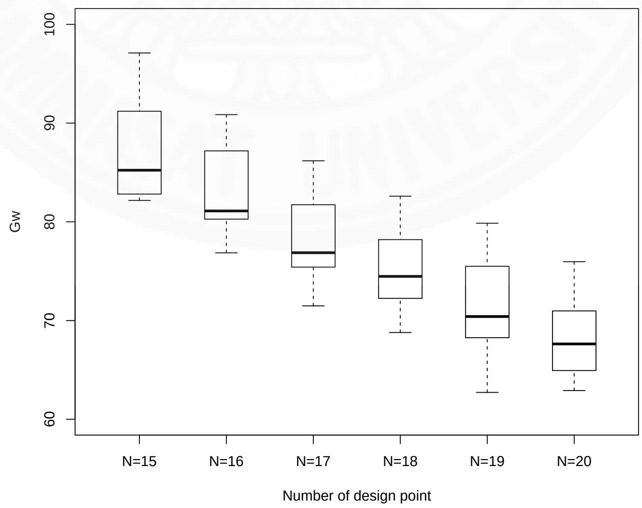


Figure 4.16: Boxplots for the weighted G-optimality criterion values in a cuboidal region for CCDs, $k = 3$

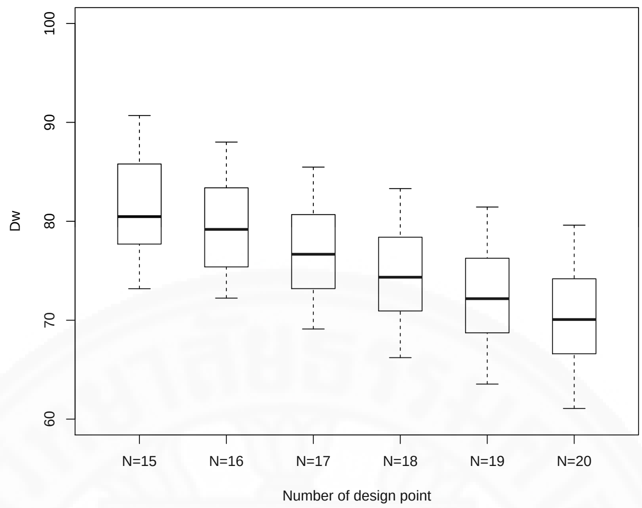


Figure 4.17: Boxplots for the weighted D-optimality criterion values in a spherical region for CCDs, $k = 3$

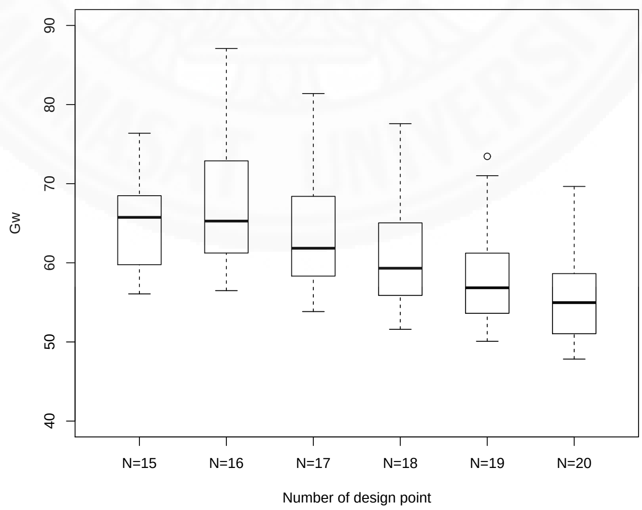


Figure 4.18: Boxplots for the weighted G-optimality criterion values in a spherical region for CCDs, $k = 3$

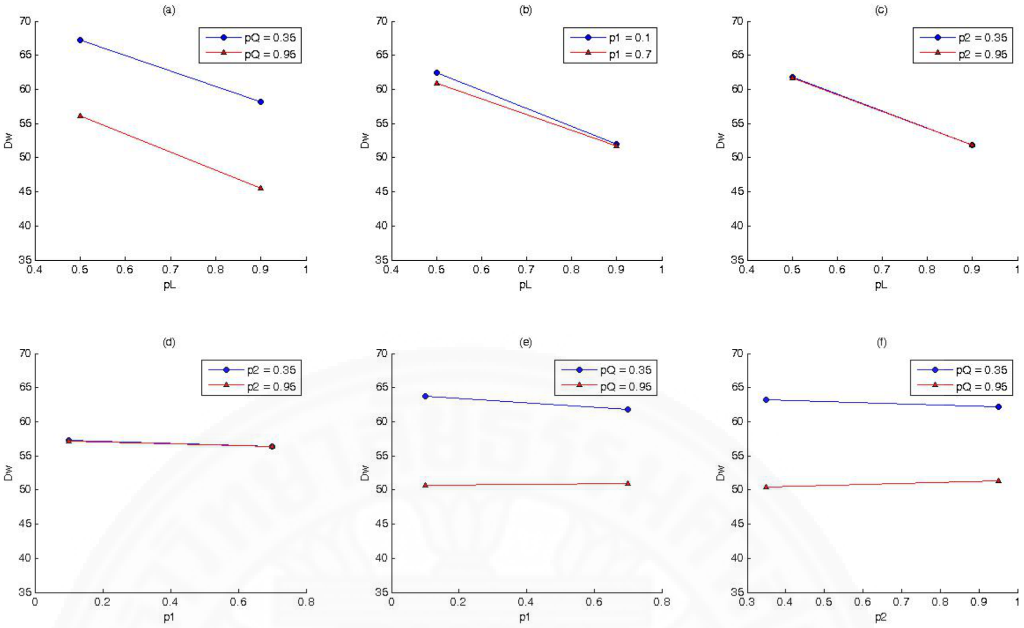


Figure 4.19: The weighted D-optimality criterion values in a cuboidal region for CCDs, $k = 3$ and $N = 15$

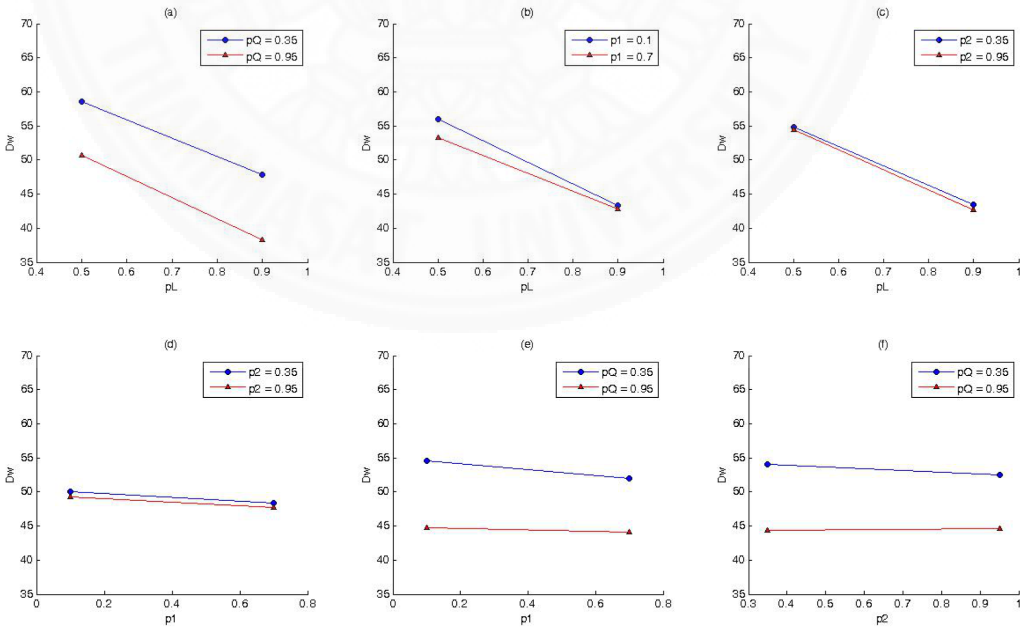


Figure 4.20: The weighted D-optimality criterion values in a cuboidal region for CCDs, $k = 3$ and $N = 20$

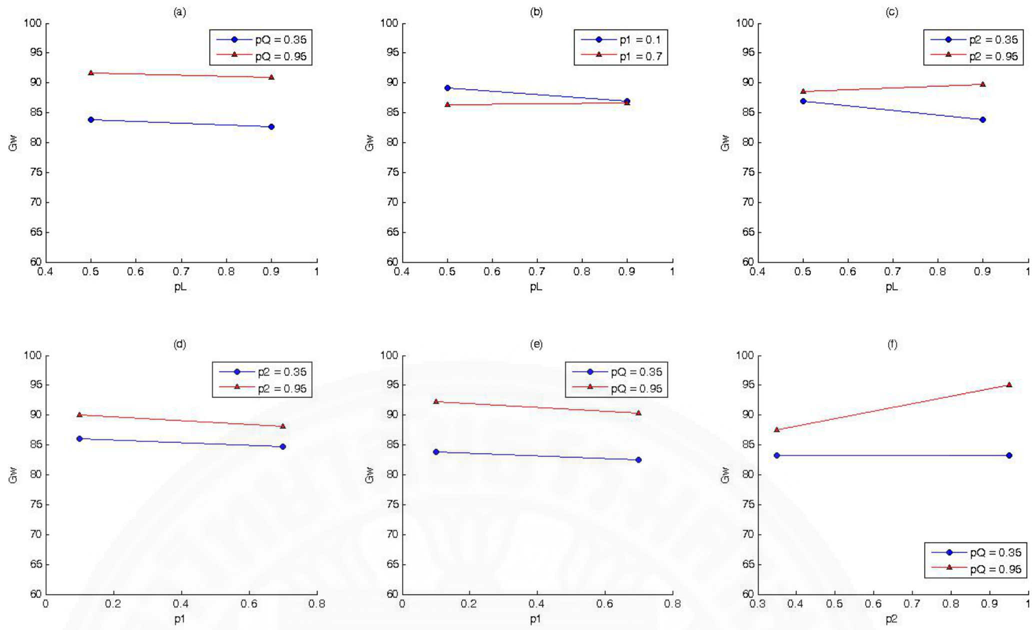


Figure 4.21: The weighted G-optimality criterion values in a cuboidal region for CCDs, $k = 3$ and $N = 15$

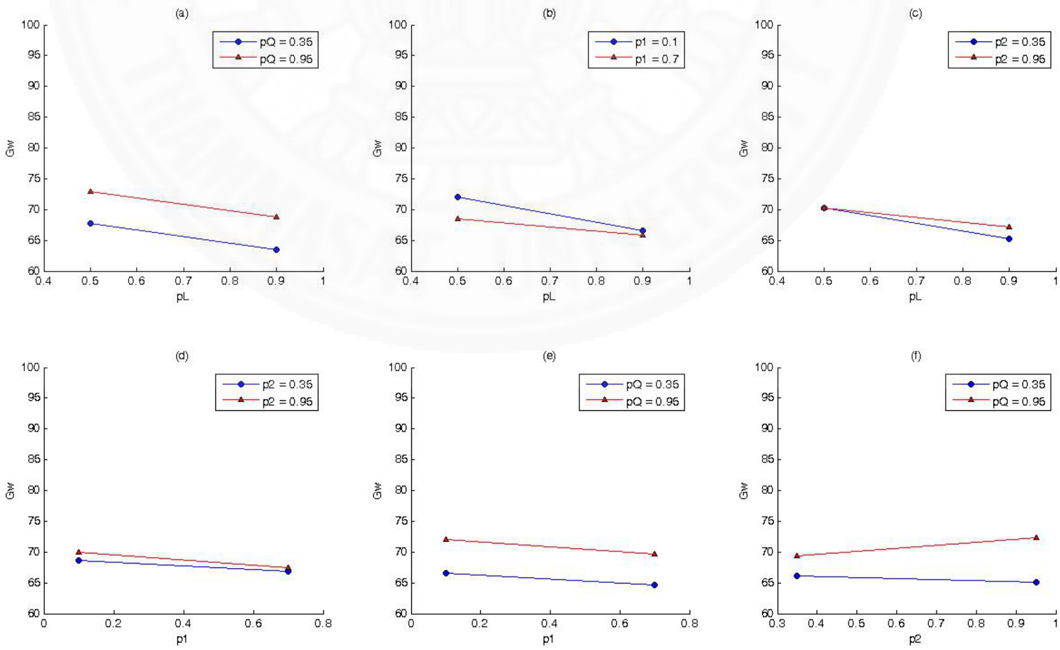


Figure 4.22: The weighted G-optimality criterion values in a cuboidal region for CCDs, $k = 3$ and $N = 20$

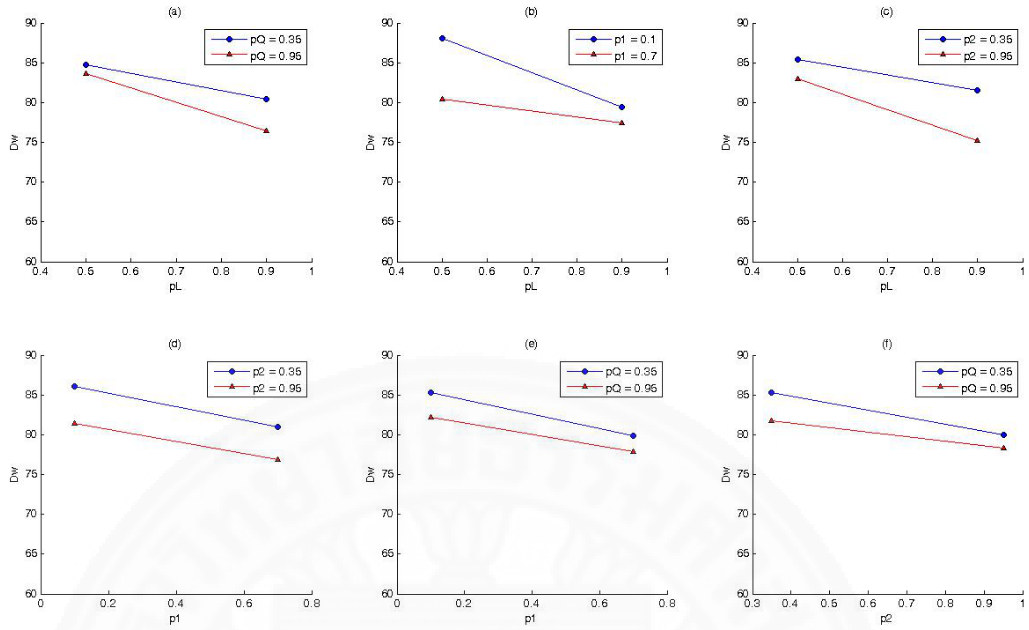


Figure 4.23: The weighted D-optimality criterion values in a spherical region for CCDs, $k = 3$ and $N = 15$

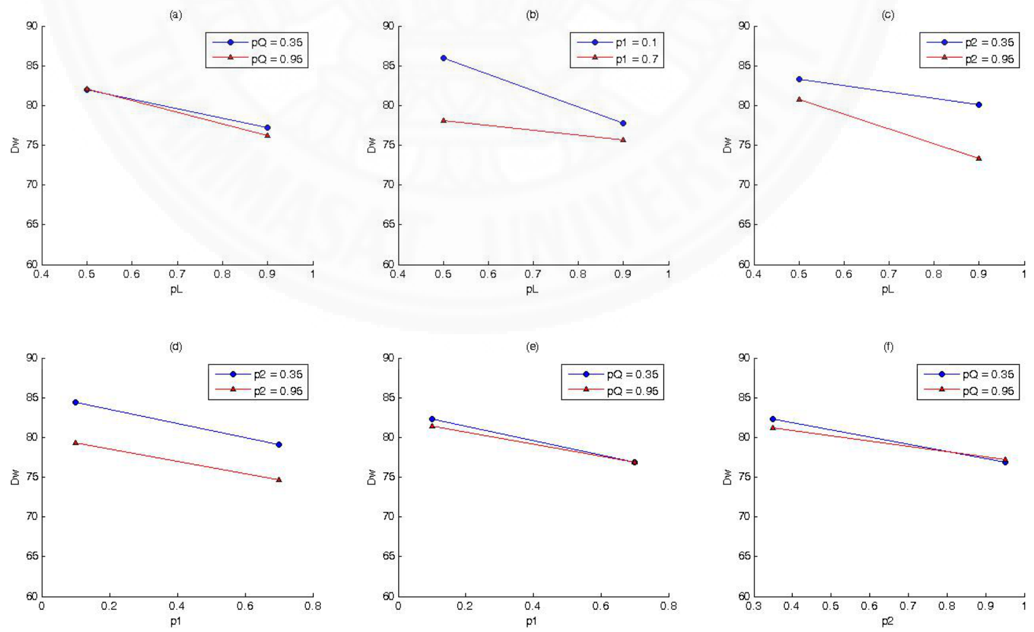


Figure 4.24: The weighted D-optimality criterion values in a spherical region for CCDs, $k = 3$ and $N = 16$

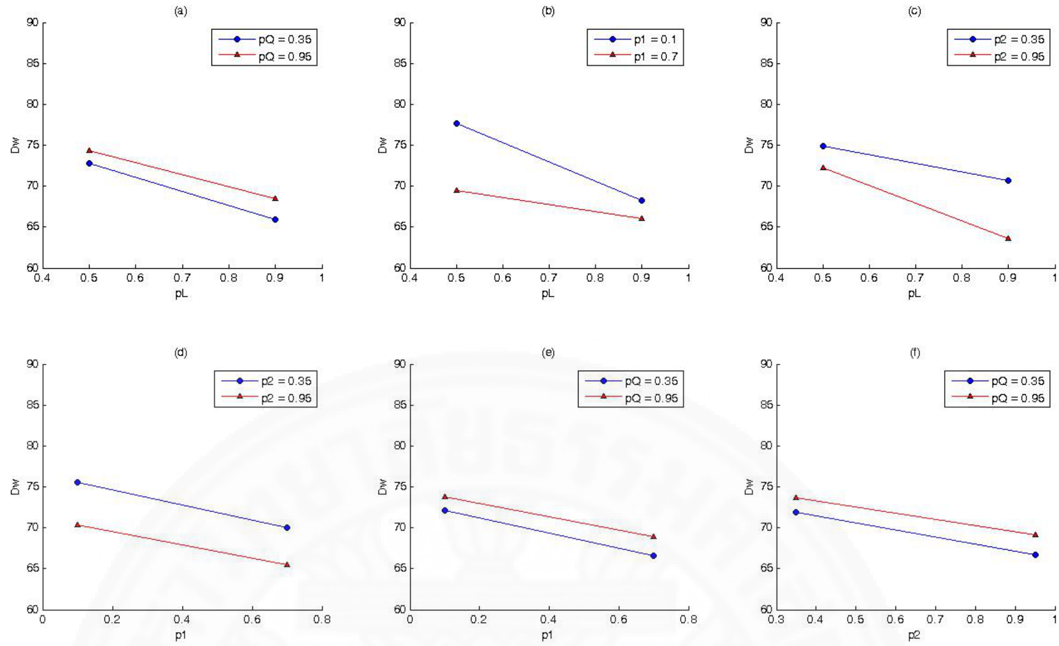


Figure 4.25: The weighted D-optimality criterion values in a spherical region for CCDs, $k = 3$ and $N = 20$

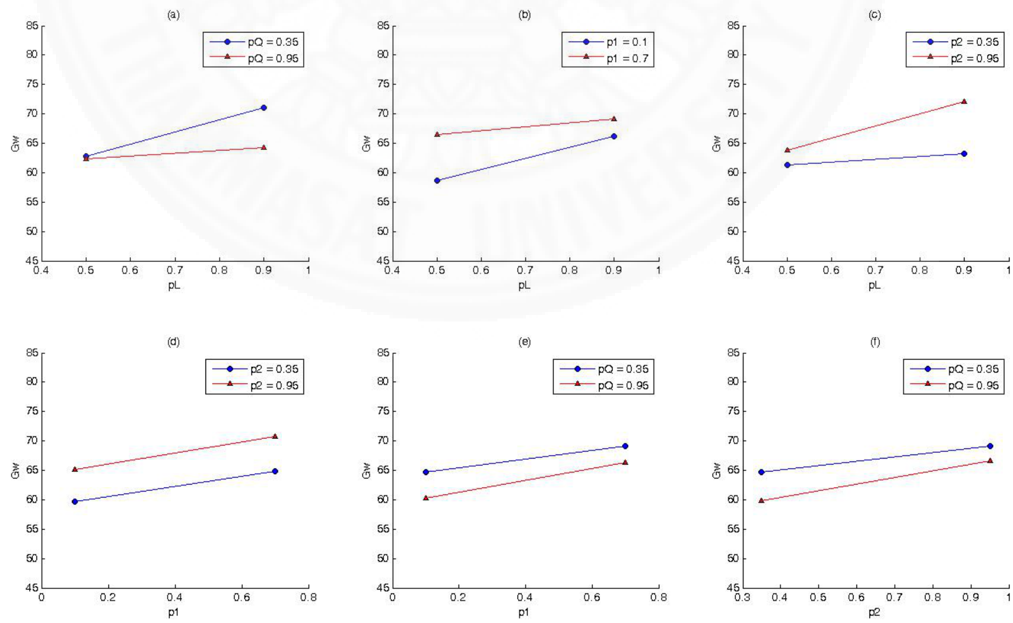


Figure 4.26: The weighted G-optimality criterion values in a spherical region for CCDs, $k = 3$ and $N = 15$

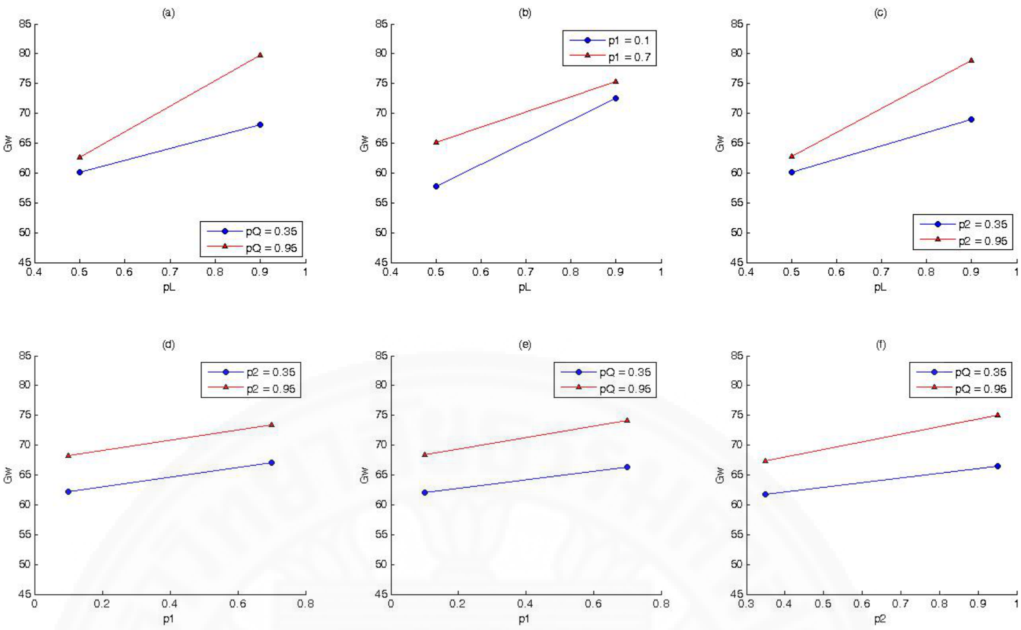


Figure 4.27: The weighted G-optimality criterion values in a spherical region for CCDs, $k = 3$ and $N = 16$

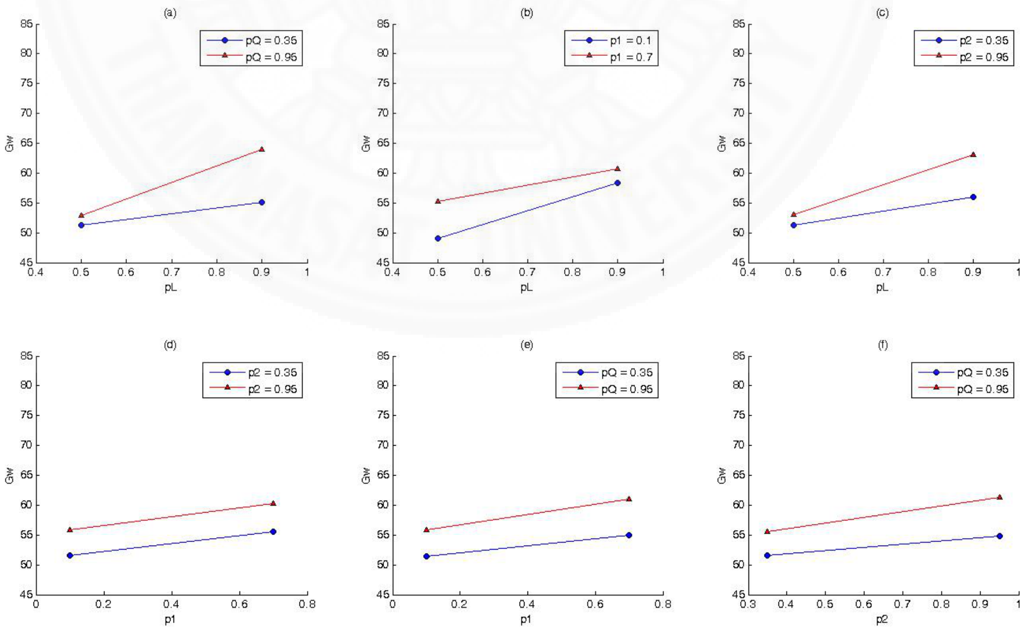


Figure 4.28: The weighted G-optimality criterion values in a spherical region for CCDs, $k = 3$ and $N = 20$

4.1.3 for $k = 4$ factors

The cuboidal CCDs

The boxplots of weighted D-optimality criterion values for the cuboidal CCDs decrease slowly when N increases (Figure 4.29). The presence of parallel lines in Figure 4.33 and 4.34 indicates no interaction between pairs of p_l, p_1, p_2 and p_q probabilities on D_w values. Similar patterns occur in all plots as N increases, and weighted D-optimality criterion values decrease. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The decrease in D_w when increasing p_q is larger than increasing p_l, p_1, or p_2. The decreases are similar which is reflected by the clear parallelism in Figure 4.33 (a), (e), (f) and Figure 4.34 (a), (e), (f). However, the decrease in D_w when increasing p_l and p_1 is at its low level is almost equal with p_1 is at its high level. This is reflected in the closeness of the segments in the Figure 4.33 (b) and Figure 4.33 (b). The D_w value decreases when p_l increases and the rate of change in D_w when p_2 is at its low level ($p_2 = 0.35$) is slightly larger than when p_2 is at its high level ($p_2 = 0.95$). This is reflected in the small lack-of-parallelism in Figure 4.33 (c) and Figure 4.33 (c). The D_w value slight increase when p_1 or p_2 increase and this is shown in the parallelism in Figure 4.33 (d) and Figure 4.33 (d).

The boxplots of weighted G-optimality criterion values for cuboidal CCDs decrease and have two outliers when N increases. The two outliers are the weighted G-optimality criterion values at ($p_l = 0.9, p_1 = 0.1, p_2 = 0.35, p_q = 0.95$) and ($p_l = 0.9, p_1 = 0.7, p_2 = 0.35, p_q = 0.95$). This is shown in Figure 4.30. The presence of parallel lines in Figure 4.35 and 4.36 indicates no interaction between pairs of p_1 and p_2 probabilities on G_w values. This is similar in all plots as N increases, and G_w values decrease. This is shown in Figure 4.35 (d) and Figure 4.36 (d). The G_w values decrease across each combination of p_q, p_1, and p_2 when p_l increases. The rate of change in G_w values at p_q and p_2 when p_l is at its low level ($p_l = 0.5$) is smaller than with p_l is at its high level ($p_l = 0.9$). This is reflected in the lack-of-parallelism in Figure 4.35 (a), (c) and Figure 4.36 (a), (c). However, the decrease in G_w when increasing p_l and p_1 is at its low level is almost equal with p_1 is at its high level. This is reflected in the parallelism in Figure 4.35 (b) and Figure 4.36 (b). The G_w values slightly decrease when p_q is low but increase when p_q is high and the rate of change in G_w values at p_1 and p_2 are low is larger than p_1 and p_2 are high level. This pattern is shown in the lack of parallel and show in Figure 4.35 (e), (f) and Figure 4.36 (e), (f).

From the results for the cuboidal CCD design, the D_w values are robust to change in p_1 and p_2 and slightly less robust as N increases. However, the D_w values are sensitive to change in p_l and p_q. The G_w values are slightly sensitive (weakly robust) to change in p_2. However, the G_w values are robust to change in p_1 and p_q and very sensitive to change in p_l.

The spherical CCDs

The boxplots of weighted D-optimality criterion values for the spherical CCDs decrease when N increases (Figure 4.31). The D_w values have only a little change across each combination of p_l, p_1, and p_2 when p_q increases. Thus, changing p_q has almost no effect. This is shown in Figure 4.37, Figure 4.38, and Figure 4.39. The presence of crossed lines indicates interaction between pairs of p_i probabilities on D_w values from $N = 25$ to 26 but the parallel line indicates no interaction after $N = 26$. However, these interaction effects are very small for $N = 25, 26$. Similar patterns occur in all plots as N increases, and weighted D-optimality criterion values decrease. This is shown in Figure 4.37 (a), (e), (f), Figure 4.38 (a), (e), (f), and Figure 4.39 (a), (e), (f). The D_w values decrease when p_l increases. The rate of change in D_w of (p_l, p_1) when p_l is low is larger than when p_l is high but the rate of change in D_w of (p_l, p_2) when p_l is low is smaller than at p_l is high. This is reflected in the lack-of-parallelism in Figure 4.37 (b), (c), Figure 4.38 (b), (c) and Figure 4.39 (b), (c). The presence of parallel lines indicates no interaction between pairs of p_1 and p_2 probabilities on D_w values. This is similar in all plots as N increases, and D_w values decrease. This is shown in Figure 4.37 (d), Figure 4.38 (d) and Figure 4.39 (d).

The boxplots of weighted G-optimality criterion values for the spherical

CCDs increase slightly from $N = 25$ to 26, but the G_w values decrease slowly after $N = 26$ (Figure 4.32). Overall there is very little effect of size N on the distribution of G_w values. The effect of changing p_l or p_q on G_w depends on the size of the CCD. The G_w values increase across each combination of p_q, p_1, and p_2 when p_l increases. The rate of change in G_w for (p_l, p_q) and (p_l, p_2) when p_l is low is <u>smaller</u> than when p_l is high, while the rate of change in G_w for (p_l, p_1) when p_l is low is <u>larger</u> than when p_l is high. This is reflected in the lack-of-parallelism in Figure 4.40 (a), (b), (c) and Figure 4.41 (a), (b), (c). The presence of parallel lines indicates no interaction between pairs of (p_1, p_2), (p_1, p_q), and (p_2, p_3) probabilities on G_w values. This is similar in all plots as N increases, and G_w values increase. This is shown in Figure 4.40 (d), (e), (f) and Figure 4.41 (d), (e), (f).

From the results for the spherical CCD design, the D_w values are <u>weakly robust</u> to change in p_l, p_1 and p_2 and <u>slightly less robust</u> as N increase. However, the D_w values are <u>robust</u> to change in p_q. The G_w values are <u>slightly sensitive</u> (weakly robust) to change in p_l, p_1 and p_q. However, the G_w values are <u>sensitive</u> to change in p_2.

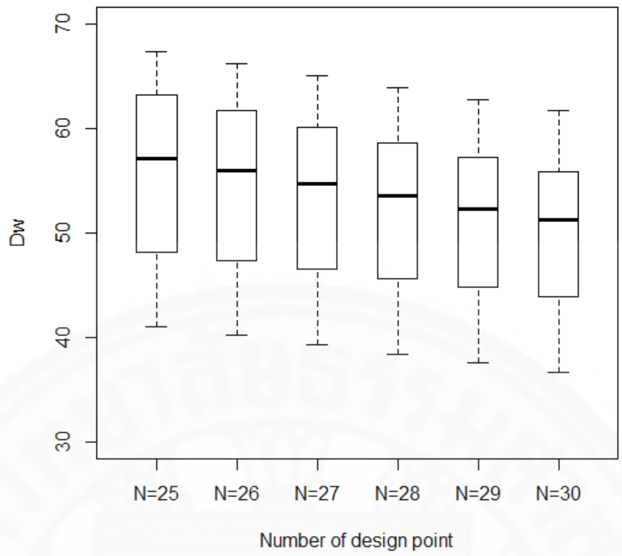


Figure 4.29: Boxplots for the weighted D-optimality criterion values in a cuboidal region for CCDs, $k = 4$

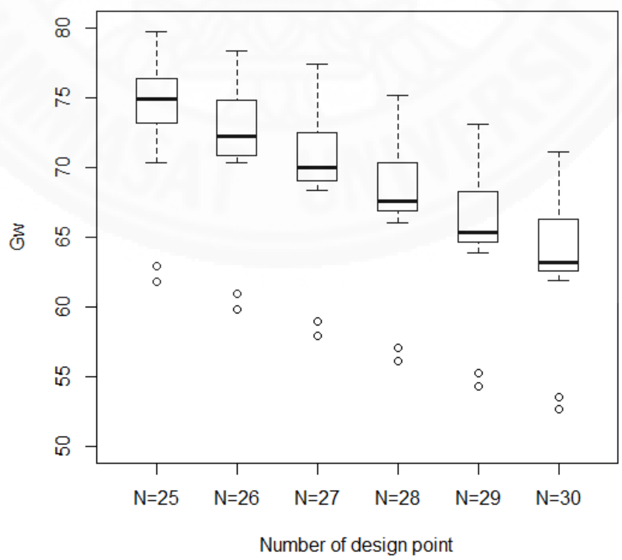


Figure 4.30: Boxplots for the weighted G-optimality criterion values in a cuboidal region for CCDs, $k = 4$

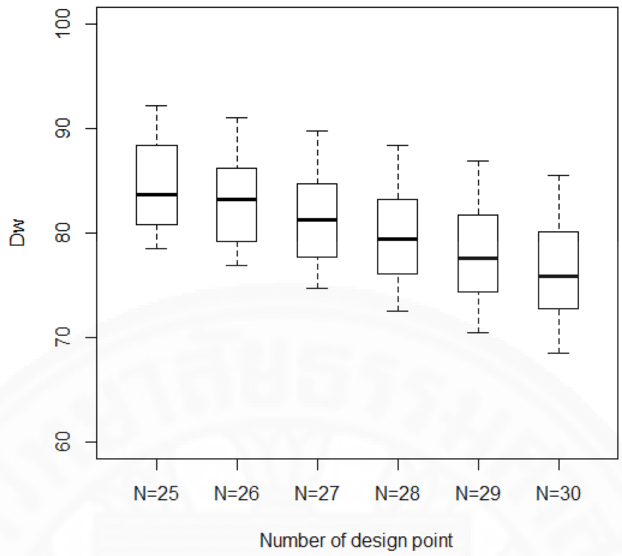


Figure 4.31: Boxplots for the weighted D-optimality criterion values in a spherical region for CCDs, $k = 4$

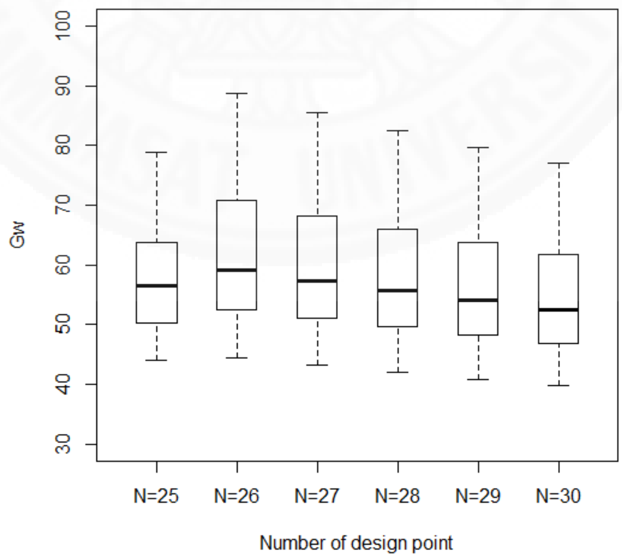


Figure 4.32: Boxplots for the weighted G-optimality criterion values in a spherical region for CCDs, $k = 4$

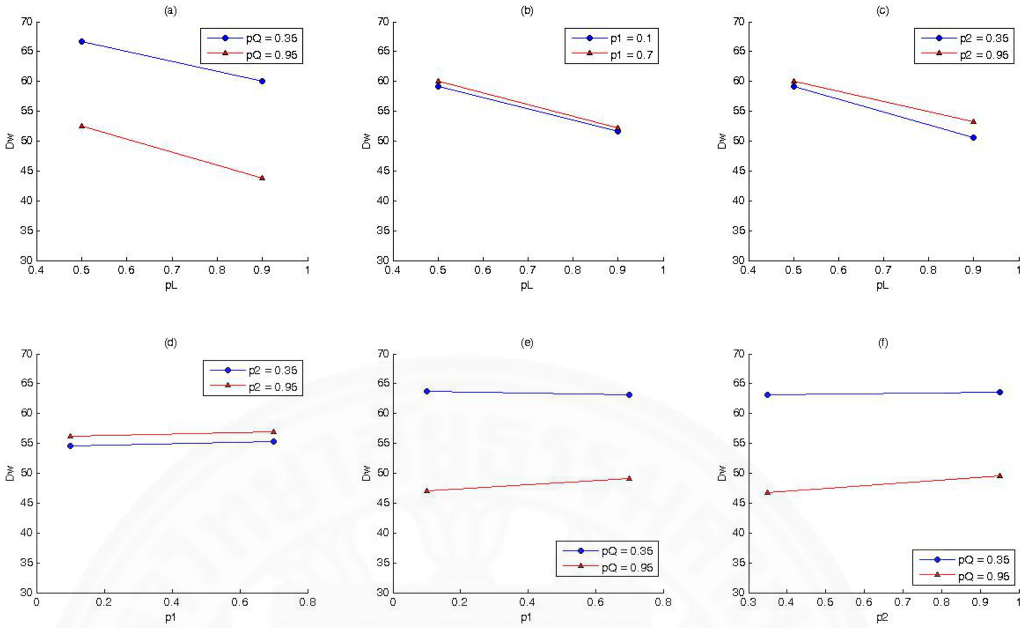


Figure 4.33: The weighted D-optimality criterion values in a cuboidal region for CCDs, $k = 4$ and $N = 25$

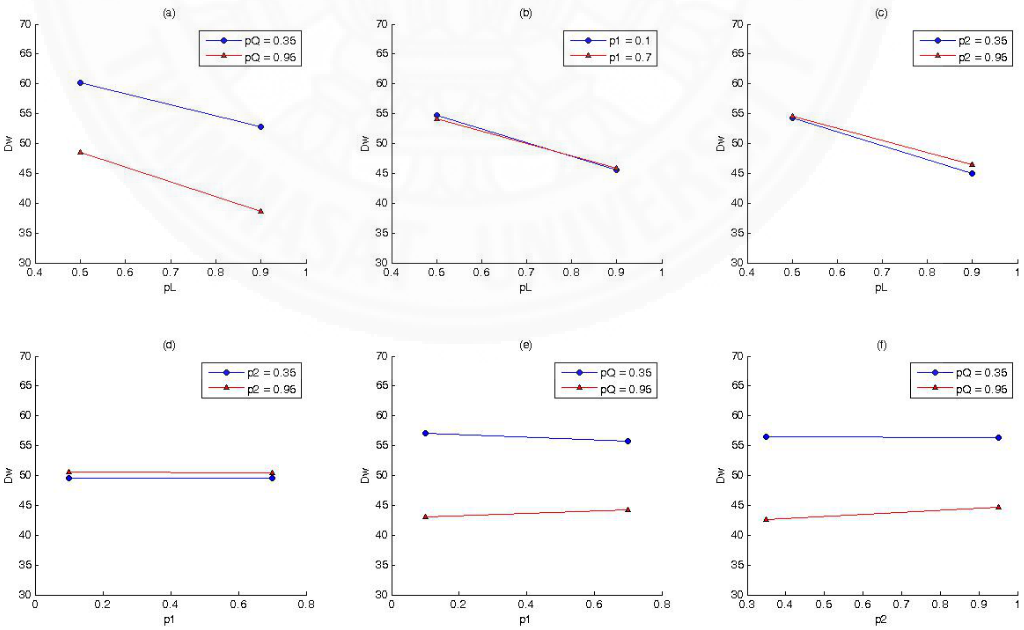


Figure 4.34: The weighted D-optimality criterion values in a cuboidal region for CCDs, $k = 4$ and $N = 30$

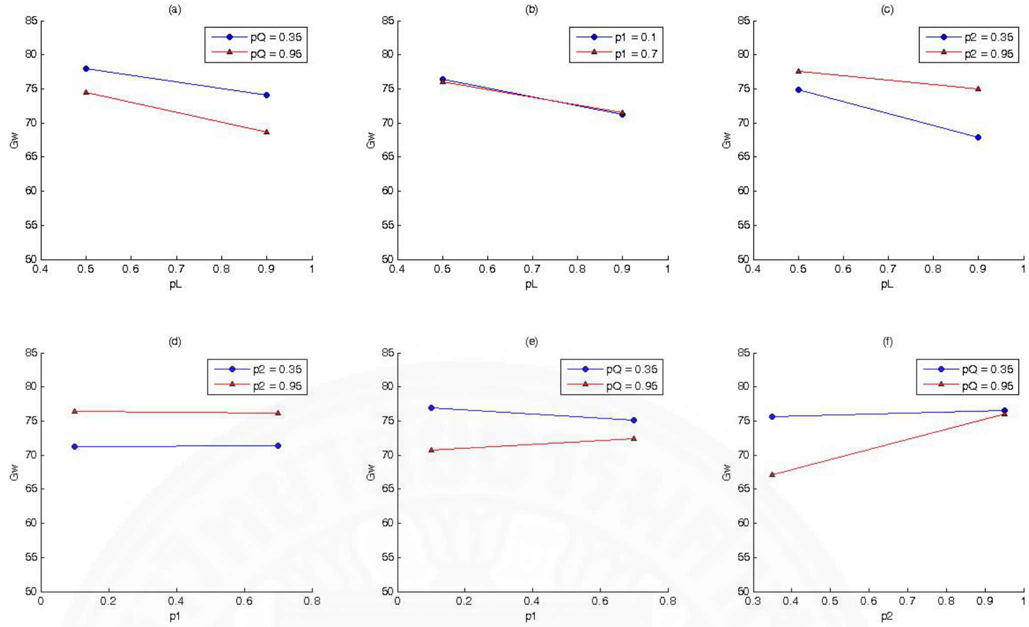


Figure 4.35: The weighted G-optimality criterion values in a cuboidal region for CCDs, $k = 4$ and $N = 25$

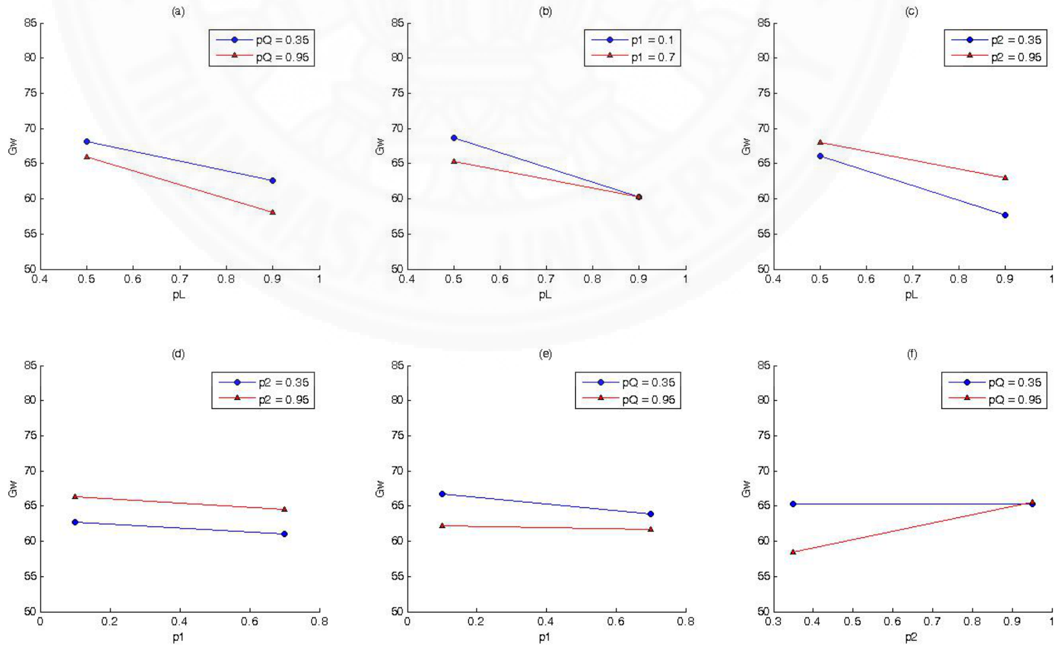


Figure 4.36: The weighted G-optimality criterion values in a cuboidal region for CCDs, $k = 4$ and $N = 30$

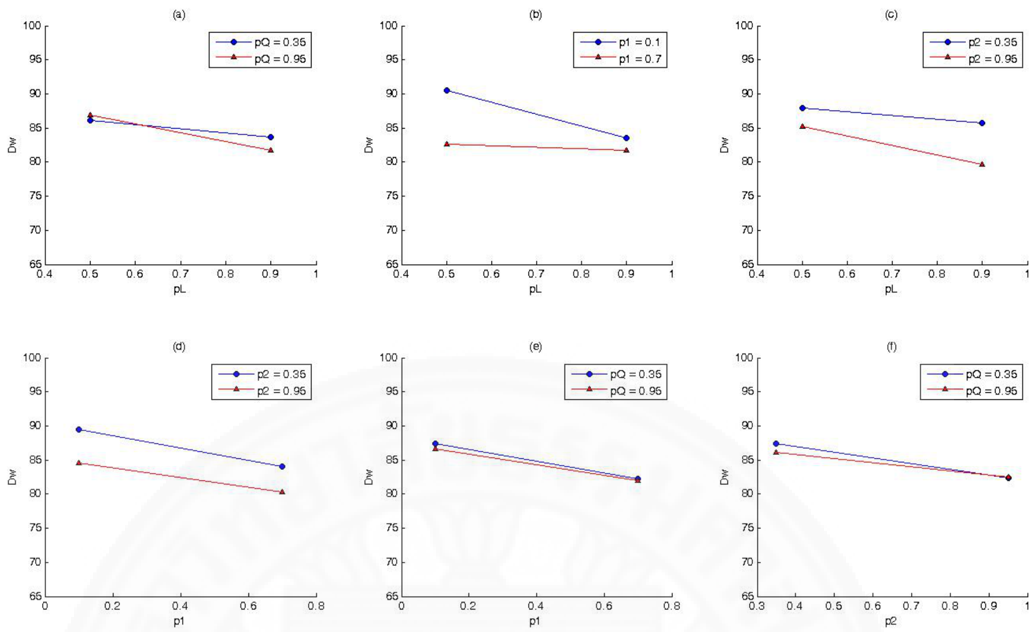


Figure 4.37: The weighted D-optimality criterion values in a spherical region for CCDs, $k = 4$ and $N = 25$

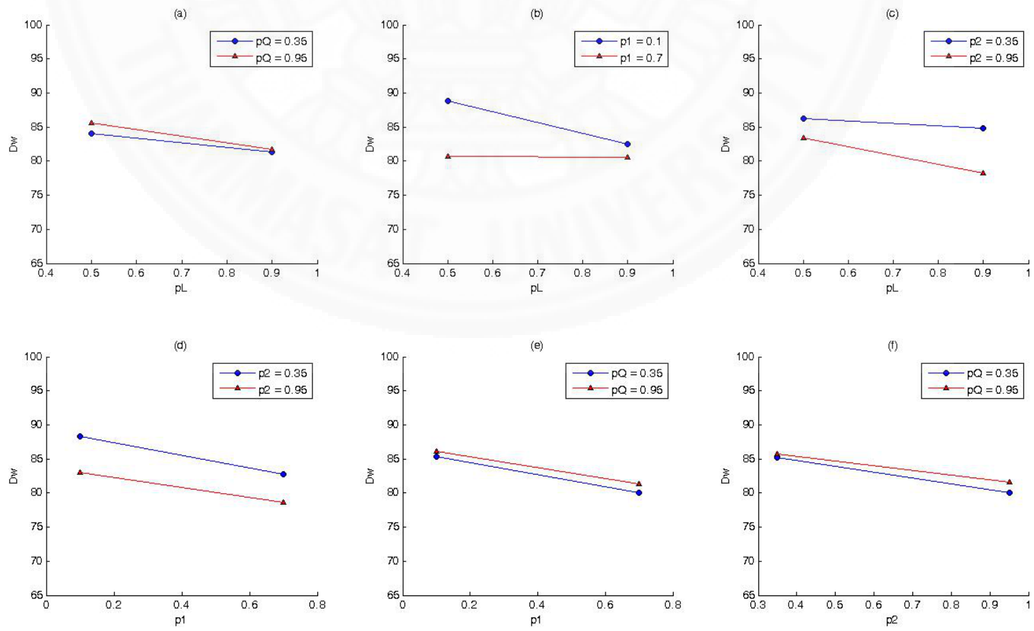


Figure 4.38: The weighted D-optimality criterion values in a spherical region for CCDs, $k = 4$ and $N = 26$

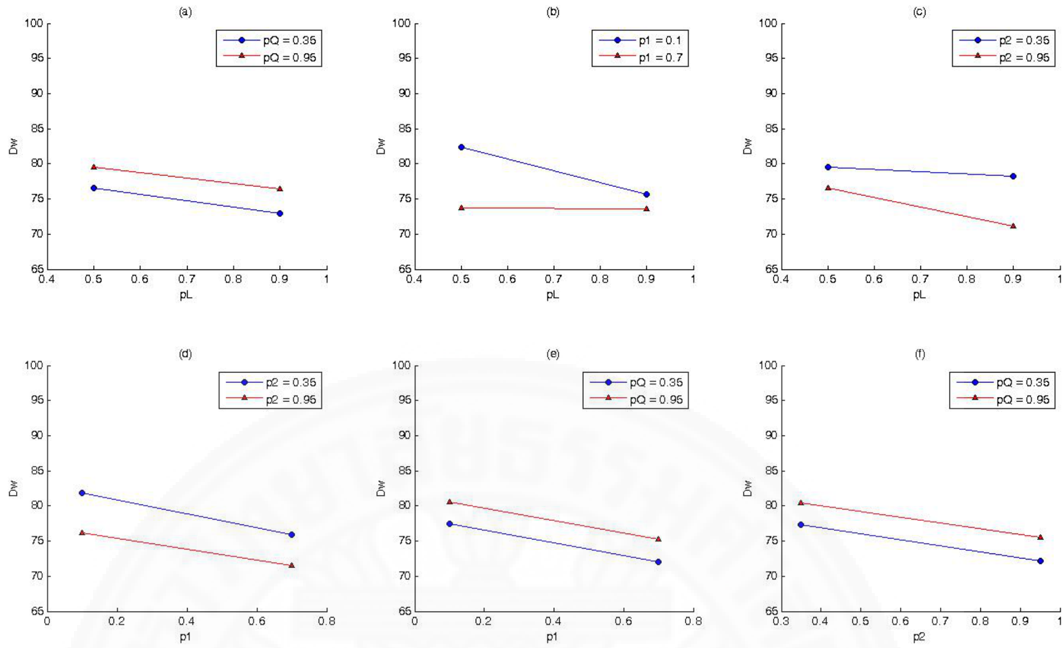


Figure 4.39: The weighted D-optimality criterion values in a spherical region for CCDs, $k = 4$ and $N = 30$

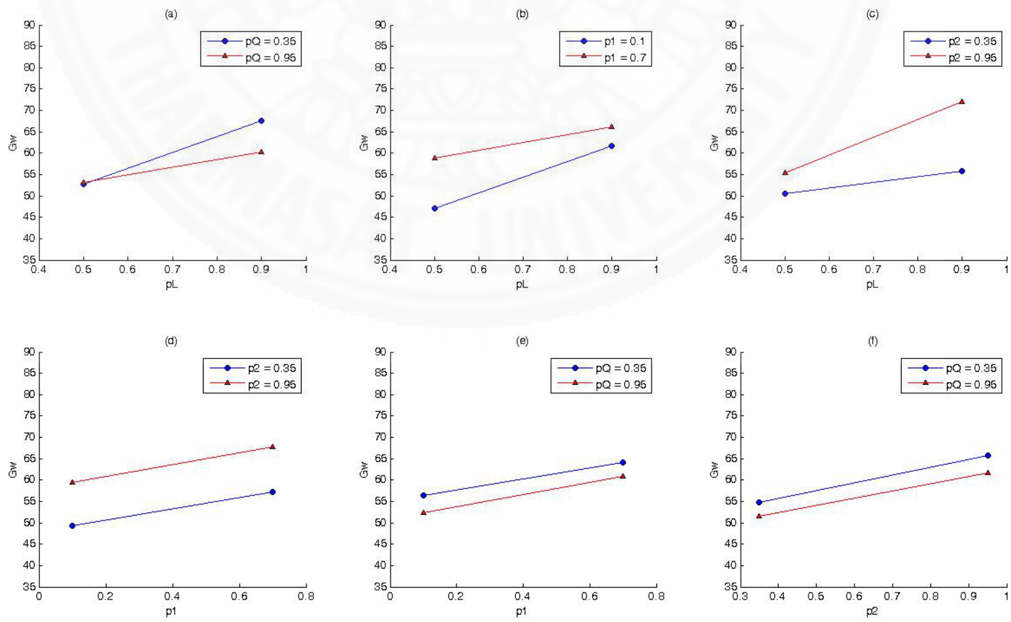


Figure 4.40: The weighted G-optimality criterion values in a spherical region for CCDs, $k = 4$ and $N = 25$

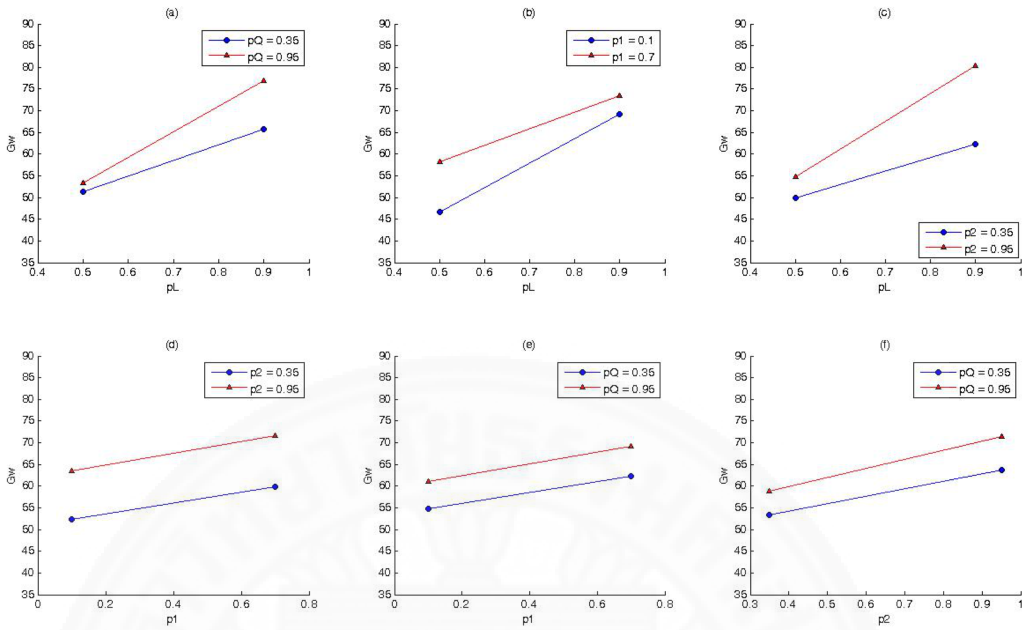


Figure 4.41: The weighted G-optimality criterion values in a spherical region for CCDs, $k = 4$ and $N = 26$

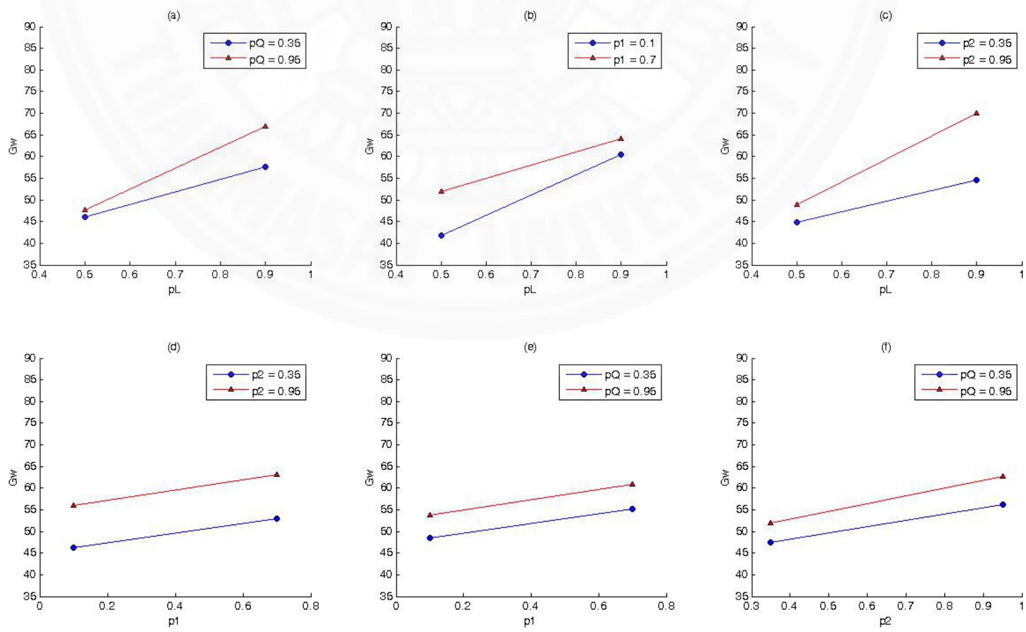


Figure 4.42: The weighted G-optimality criterion values in a spherical region for CCDs, $k = 4$ and $N = 30$

4.2 The results of the weighted D and G-optimality criteria values for BBDs

4.2.1 for $k = 3$ factors

The cuboidal BBDs

The boxplots of weighted D-optimality criterion values for the cuboidal BBDs decrease when N increases (Figure 4.43). The presence of parallel lines in Figure 4.47 and 4.48 indicates no interaction between p_l and p_q and between p_1 and p_2 probabilities on D_w values. Similar patterns occur in all plots as N increases, and weighted D-optimality criterion values decrease. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The decrease in D_w when increasing p_l is larger than when increasing p_1 and p_2. This is shown in Figure 4.47 (a), (e), (f) and Figure 4.48 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in D_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.3$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.47 (b), (c) and Figure 4.48 (b), (c). There is only a small decrease in D_w when increasing p_1 or p_2. The small decreases are similar which is reflected by the clear parallelism in Figure 4.47 (d) and Figure 4.48 (d). Thus D_w is quite robust to changes in p_1 or p_2.

The boxplots of weighted G-optimality criterion values for the cuboidal BBDs decrease when N increases (Figure 4.44). When $N = 13$, the decrease in G_w when increasing p_l is smaller when p_q is at its low level ($p_q = 0.35$) than when it is at its high level ($p_q = 0.95$). Also, the decrease in G_w when increasing p_2 is larger when p_q is at its low level ($p_q = 0.35$) than when it is at its high level ($p_q = 0.95$). This is reflected in the lack-of-parallelism in Figure 4.49 (f). But $N > 13$, the presence of parallel lines indicates no interaction between pairs of p_1, p_2 and p_q probabilities on G_w values. Similar patterns occur in all plots as N increases, and weighted G-optimality criterion values decrease. Increasing p_l, p_1, and p_2 lowers G_w values, but there is the increase in G_w when increasing p_q. This is shown in Figure 4.49 (a), (e), (f), Figure 4.50 (a), (e), (f), and Figure 4.51 (a), (e), (f). However, the decrease in G_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in G_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.49 (b), (c), Figure 4.50 (b), (c), and Figure 4.51 (b), (c). The values of G_w decrease when p_1 or p_2 increased, but with larger decrease in G_w when increasing p_2. The dereases are similar when increasing p_1 for both p_2 levels which is reflected by the parallelism in Figure 4.49 (d), Figure 4.50 (d), and Figure 4.51 (d).

From the results for the cuboidal BBD design, the D_w values are robust to change in p_1 and p_2 and slightly less robust as N increase. However, the D_w values are sensitive to change in p_l and p_q and slightly more robust to change in p_q as N increase. The G_w values are slightly sensitive (weakly robust) to change in p_1, p_2 and p_q. However, the G_w values are very sensitive to change in p_l.

The spherical BBDs

The boxplots of weighted D-optimality criterion values for spherical BBDs decrease when N increases (Figure 4.45). Similar patterns occur in all plots as N increases, and weighted D-optimality criterion values decrease. The rate of change in D_w decreases when N increases. The decrease in D_w when increasing p_l is larger than increasing p_1 and p_2. This is shown in Figure 4.52 (a), (e), (f) and Figure 4.53 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in D_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.52 (b), (c) and Figure 4.53 (b), (c). There is only a small decrease in D_w when increasing p_1 or p_2. The presence of parallel lines indicates no interaction between pairs of p_i probabilities on D_w values which is reflected by the clear parallelism in Figure 4.52 (d) and Figure 4.53 (d). Thus

D_w is quite robust to change in p_1 or p_2.

The boxplots of weighted G-optimality criterion values for spherical BBDs decrease when N increases (Figure 4.46). The increase in G_w when increasing p_l is smaller when p_q is at its low level ($p_q = 0.35$) than when it is at its high level ($p_q = 0.95$). This is reflected in the lack-of-parallelism in Figure 4.54 (a) and Figure 4.56 (a). While the presence of parallel lines indicates no interaction between pairs of p_1, p_2 and p_q probabilities on G_w values and is shown in Figure 4.54 (d), (e), (f) and Figure 4.56 (d), (e), (f). Similar patterns occur in all plots as N increases, and weighted G-optimality criterion values increase. Increasing p_1 and p_2 increases G_w values, but there is a decrease in G_w when increasing p_q. This is shown in Figure 4.54 (a), (e), (f) and Figure 4.56 (a), (e), (f). However, the increase in G_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the increase in G_w when increase p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.54 (b), (c) and Figure 4.56 (b), (c).

From the results for the spherical BBD design, the D_w values are robust to change in p_1 and p_2 and slightly less robust as N increase. However, the D_w values are sensitive to change in p_l and p_q and slightly more robust in p_q as N increase. The G_w values are slightly sensitive (weakly robust) to change in p_l, p_1 and p_2. However, the G_w values are robust to change in p_q.

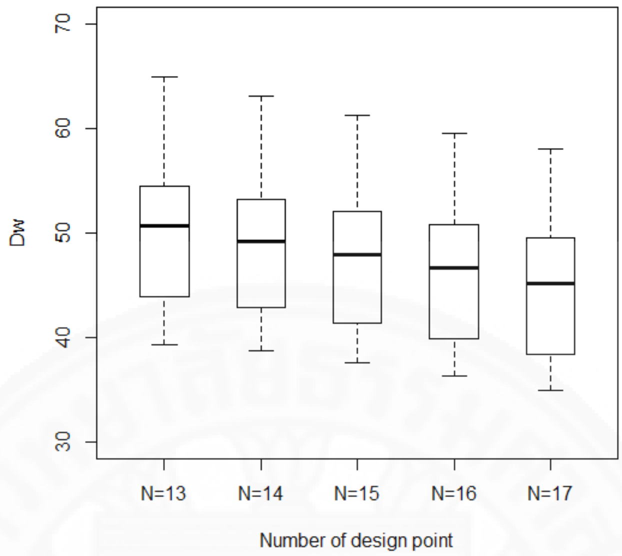


Figure 4.43: Boxplots for the weighted D-optimality criterion values in a cuboidal region for BBDs, $k = 3$

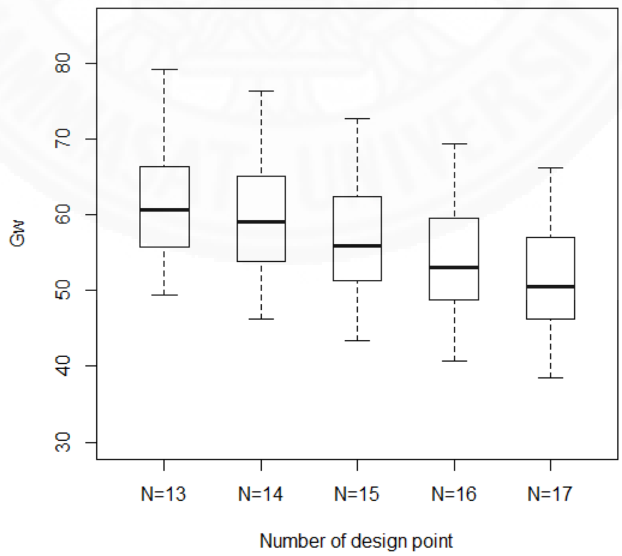


Figure 4.44: Boxplots for the weighted G-optimality criterion values in a cuboidal region for BBDs, $k = 3$

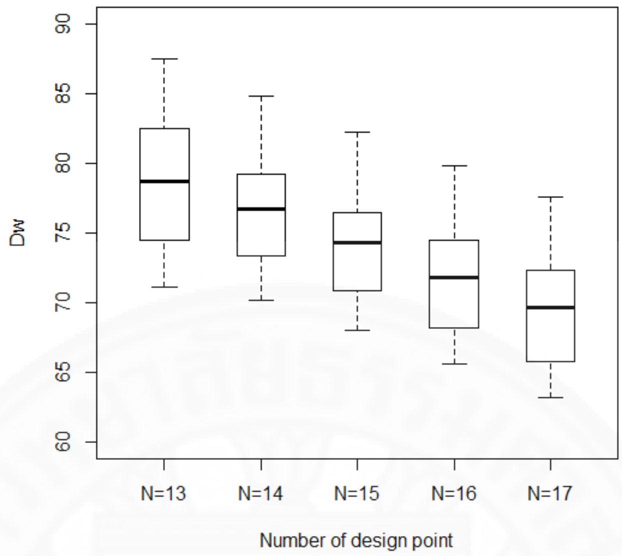


Figure 4.45: Boxplots for the weighted D-optimality criterion values in a spherical region for BBDs, $k = 3$

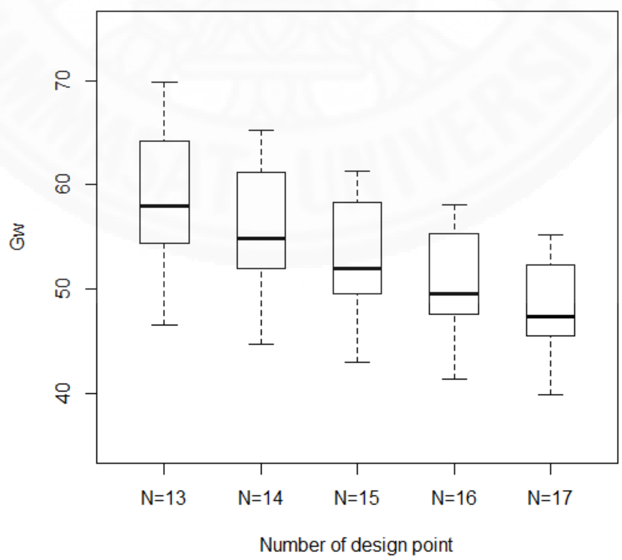


Figure 4.46: Boxplots for the weighted G-optimality criterion values in a spherical region for BBDs, $k = 3$

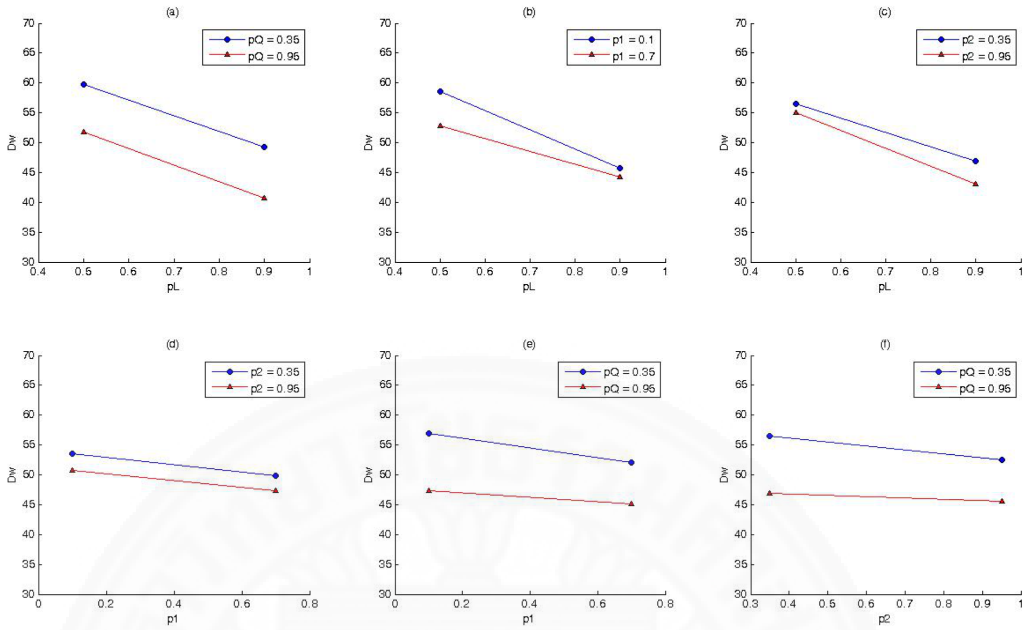


Figure 4.47: The weighted D-optimality criterion values in a cuboidal region for BBDs, $k = 3$ and $N = 13$

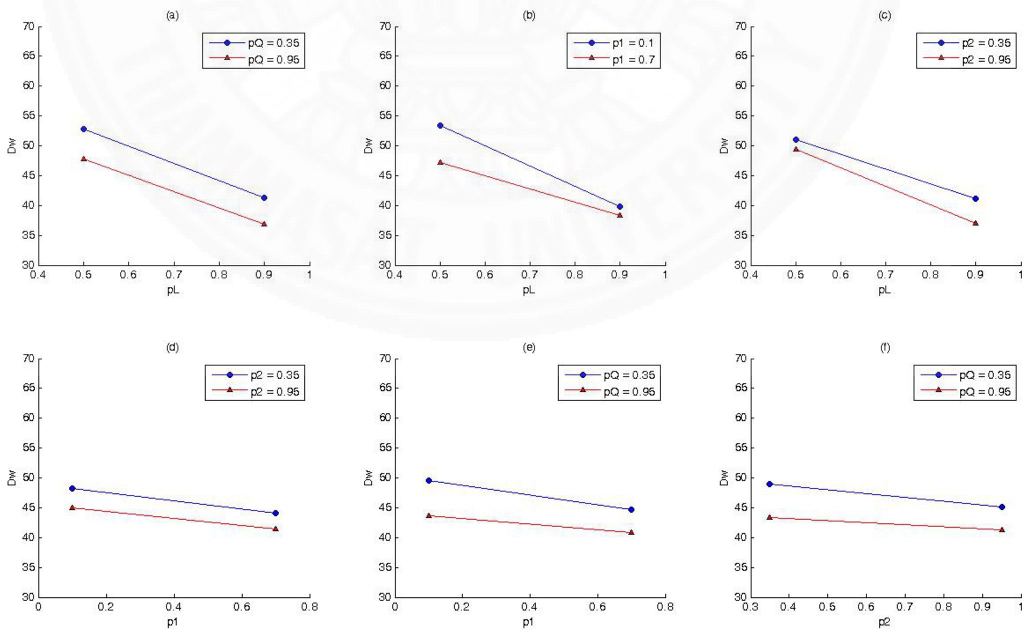


Figure 4.48: The weighted D-optimality criterion values in a cuboidal region for BBDs, $k = 3$ and $N = 17$

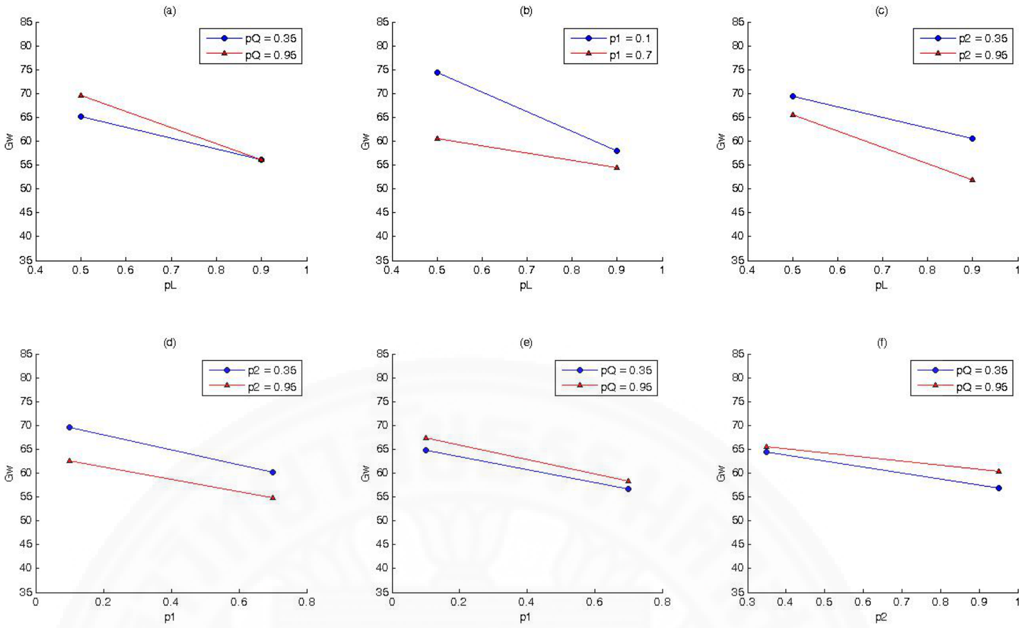


Figure 4.49: The weighted G-optimality criterion values in a cuboidal region for BBDs, $k = 3$ and $N = 13$

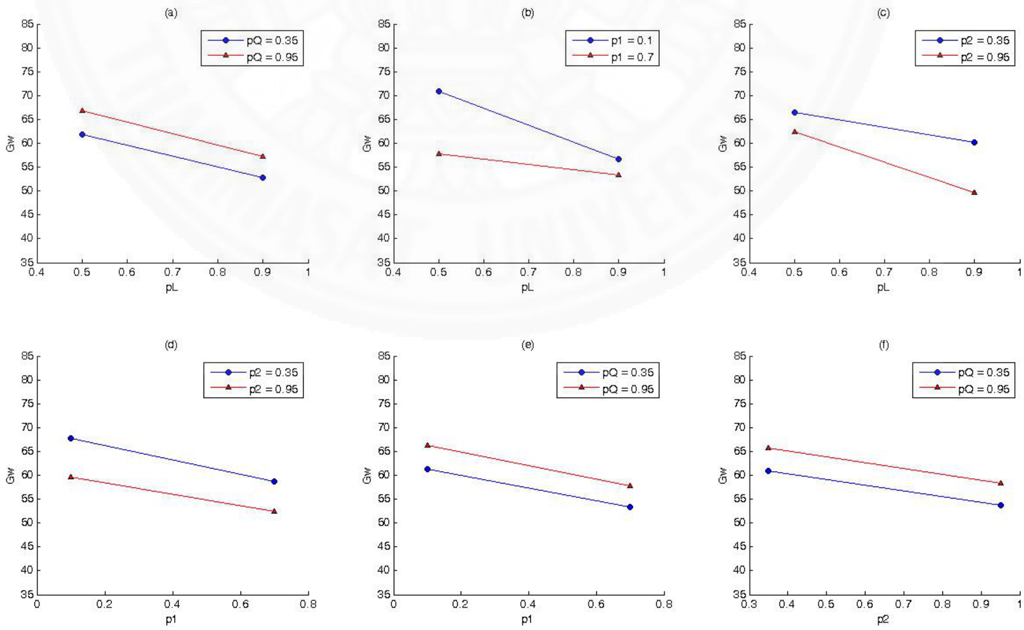


Figure 4.50: The weighted G-optimality criterion values in a cuboidal region for BBDs, $k = 3$ and $N = 14$

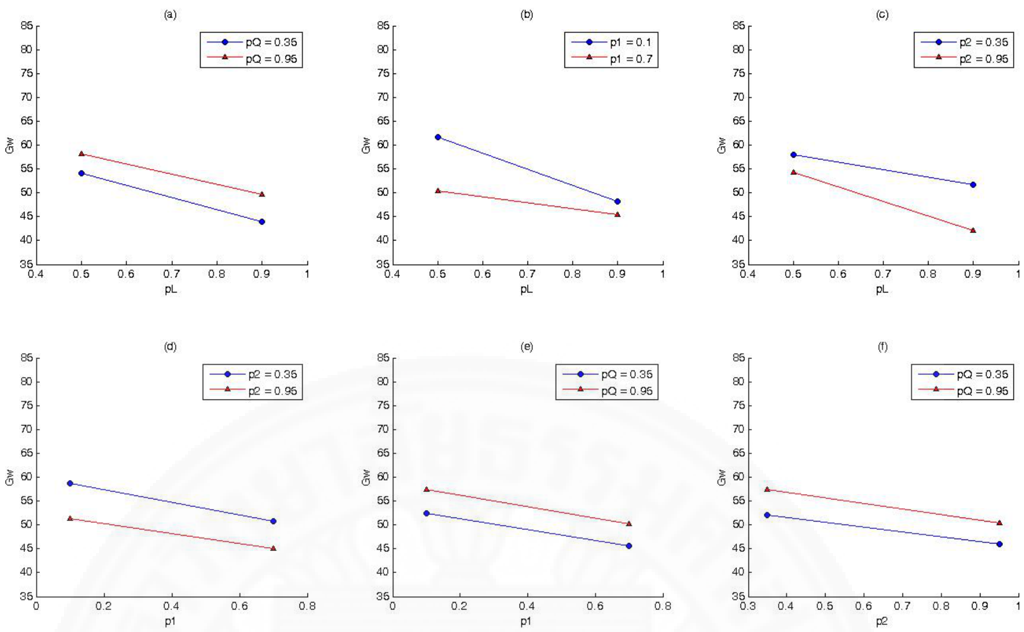


Figure 4.51: The weighted G-optimality criterion values in a cuboidal region for BBDs, $k = 3$ and $N = 17$

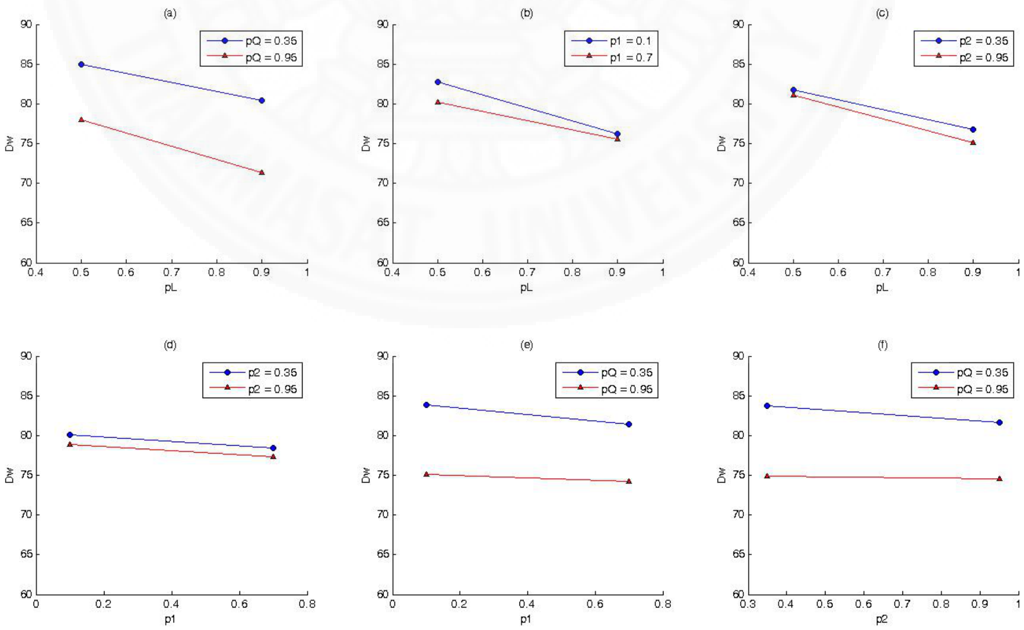


Figure 4.52: The weighted D-optimality criterion values in a spherical region for BBDs, $k = 3$ and $N = 13$

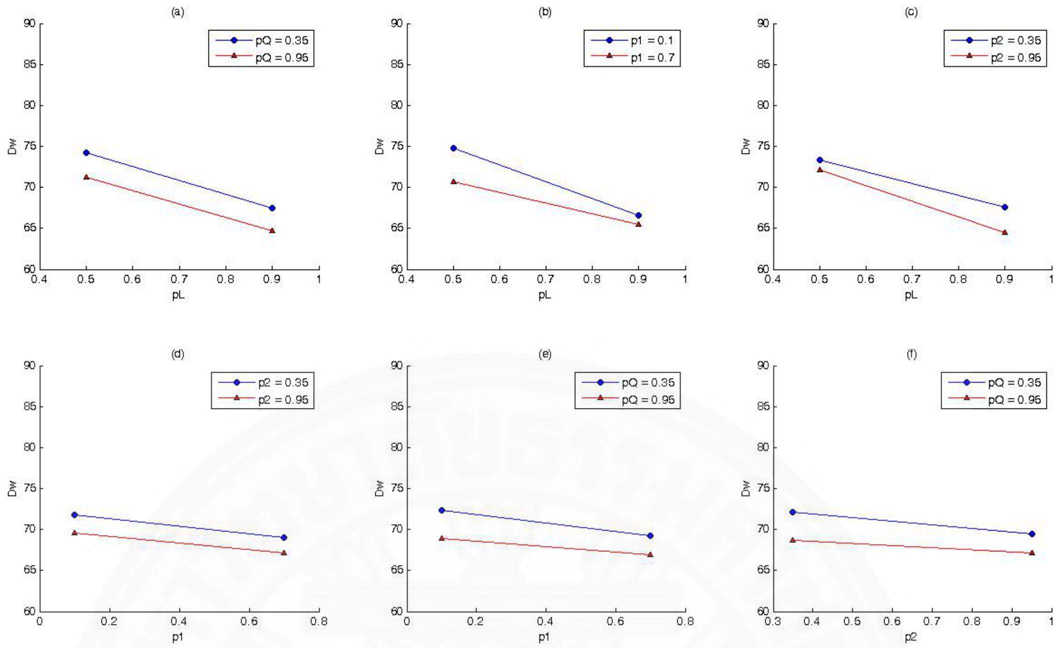


Figure 4.53: The weighted D-optimality criterion values in a spherical region for BBDs, $k = 3$ and $N = 17$

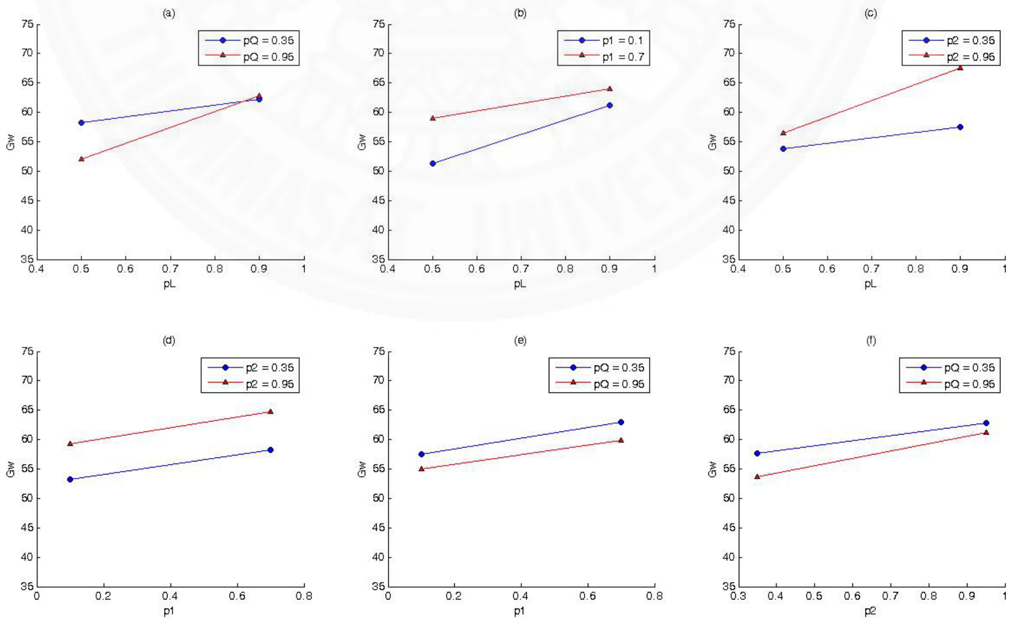


Figure 4.54: The weighted G-optimality criterion values in a spherical region for BBDs, $k = 3$ and $N = 13$

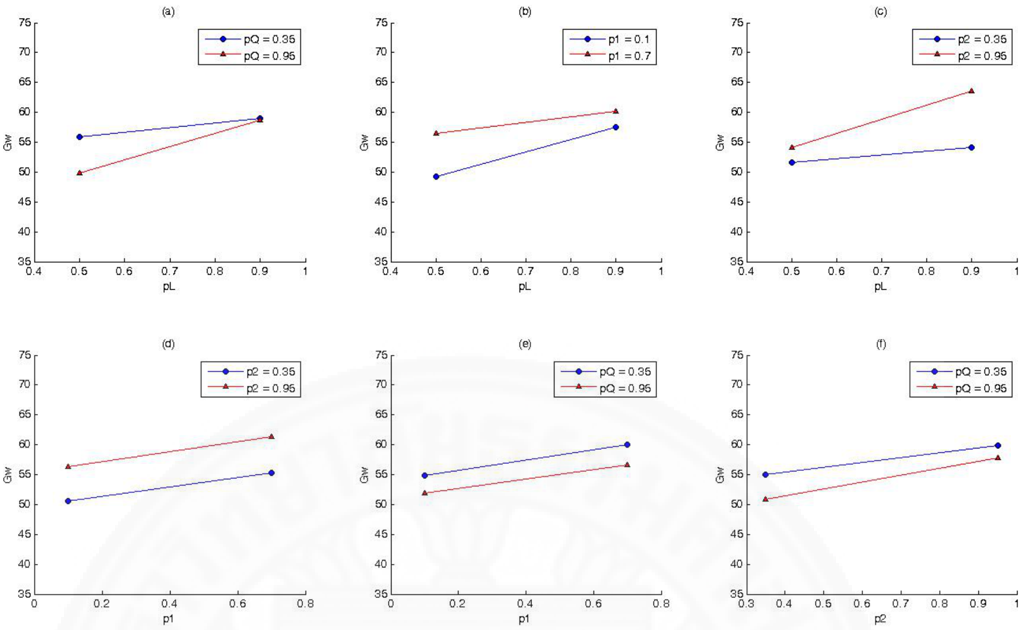


Figure 4.55: The weighted G-optimality criterion values in a spherical region for BBDs, $k = 3$ and $N = 14$

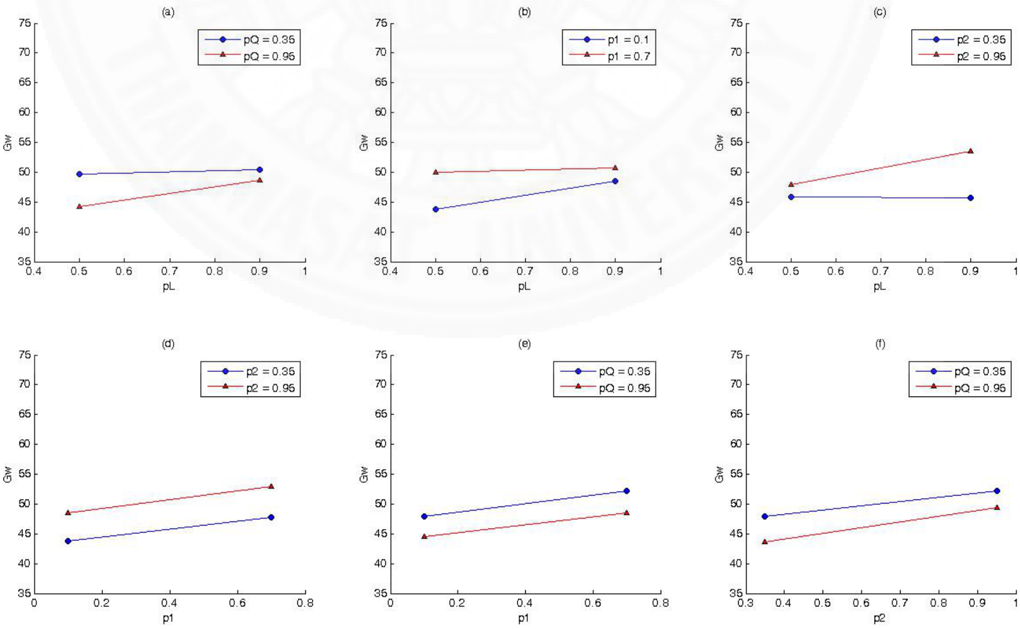


Figure 4.56: The weighted G-optimality criterion values in a spherical region for BBDs, $k = 3$ and $N = 17$

4.2.2 for $k = 4$ factors

The cuboidal BBDs

The boxplots of weighted D-optimality criterion values for the cuboidal BBDs slightly decrease when N increases (Figure 4.57). The values of D_w decrease when p_l or p_q increase, but with a larger decrease in D_w when increasing p_q. The decrease is similar when increasing p_l for both p_q levels which is reflected by the parallelism in Figure 4.61 (a) and Figure 4.62 (a). Thus D_w is more robust to changes in p_l than to change in p_q. The decrease in D_w when increasing p_1 or p_2 is larger when p_q is at its low level ($p_q = 0.35$) than when it is at its high level ($p_q = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.61 (e), (f) and Figure 4.62 (e), (f). However, the decrease in D_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in D_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in Figure 4.61 (b), (c) and Figure 4.62 (b), (c). The values of D_w decrease when p_1 or p_2 is increased, but with a larger decrease in D_w when increasing p_2. The decreases are similar when increasing p_1 for both p_2 levels which is reflected by the parallelism in Figure 4.61 (d) and Figure 4.62 (d). Thus D_w is more robust to change in p_1 than to change in p_2.

The boxplots of weighted G-optimality criterion values for the cuboidal BBDs slightly decrease when N increases (Figure 4.58). The decrease in G_w when increasing p_l is smaller when p_q is at its low level ($p_q = 0.35$) than when it is at its high level ($p_q = 0.95$). Also, the decrease in G_w when increasing p_2 is larger when p_q is at its low level ($p_q = 0.35$) than when it is at its high level ($p_q = 0.95$). This is reflected in the lack-of-parallelism. The value of G_w decreases when p_1 or p_q are increased, but with a larger decrease in G_w when increasing p_1. The decreases are similar when increasing p_1 for both p_q levels which is reflected by the parallelism. The G_w values when p_q is at its low level ($p_q = 0.35$) decrease more than when it is at its high level ($p_q = 0.95$), and after $N = 27$, the G_w values when p_q is at its low level ($p_q = 0.35$) are lower than when it is at its high level

($p_q = 0.95$). This pattern is shown in Figure 4.63 (a), (e), (f) and Figure 4.64 (a), (e), (f). However, the decrease in G_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). This is reflected in the lack-of-parallelism in Figure 4.63 (b) and Figure 4.64 (b). The presence of parallel lines indicates no interaction between pairs of (p_l, p_2) and (p_1, p_2) on G_w values. Similar patterns occur in all plots as N increases. Increasing p_l, p_1, and p_2 decreases the G_w values. This is shown in Figure 4.63 (c), (d) and Figure 4.64 (c), (d).

From the results for the cuboidal BBD design, the D_w values are robust to change in p_1, p_2, and p_q and slightly less robust as N increases. However, the D_w values are sensitive to change in p_l. The G_w values are slightly sensitive (weakly robust) to change in p_2 and p_q. However, the G_w values are very sensitive to change in p_l and p_1.

The spherical BBDs

The boxplots of weighted D-optimality criterion values for the spherical BBDs decrease when N increase (Figure 4.59). The value of D_w decreases when p_l is increased, but it increases when p_q is increased. The decrease is similar when increasing p_l for both p_q levels which is reflected by the parallelism in Figure 4.65 (a), (e), (f) and Figure 4.66 (a), (e), (f). Thus D_w is more robust to change in p_l, p_1, and p_2 than to change in p_q. However, the decrease in D_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in D_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in Figure 4.65 (b), (c) and Figure 4.66 (b), (c). The values of D_w decrease when p_1 or p_2 increases, but with a larger decrease in D_w when increasing p_2. The decreases are similar when increasing p_1 for both p_2 levels which is reflected by the parallelism in Figure 4.65 (d) and Figure 4.66 (d). Thus D_w is more robust to change in p_1 than to change in p_2.

The boxplots of weighted G-optimality criterion values for the spherical

BBDs increase from $N = 25$ to $N = 26$, but the G_w values decrease slowly after $N = 26$ (Figure 4.60). The effect of changing p_l or p_q on G_w depends on the size of the BBDs. The G_w values increase across each combination of p_q, p_1, and p_2 when p_l increases. The rate of change in G_w for (p_l, p_q) and (p_l, p_2) at the low level of p_l is smaller than the high level of p_l. The rate of change in G_w for (p_l, p_1) at the low level of p_l is larger than the high level of p_l. This is reflected in the lack-of-parallelism in Figure 4.67 (a), (b), (c) and Figure 4.68 (a), (b), (c). The presence of parallel lines indicates no interaction between pairs of (p_1, p_2), (p_1, p_q) and (p_2, p_q) probabilities on G_w values. Similar patterns occur in all plots as N increases, and G_w values increase. When $N = 25$, the G_w values increase when p_1 or p_2 increase, but decrease when p_q increases. However, after $N = 25$ the G_w values increase when p_1, p_2, and p_q increase. This is shown in Figure 4.67 (d), (e), (f) and Figure 4.68 (d), (e), (f).

From the results for the spherical BBD design, the D_w values are robust to change in p_l and p_q, and slightly less robust as N increase. However, the D_w values are slightly sensitive to change in p_1 and p_2. The G_w values are slightly sensitive (weakly robust) to change in p_q. However, the G_w values are very sensitive to change in p_l, p_1 and p_2.

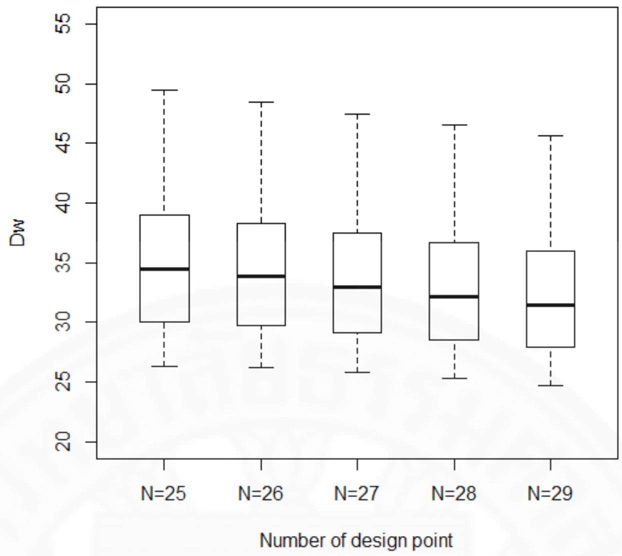


Figure 4.57: Boxplot for the weighted D-optimality criterion values in cuboidal region for BBDs, $k = 4$

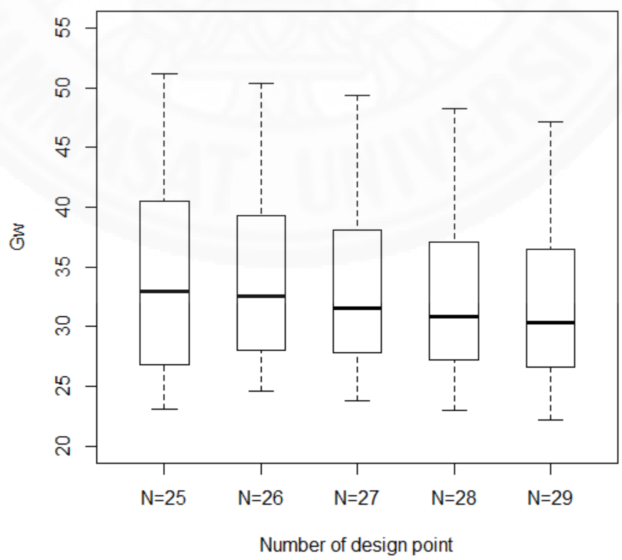


Figure 4.58: Boxplot for the weighted G-optimality criterion values in cuboidal region for BBDs, $k = 4$

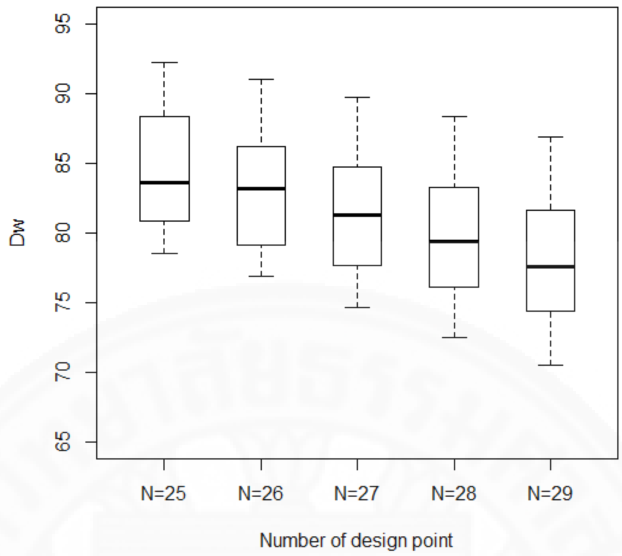


Figure 4.59: Boxplot for the weighted D-optimality criterion values in spherical region for BBDs, $k = 4$

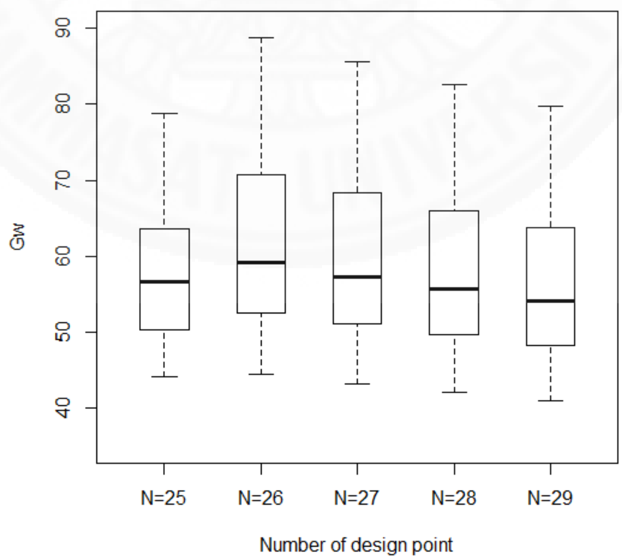


Figure 4.60: Boxplot for the weighted G-optimality criterion values in spherical region for BBDs, $k = 4$

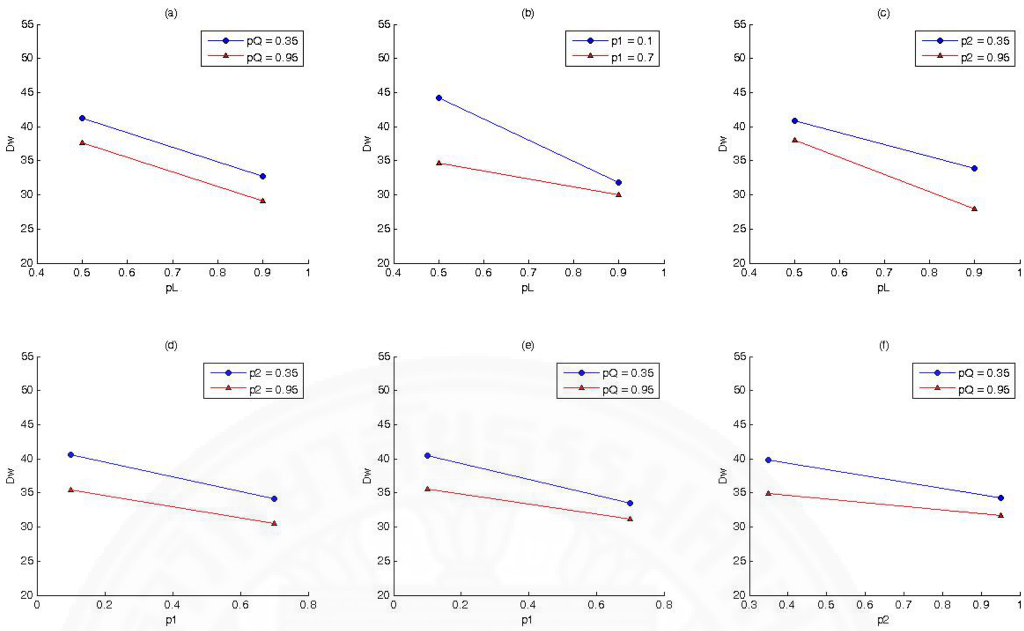


Figure 4.61: The weighted D-optimality criterion values in cuboidal region for BBDs, $k = 4$ and $N = 25$

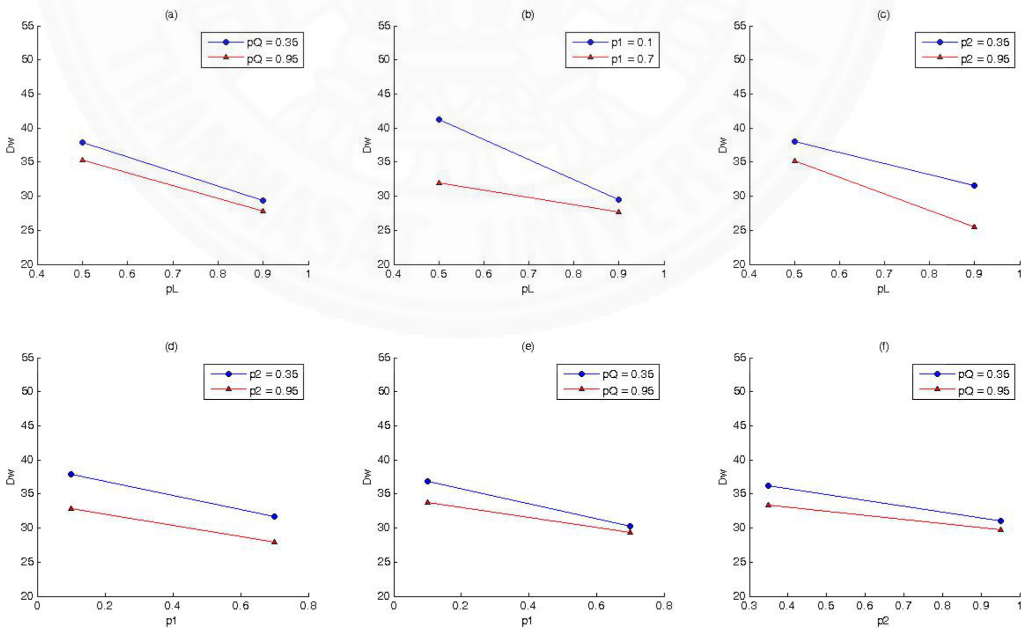


Figure 4.62: The weighted D-optimality criterion values in cuboidal region for BBDs, $k = 4$ and $N = 29$

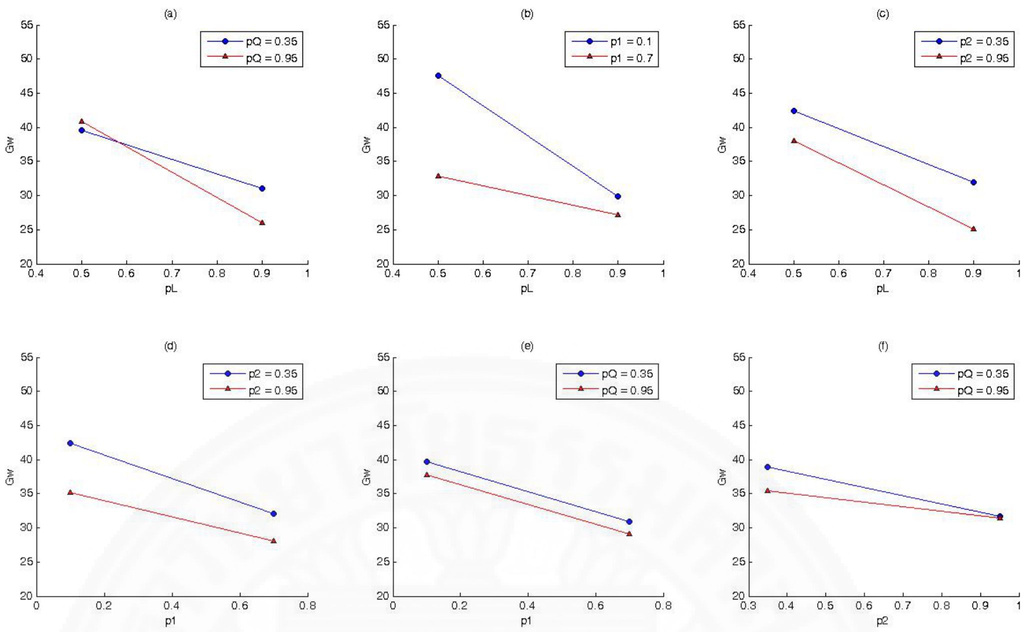


Figure 4.63: The weighted G-optimality criterion values in cuboidal region for BBDs, $k = 4$ and $N = 25$

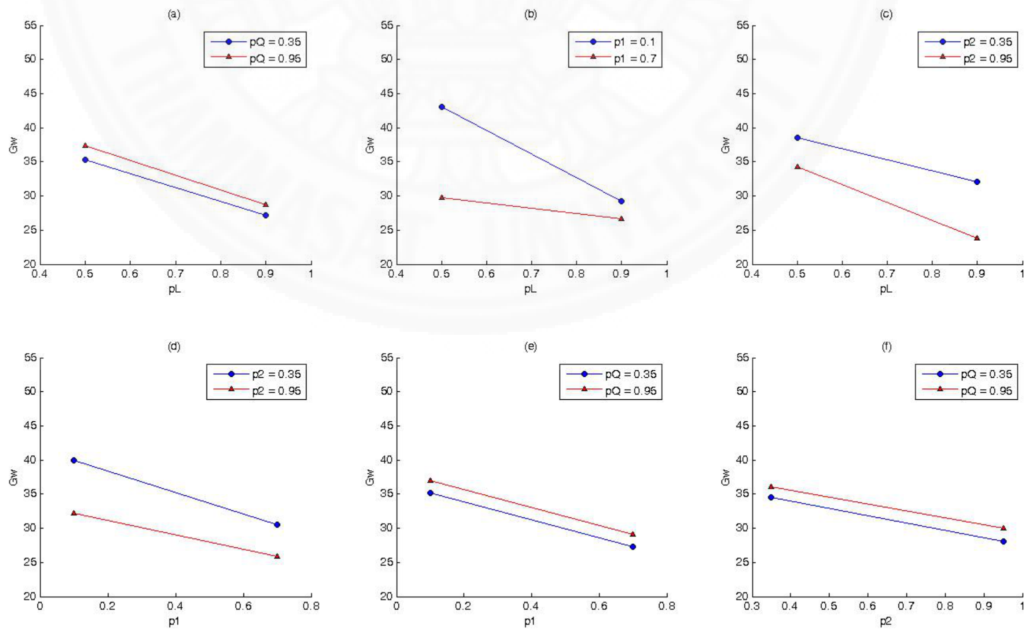


Figure 4.64: The weighted G-optimality criterion values in cuboidal region for BBDs, $k = 4$ and $N = 29$

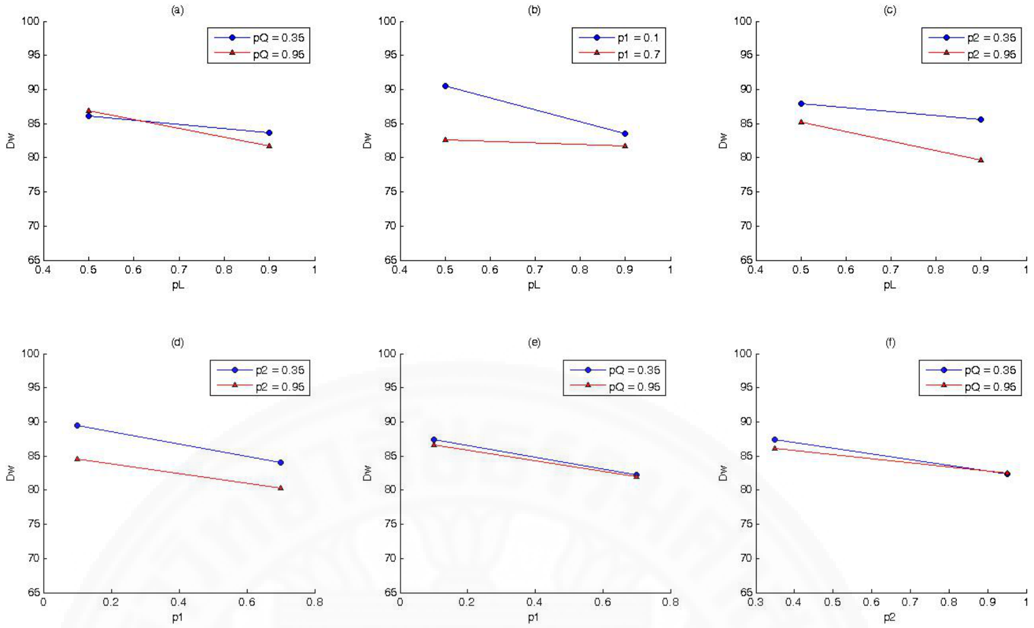


Figure 4.65: The weighted D-optimality criterion values in spherical region for BBDs, $k = 4$ and $N = 25$

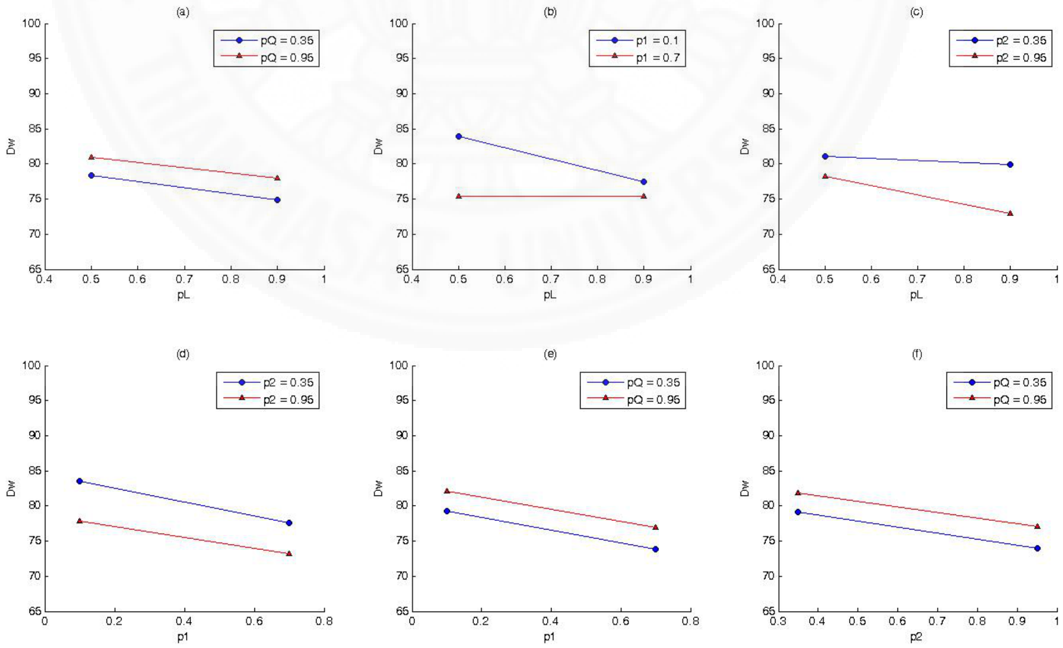


Figure 4.66: The weighted D-optimality criterion values in spherical region for BBDs, $k = 4$ and $N = 29$

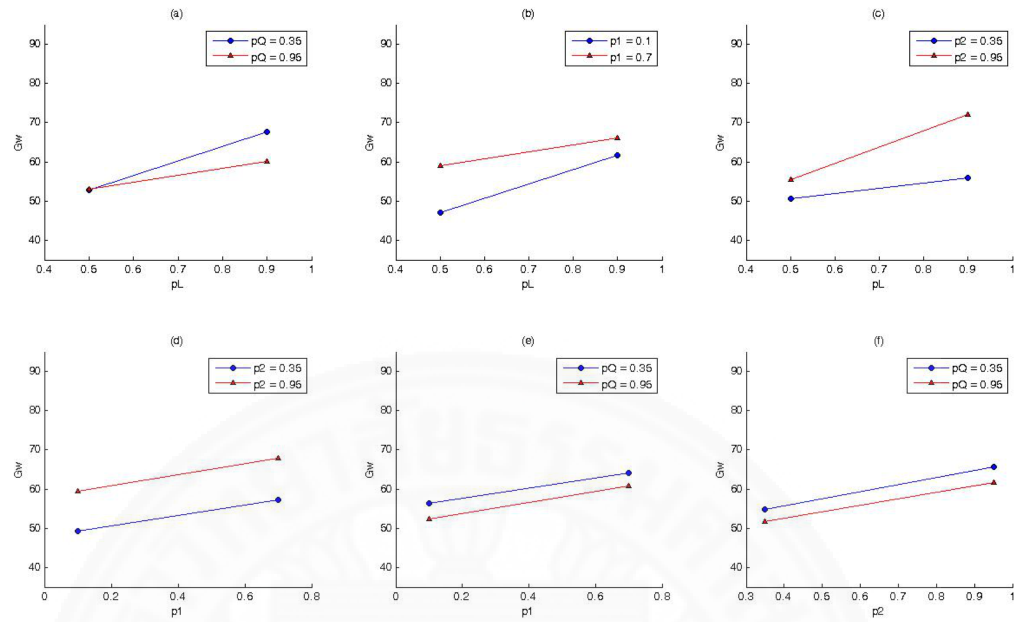


Figure 4.67: The weighted G-optimality criterion values in spherical region for BBDs, $k = 4$ and $N = 25$

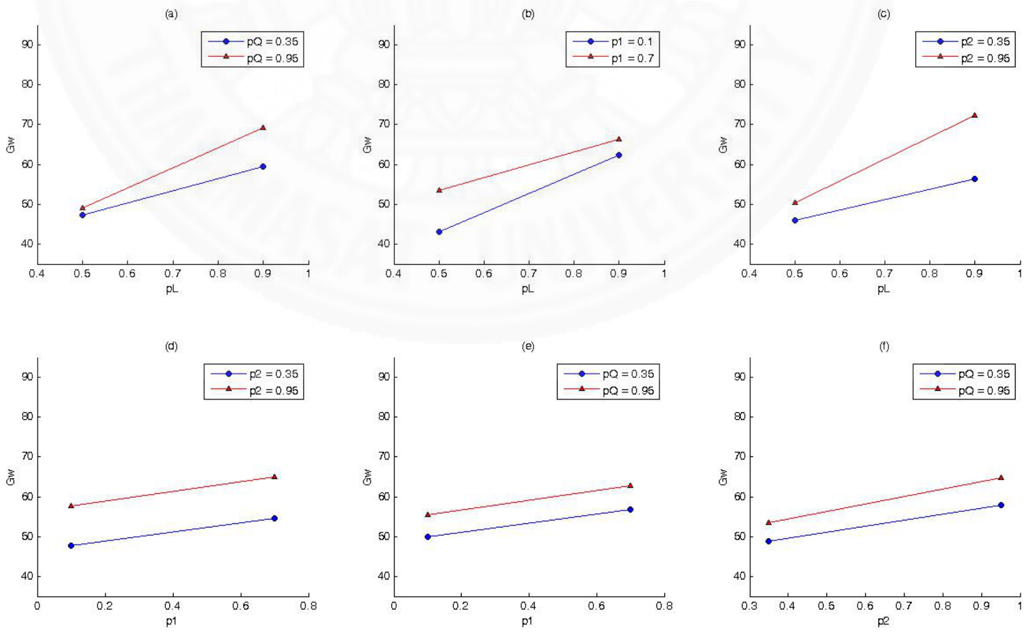


Figure 4.68: The weighted G-optimality criterion values in spherical region for BBDs, $k = 4$ and $N = 29$

4.3 The results of the weighted D and G-optimality criteria values for computer generated designs

4.3.1 for $k = 2$ factors

The boxplots of weighted D-optimality criterion values for the cuboidal region for generated by OPTEX procedure in SAS (light shade) are almost equal, but for the GA designs (dark shade) there is slight increase when N increases. The D_w values of GA designs are higher than OPTEX designs. This is shown in Figure 4.69.

D_w for OPTEX procedure designs in the cuboidal region

The presence of parallel lines in Figure 4.73 and 4.74 indicates no interaction between pairs of p_i probabilities on D_w values. There are similar patterns in all plots as N increases, and weighted D-optimality criterion values decrease. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The decrease in D_w when increasing p_q is larger than increasing p_1 and p_2. This is shown in Figure 4.73 (a), (e), (f) and Figure 4.74 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger than increasing p_1 or p_2. This is reflected by the nearly parallel lines in Figure 4.73 (b), (c) and Figure 4.74 (b), (c). There are small decreases in D_w when increasing p_1 or p_2. The decreases are similar which is reflected by the clear horizontal parallelism in Figure 4.73 (d), (e), (f) and Figure 4.74 (d), (e), (f). Thus D_w is quite robust to changes in p_1 or p_2.

D_w for GA designs in the cuboidal region

The presence of parallel lines in Figure 4.75 and 4.76 indicates no interaction between pairs of p_i probabilities on D_w values. There are similar patterns in all plots as N increases, and weighted D-optimality criterion values decrease. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The decrease in D_w when increasing p_q is larger than increasing p_1

and p_2. This is shown in Figure 4.75 (a), (e), (f) and Figure 4.76 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger than increasing p_1 or p_2, and the G_w values are almost equal for both levels of p_1 and p_2. This is reflected in the clear parallelism in Figure 4.75 (b), (c) and Figure 4.76 (b), (c). There is a little increase in D_w when increasing p_1 or p_2. The increases are similar which is reflected by the clear parallelism in Figure 4.75 (d), (e), (f) and Figure 4.76 (d), (e), (f). Thus D_w is quite robust to changes in p_1 or p_2.

The boxplots of weighted G-optimality criterion values for the cuboidal region for both OPTEX designs (light shade) and GA designs (dark shade) slightly increase when N increases, but it is highest at $N = 9$ and then decreased after $N = 9$. The G_w values of GA designs are higher than OPTEX designs. This is shown in Figure 4.70.

G_w for OPTEX procedure designs in the cuboidal region

The G_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases, but the presence of parallel lines in Figure 4.77 and 4.78 indicates no interaction between pairs of all p_i probabilities on G_w values when $N \geq 9$. This is shown in Figure 4.77 (a), (e), (f), Figure 4.78 (a), (e), (f) and Figure 4.79 (a), (e), (f). However, the decrease in G_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in G_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.77 (b), (c), Figure 4.78 (b), (c) and Figure 4.79 (b), (c). When $N < 9$, the values of G_w decrease when p_1 increased, but G_w value increase when p_2 increased. For $N \geq 9$, the values of G_w decrease when p_1 or p_2 increased. The decreases are similar when increasing p_1 for both p_2 levels which is reflected by the parallelism in Figure 4.77 (d), Figure 4.78 (d) and Figure 4.79 (d).

G_w for GA designs in the cuboidal region

The G_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. For $N = 6$, the G_w values decrease when p_q increases, but for $N \geq 7$ G_w values increase when p_q increase. The presence of parallel lines when $N \geq 9$ indicates no interaction between pairs of p_i probabilities on G_w values. This is shown in Figure 4.80 (a), (e), (f), Figure 4.81 (a), (e), (f) and Figure 4.82 (a), (e), (f). However, the decrease in G_w when increasing p_l is larger than when increasing p_1 or p_2. For $N = 9$, the decrease in G_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in G_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.80 (b), (c), Figure 4.81 (b), (c) and Figure 4.82 (b), (c). The values of G_w slightly decrease when p_1 increases, but G_w values increase when p_2 increases. For $N = 9$, the values of G_w decrease when p_1 or p_2 increases. The decreases are similar when increasing p_1 for both p_2 levels which is reflected by the parallelism in Figure 4.80 (d), Figure 4.81 (d) and Figure 4.82 (d).

For the cuboidal computer generated designs (OPTEX procedure and GAs), the results are the same for the OPTEX designs and GA designs. The D_w values are robust to change in p_1 and p_2, and slightly less robust as N increases. However, the D_w values are sensitive to change in p_l and p_q. The G_w values are slightly sensitive (weakly robust) to change in p_1, p_2 and p_q, but are sensitive to p_q for $N \geq 9$. However, the G_w values are very sensitive to change in p_l.

The boxplots of weighted D-optimality criterion values for the spherical region for GA designs (dark shade) are very similar for all N. While the D_w values for the OPTEX designs (light shade) decrease at $N = 9$ but increase after $N = 9$. The D_w values of GA designs are higher than for OPTEX designs. This is shown in Figure 4.71.

D_w for OPTEX procedure designs in the spherical region

The presence of near parallel lines in Figure 4.83 and 4.84 indicates no interaction between pairs of p_i probabilities on D_w values. Similar patterns occur in all plots as N increases, and weighted D-optimality criterion values decrease. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The decrease in D_w when increasing p_l is larger than when increasing p_1 and p_2. This is shown in Figure 4.83 (a), (e), (f) and Figure 4.84 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in D_w when increasing p_l is similar when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.83 (b), (c) and Figure 4.84 (b), (c). There is only a small decrease in D_w when increasing p_1 or p_2. The decreases are similar which is reflected by the clear parallelism in Figure 4.83 (d) and Figure 4.84 (d). Thus D_w is quite robust to changes in p_1 or p_2.

D_w for GA designs in the spherical region

The presence of near parallel lines in Figure 4.85 and 4.86 indicates no interaction between pairs of p_i probabilities on D_w values. Similar patterns occur in all plots as N increases, and weighted D-optimality criterion values decrease. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The decrease in D_w when increasing p_l is larger than increasing p_1 and p_2. This is shown in Figure 4.85 (a), (e), (f) and Figure 4.86 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in D_w when increasing p_l is similar when p_2 is at its low level ($p_2 = 0.35$) and when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.85 (b), (c) and Figure 4.86 (b), (c). There is only a small decrease in D_w when increasing p_1 or p_2. The decreases are similar which is reflected by the near parallelism in Figure 4.85 (d) and Figure 4.86 (d). Thus D_w is quite robust to changes in p_1 or p_2.

The boxplots of weighted G-optimality criterion values for the spherical region for OPTEX designs (light shade) have the wave shape with the G_w values lowest at $N = 9$. The G_w values for GA designs (dark shade) is slight increase when N increase. The G_w values of GAs is higher than OPTEX procedure. This is shown in Figure 4.72.

G_w for OPTEX procedure designs in the spherical region

The effect of changing p_l or p_q on G_w depends on the size of the OPTEX procedure design. When the number of points increases from $N = 7$ to $N = 10$, the positive effect of increasing p_l on G_w at the high level of p_q becomes smaller until there is almost no effect on G_w. However, there is an increasing negative effect on G_w when increasing p_l at the low level of p_q as the number of points increases from $N = 7$ to $N = 10$. An example can be seen in the comparison of the plots in Figure 4.88 (a) and Figure 4.89 (a). An exception occurs when changing from 6 to 7 points. If one additional number point is added to the OPTEX procedure design ($N = 6, 7$), the interaction effect between p_l and p_q on G_w is reversed. This can be seen in Figures 4.87 (a) and Figure 4.88 (a).

The rate of change in G_w when increasing p_l is smaller when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the rate of change in G_w when increasing p_l is larger when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). For $N = 6$, G_w values decrease, and when $N > 6$, G_w values increase and then gradually decreases as N increases. This is reflected in the lack-of-parallelism in the Figure 4.87 (b), (c), Figure 4.88 (b), (c) and Figure 4.89 (b), (c).

The G_w values increase as p_1, p_2, and N increase. For $N = 6$, G_w values decrease when p_q increases. For $N > 6$, G_w values increase when p_q increases for all levels of p_1 and p_2. In the case of the pair p_1 and p_q, values of G_w changed in a parallel manner. That is, the parallel lines indicates no interaction between pairs of p_1 and p_q on G_w values. Also, the increase in G_w when increasing p_q is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.87 (e), (f), Figure 4.88 (e), (f) and Figure 4.89 (e), (f). There is a small increase in G_w when increasing p_1 or p_2. The increases are similar which is reflected by the clear parallelism in Figure 4.87 (d) to Figure 4.89 (d). Thus G_w is quite robust to changes in p_1 or p_2.

G_w for GA designs in the spherical region

When p_l increase, the G_w values decrease as p_q is at its low level ($p_q = 0.35$), but it increase as p_q is at its high level ($p_q = 0.95$) and the G_w values decrease as N increase. It shows in Figure 4.90 (a), Figure 4.91 (a), and Figure 4.92 (a).

The rate of change in G_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the rate of change in G_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). For $N = 6$, G_w values increase, and when $N > 6$, G_w values decrease and then gradually increase as N increases. This is reflected in the lack-of-parallelism in the Figure 4.90 (b), (c), Figure 4.91 (b), (c) and Figure 4.92 (b), (c).

The G_w values increase as p_1, p_2, and N are increase. For $N = 6$, G_w values increase when p_q increases. For $N > 6$, G_w values decrease when p_q increases for all level of p_1 and p_2 except p_2 is at its high level ($p_2 = 0.95$). In the case of the pair p_1 and p_q, values of G_w has changed in a parallel manner. That is, the parallel lines indicates no interaction between pairs of p_1 and p_q on G_w values. When $N = 6$, the increase in G_w when increasing p_q is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$), but $N > 6$ the G_w values increase when p_q is at its high level ($p_q = 0.95$) and it slightly decrease when p_q is at its low level ($p_q = 0.35$). This is reflected in the lack-of-parallelism in the Figure 4.90 (e), (f), Figure 4.91 (e), (f) and Figure 4.92 (e), (f). There is a small increase in G_w when increasing p_1 or p_2. The increases are similar which is reflected by the clear parallelism in Figure 4.90 (d) to Figure 4.92 (d). Thus G_w

is quite robust to changes in p_1 or p_2.

For the spherical computer generated designs (OPTEX procedure and GAs), the results are similar for OPTEX designs and GA designs. The D_w values are robust to change in p_1 and p_2. However, the D_w values are sensitive to change in p_l and p_q. For the OPTEX designs, the G_w values are slightly sensitive (weakly robust) to change in p_1, p_2 and p_q. However, the G_w values are very sensitive to change in p_l. In case of GA design, the G_w values are slightly sensitive (weakly robust) to change in p_1 and p_2. However, the G_w values are very sensitive to change in p_l and p_q.

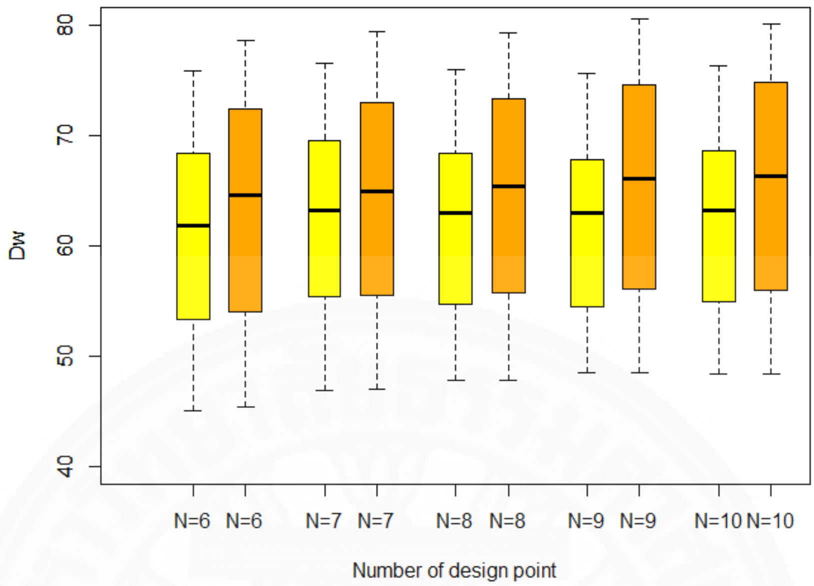


Figure 4.69: Boxplot for the weighted D-optimality criterion values in cuboidal region for OPTEX designs (light shade) and GA designs (dark shade), $k = 2$

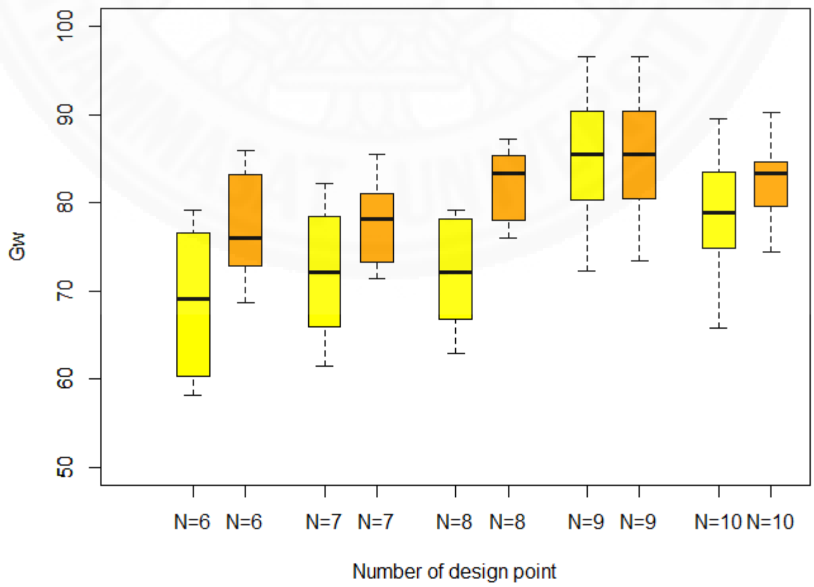


Figure 4.70: Boxplot for the weighted G-optimality criterion values in cuboidal region for OPTEX designs (light shade) and GA designs (dark shade), $k = 2$

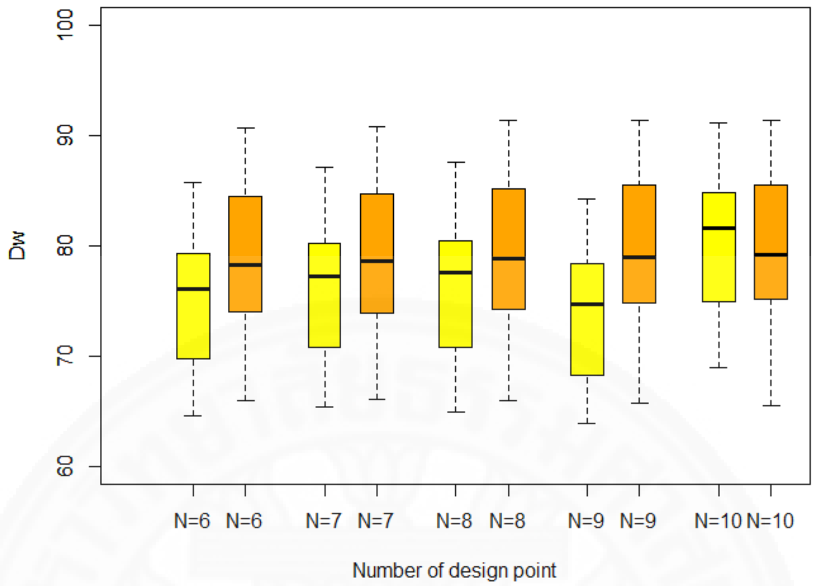


Figure 4.71: Boxplot for the weighted D-optimality criterion values in spherical region for OPTEX designs (light shade) and GA designs (dark shade), $k = 2$

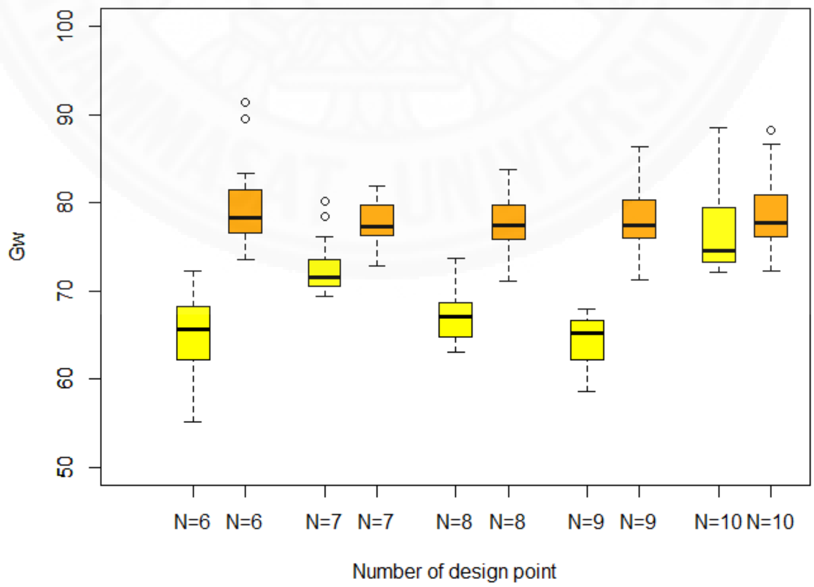


Figure 4.72: Boxplot for the weighted G-optimality criterion values in spherical region for OPTEX designs (light shade) and GA designs (dark shade), $k = 2$

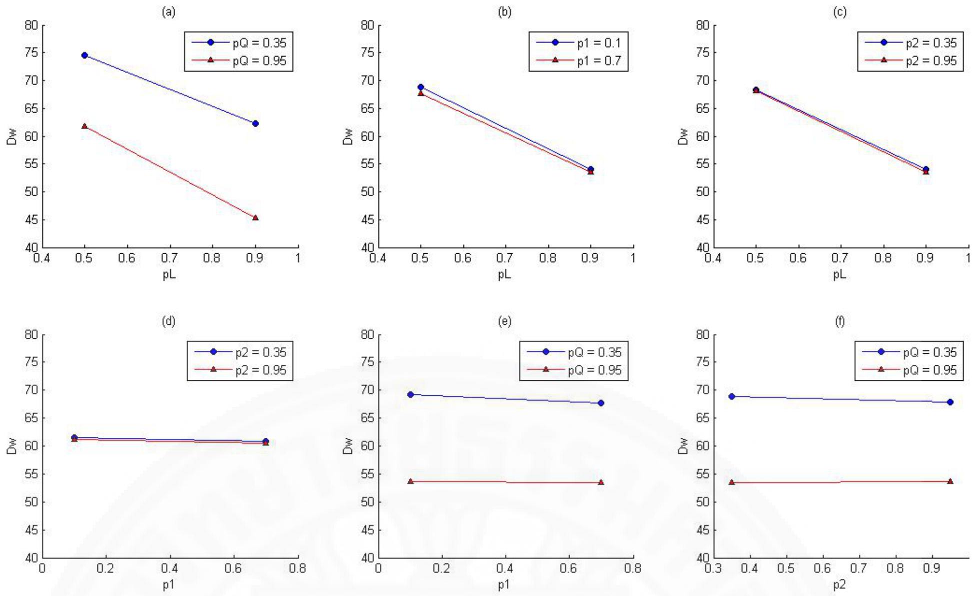


Figure 4.73: The weighted D-optimality criterion values in cuboidal region for OPTEX procedure designs for $k = 2$ and $N = 6$

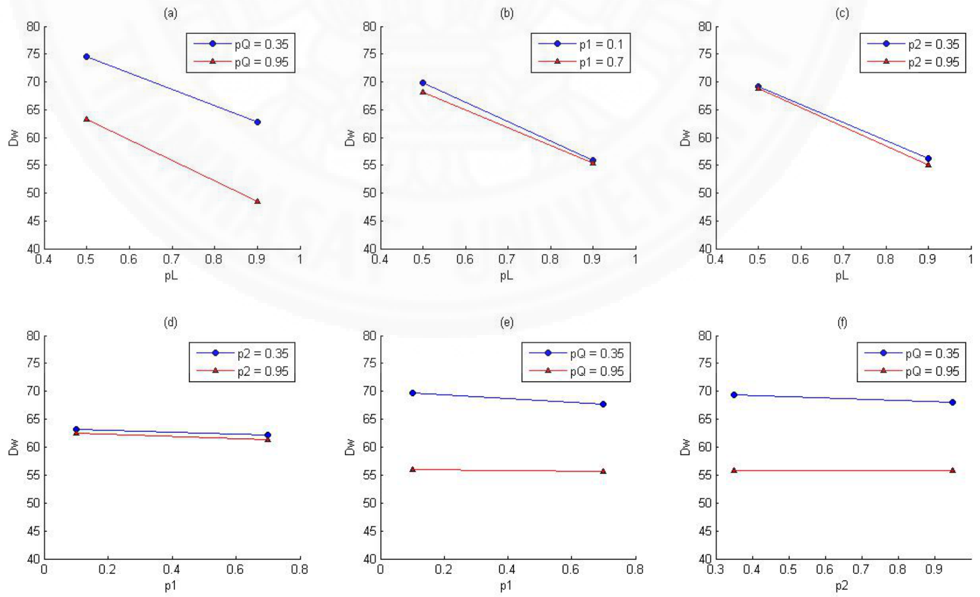


Figure 4.74: The weighted D-optimality criterion values in cuboidal region for OPTEX procedure designs for $k = 2$ and $N = 10$

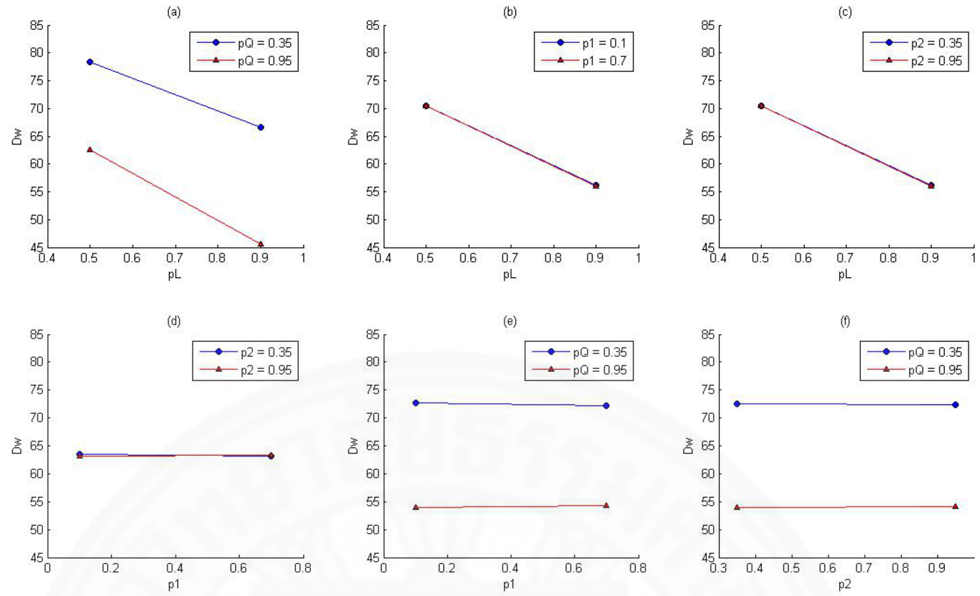


Figure 4.75: The weighted D-optimality criterion values in cuboidal region for GA designs for $k = 2$ and $N = 6$

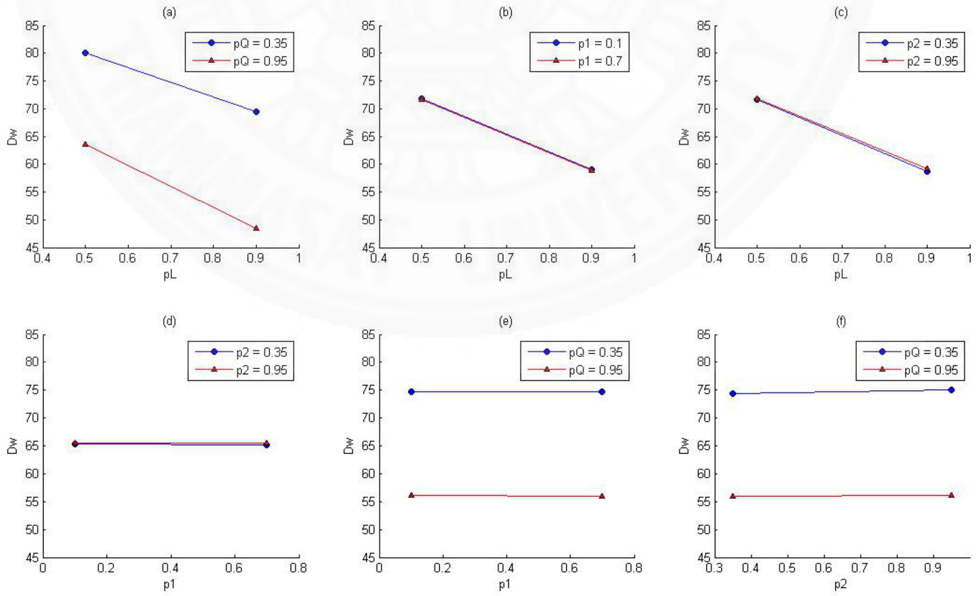


Figure 4.76: The weighted D-optimality criterion values in cuboidal region for GA designs for $k = 2$ and $N = 10$

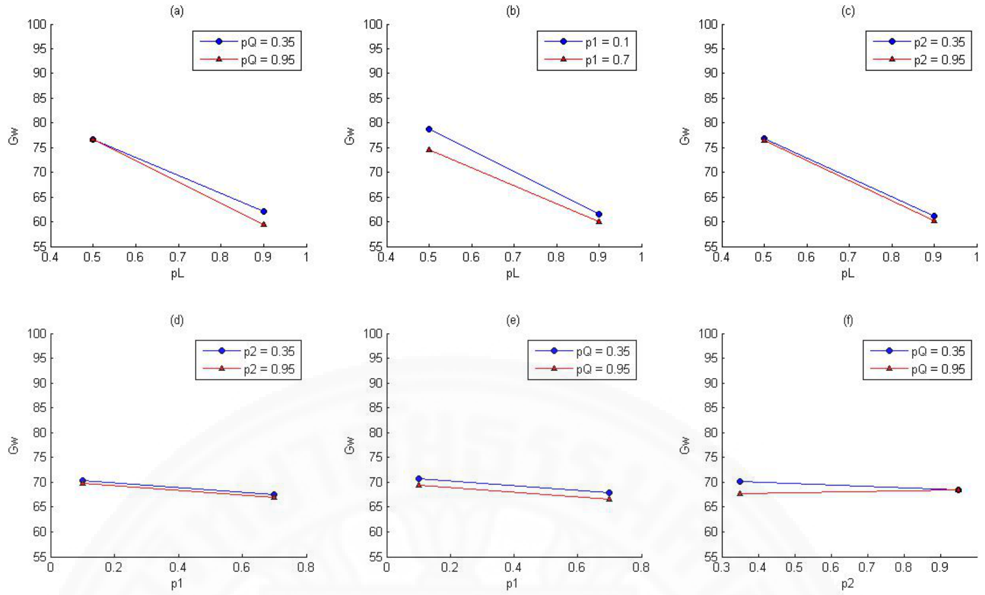


Figure 4.77: The weighted G-optimality criterion values in cuboidal region for OPTEX procedure designs for $k = 2$ and $N = 6$

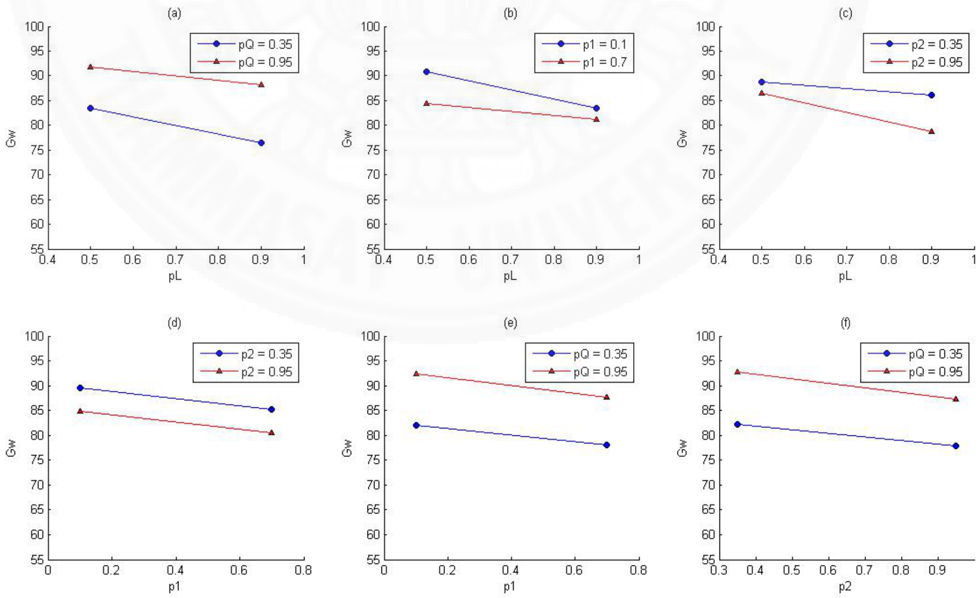


Figure 4.78: The weighted G-optimality criterion values in cuboidal region for OPTEX procedure designs for $k = 2$ and $N = 9$

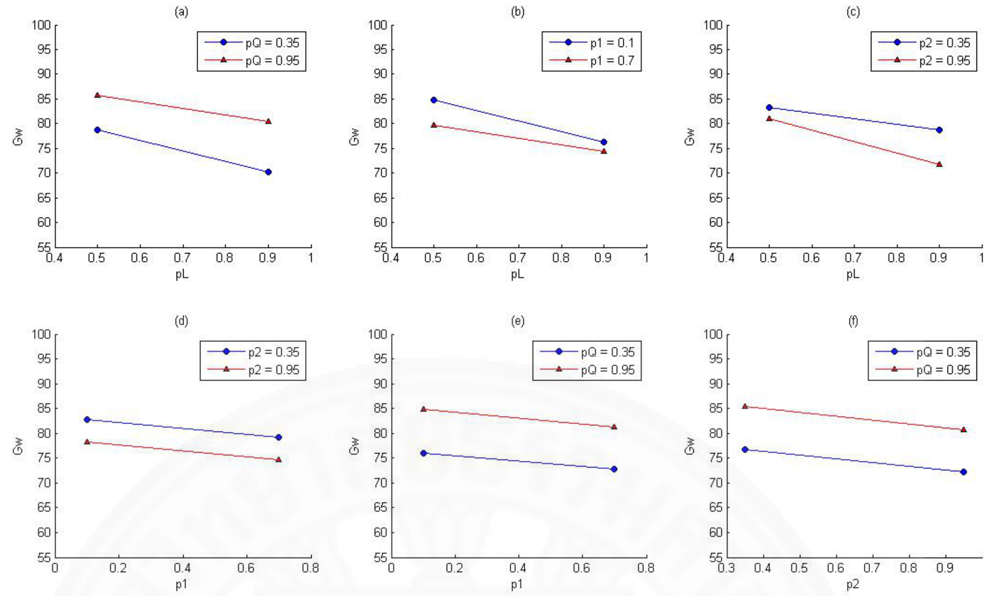


Figure 4.79: The weighted G-optimality criterion values in cuboidal region for OPTEX procedure designs for $k = 2$ and $N = 10$

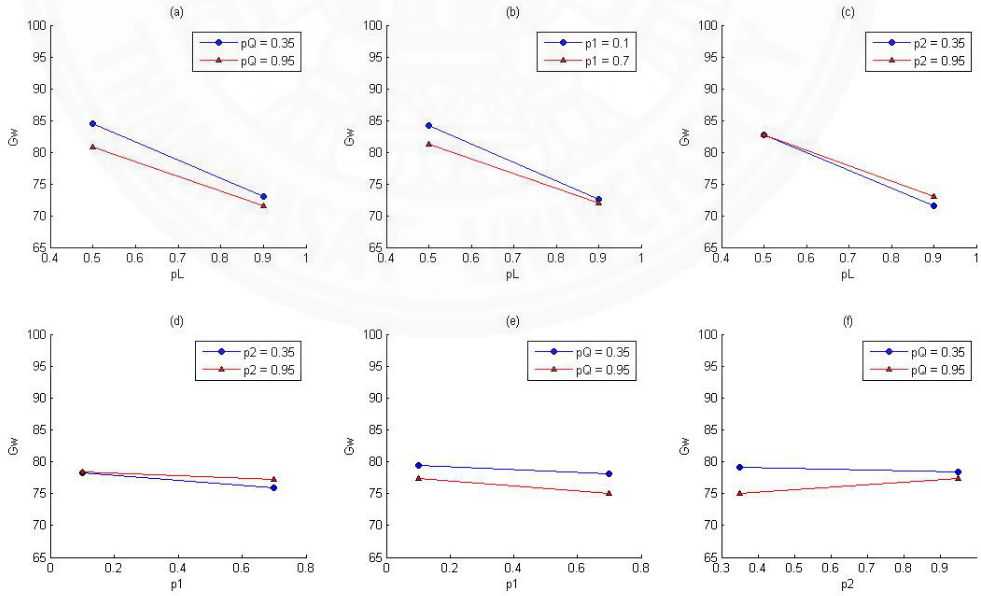


Figure 4.80: The weighted G-optimality criterion values in cuboidal region for GA designs for $k = 2$ and $N = 6$

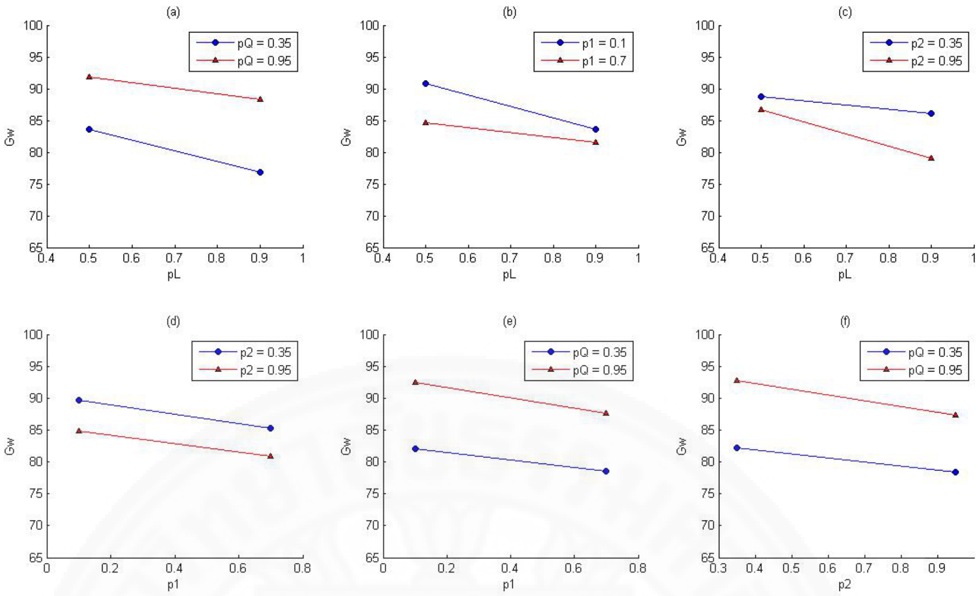


Figure 4.81: The weighted G-optimality criterion values in cuboidal region for GA designs for $k = 2$ and $N = 9$

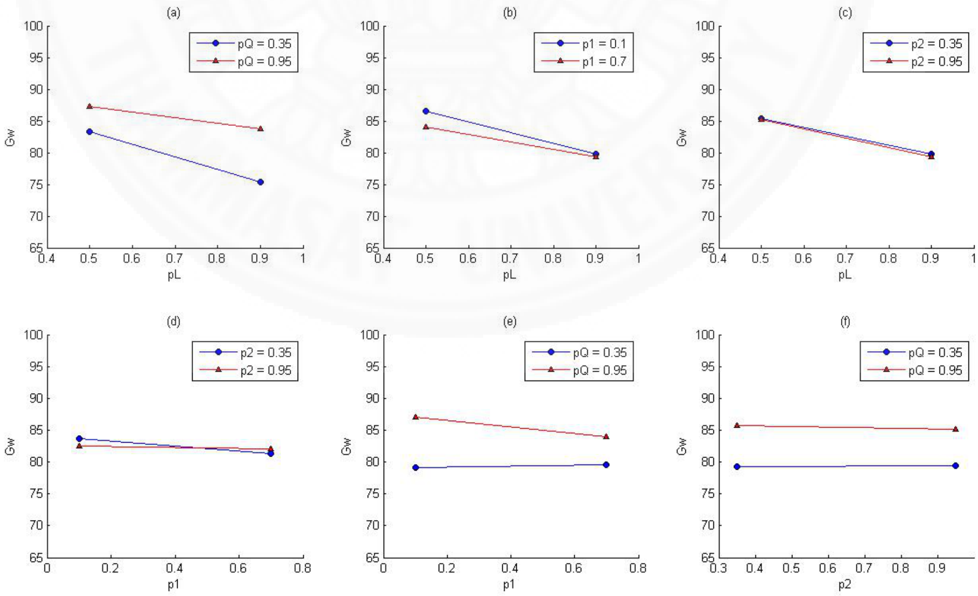


Figure 4.82: The weighted G-optimality criterion values in cuboidal region for GA designs for $k = 2$ and $N = 10$

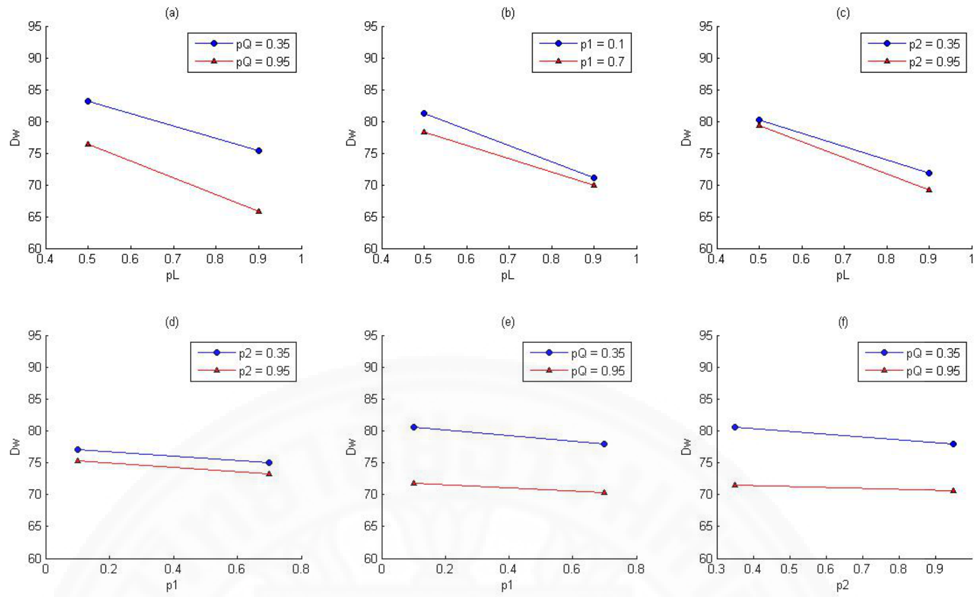


Figure 4.83: The weighted D-optimality criterion values in spherical region for OPTEX procedure designs for $k = 2$ and $N = 6$

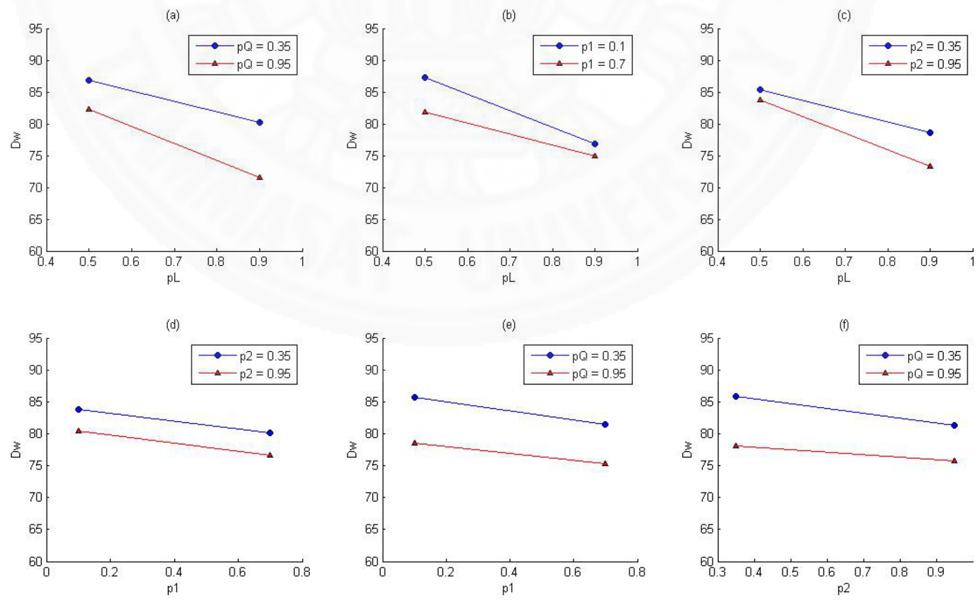


Figure 4.84: The weighted D-optimality criterion values in spherical region for OPTEX procedure designs for $k = 2$ and $N = 10$

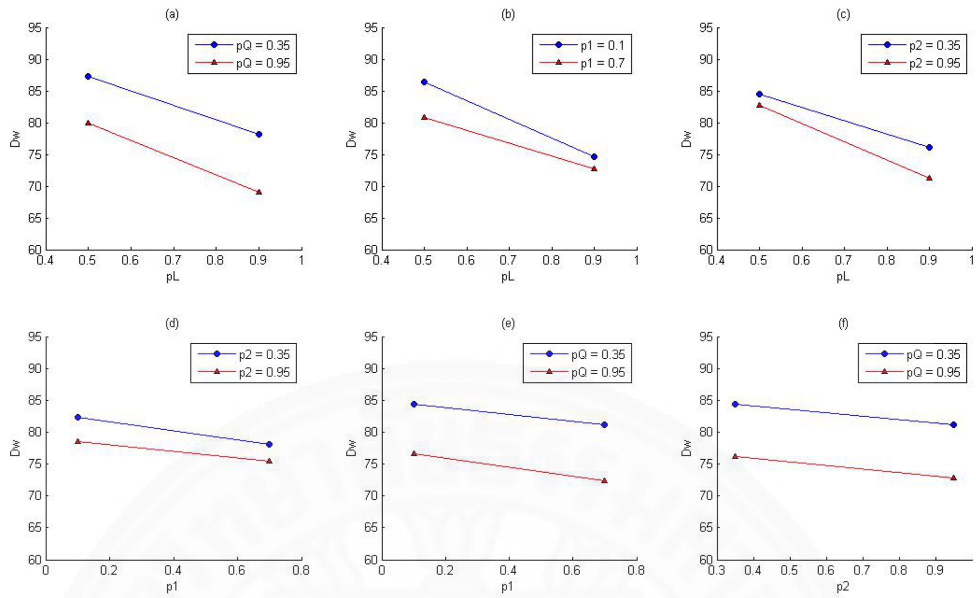


Figure 4.85: The weighted D-optimality criterion values in spherical region for GA designs for $k = 2$ and $N = 6$

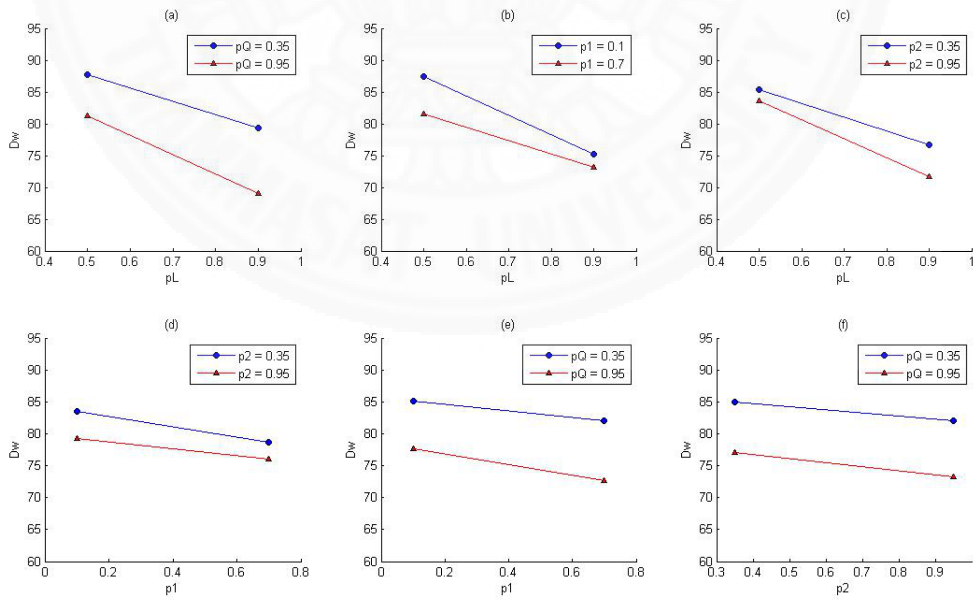


Figure 4.86: The weighted D-optimality criterion values in spherical region for GA designs for $k = 2$ and $N = 10$

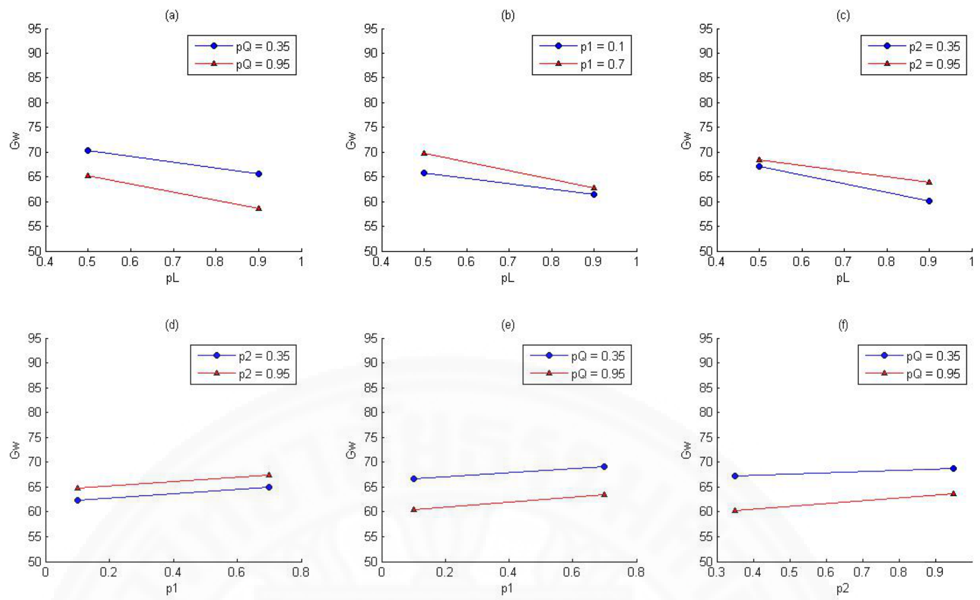


Figure 4.87: The weighted G-optimality criterion values in spherical region for OPTEX procedure designs for $k = 2$ and $N = 6$

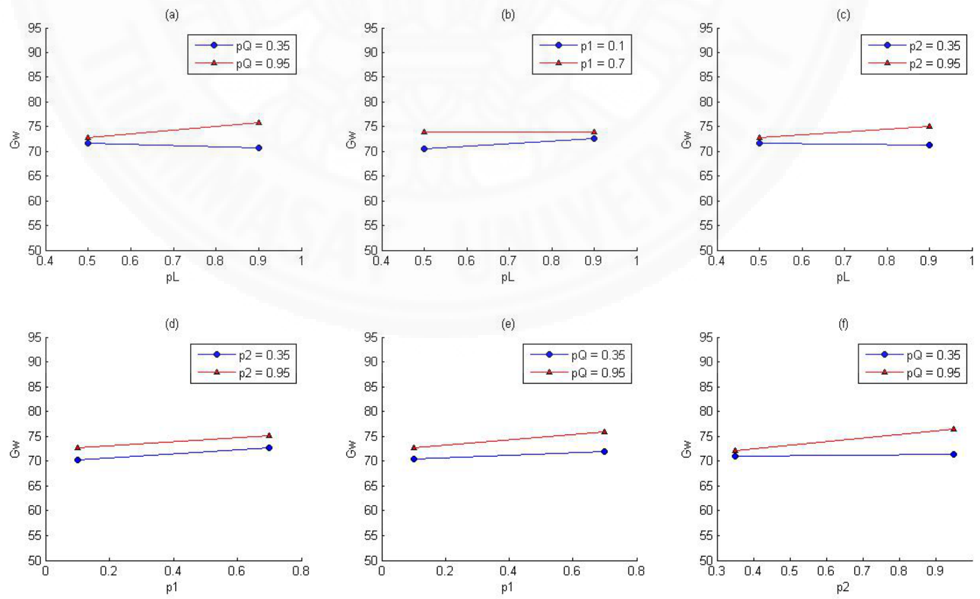


Figure 4.88: The weighted G-optimality criterion values in spherical region for OPTEX procedure designs for $k = 2$ and $N = 7$

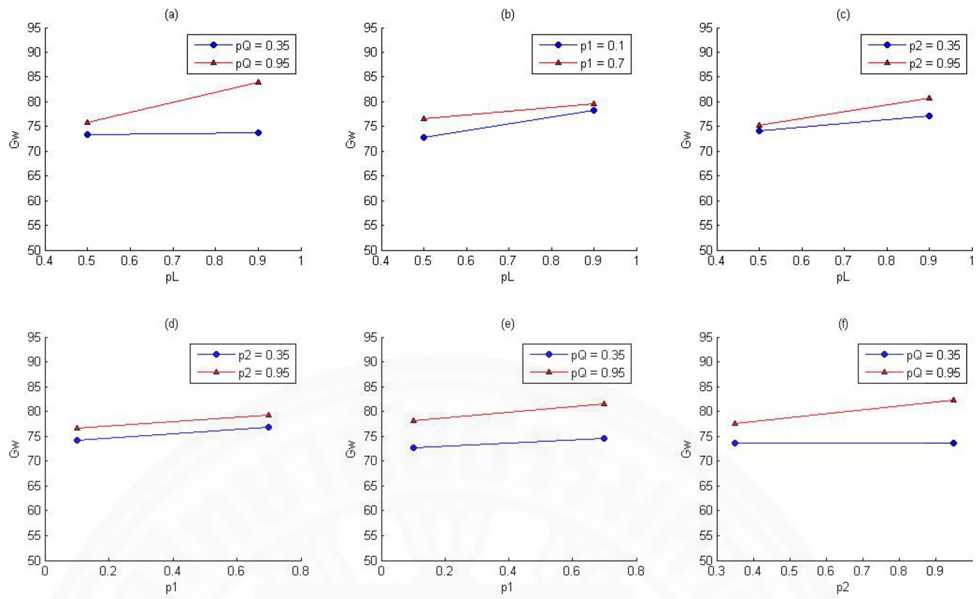


Figure 4.89: The weighted G-optimality criterion values in spherical region for OPTEX procedure designs for $k = 2$ and $N = 10$

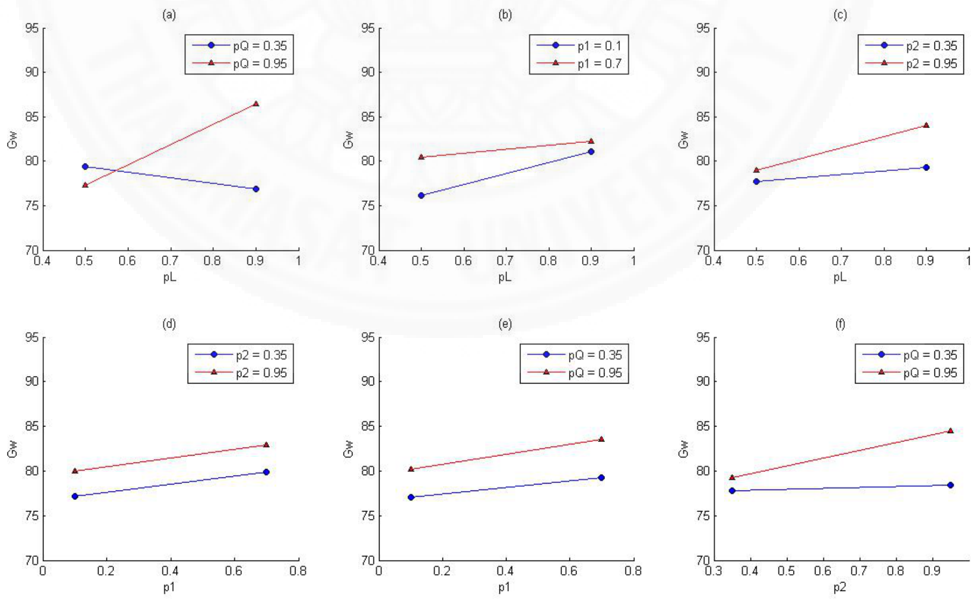


Figure 4.90: The weighted G-optimality criterion values in spherical region for GA designs for $k = 2$ and $N = 6$

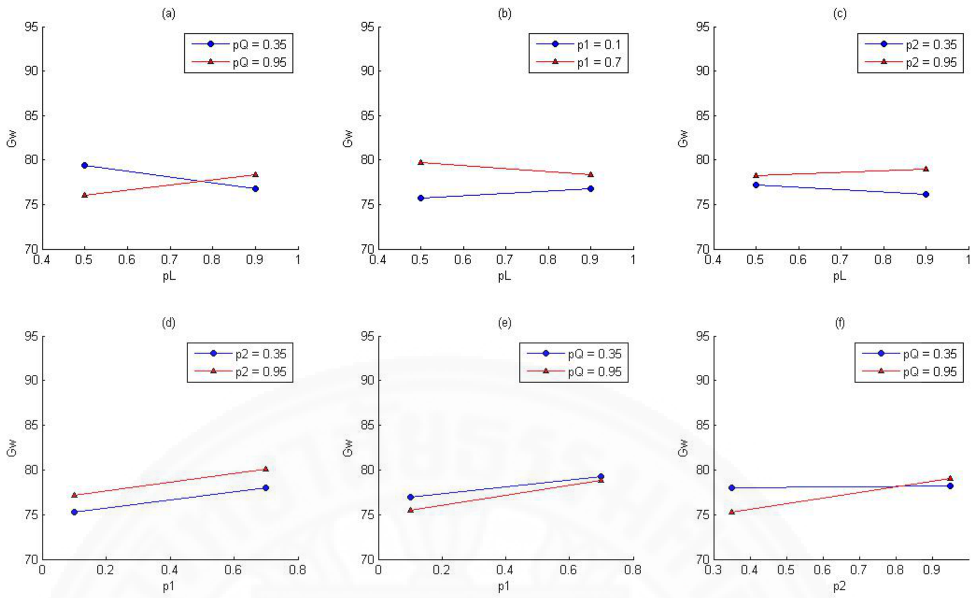


Figure 4.91: The weighted G-optimality criterion values in spherical region for GA designs for $k = 2$ and $N = 7$

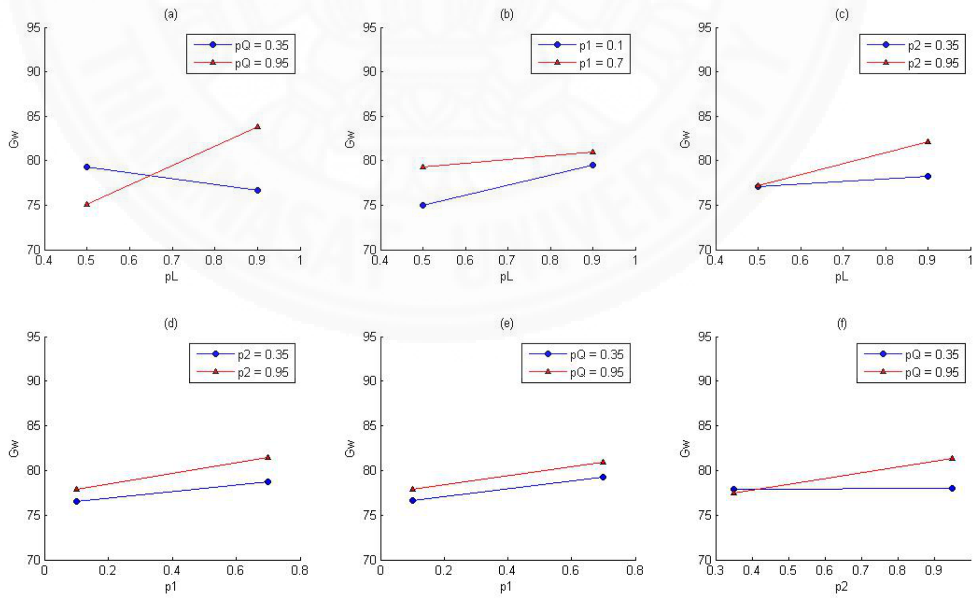


Figure 4.92: The weighted G-optimality criterion values in spherical region for GA designs for $k = 2$ and $N = 10$

4.3.2 for $k = 3$ factors

The boxplots of weighted D-optimality criterion values for the cuboidal region for OPTEX designs (light shade) indicate a slight increase from $N = 10$ to $N = 11$, and are then decrease slightly for $N \geq 11$. The boxplots for the GA designs (dark shade) are almost equal for all N. The D_w values for GA designs are higher than for OPTEX designs. This is shown in Figure 4.93.

D_w for OPTEX procedure designs in the cuboidal region

The presence of parallel lines in Figure 4.97 and 4.98 indicates no interaction between pairs of p_i probabilities on D_w values. There are similar patterns in all plots as N increases, and weighted D-optimality criterion values decrease. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The decrease in D_w when increasing p_q is larger than increasing p_1 and p_2. This is shown in Figure 4.97 (a), (e), (f) and Figure 4.98 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger than increasing p_1 or p_2. This is reflected by the nearly parallel in the Figure 4.97 (b), (c) and Figure 4.98 (b), (c). There are small decreases in D_w when increasing p_1 or p_2. The decreases are similar which is reflected by the clear horizontal parallelism in Figure 4.97 (d), (e), (f) and Figure 4.98 (d), (e), (f). Thus D_w is quite robust to changes in p_1 or p_2.

D_w for GA designs in the cuboidal region

The presence of parallel lines in Figure 4.99 and 4.100 indicates no interaction between pairs of p_i probabilities on D_w values. Similar patterns in all plots as N increase, and weighted D-optimality criterion values decrease. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The decrease in D_w when increasing p_q is larger than increasing p_1 and p_2. This is shown in Figure 4.99 (a), (e), (f) and Figure 4.100 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger than when increasing p_1 or p_2, and the D_w values are almost equal for both levels of p_1 and p_2. This is reflected in the clear parallelism in the Figure 4.99 (b), (c) and Figure 4.100 (b), (c). There are small increases in D_w when increasing p_1 or p_2. The increases are similar which is reflected by the clear parallelism in Figure 4.99 (d), (e), (f) and Figure 4.100 (d), (e), (f). Thus D_w is quite robust to changes in p_1 or p_2.

The boxplots of weighted G-optimality criterion values for the cuboidal region for both OPTEX designs (light shade) and GA designs (dark shade) increase when N increases. The G_w values for GA designs are higher than for OPTEX designs, but are almost equal at $N = 14$. This is shown in Figure 4.94.

G_w for OPTEX procedure designs in the cuboidal region

The presence of parallel lines in Figure 4.101, 4.102, and 4.103 indicates no interaction between pairs of p_l and p_q probabilities on G_w values. For $N = 10$ to 12, the G_w values decrease when p_q increases, but the G_w values increase when p_q increases as $N \geq 13$. For p_1 and p_2 increasing, the G_w values increase when p_q is at its low level ($p_q = 0.35$), but decrease when p_q is at its high level ($p_q = 0.95$). This is shown in Figure 4.101 (a), (e), (f), Figure 4.102 (a), (e), (f) and Figure 4.103 (a), (e), (f). However, the decrease in G_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). This is reflected in the lack-of-parallelism. Also, the decrease in G_w when increasing p_l are similar for both levels of p_2.This is reflected in the near parallelism. This is shown in the Figure 4.101 (b), (c), Figure 4.102 (b), (c) and Figure 4.103 (b), (c). There are small decreases in G_w when increasing p_1 or p_2. The decreases are similar which is reflected by the clear horizontal parallelism in Figure 4.101 (d), Figure 4.102 (d) and Figure 4.103 (d). Thus G_w is quite robust to changes in p_1 or p_2.

G_w for GA designs in the cuboidal region

For $N = 10$, the G_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. For $N = 11$ to $N = 12$, the G_w values increase across each combination of p_1 and p_2 when p_q increases. For $N \geq 13$, the G_w values increase in G_w values when p_q increases. The presence of parallel lines when $N = 14$ indicates no interaction between pairs of p_i probabilities on G_w values. This is shown in Figure 4.104 (a), (e), (f), Figure 4.105 (a), (e), (f), Figure 4.106 (a), (e), (f) and Figure 4.107 (a), (e), (f). However for $N = 10$, the decrease in G_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$), but for $N > 10$ the decrease in G_w when increasing p_l is smaller when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$) and The G_w values increase when p_1 increases. Also for $N = 10$, the decrease in G_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$), but for $N > 10$ the decrease in G_w when increasing p_l is larger when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism and the G_w values are close to near parallelism as N increase. It shows in the Figure 4.104 (b), (c), Figure 4.105 (b), (c) and Figure 4.107 (b), (c). For $N = 10$, the decreases are similar when increasing p_1 for both p_2 levels. For $N > 10$, the values of G_w are similar when p_1 and p_2 increased which is reflected by the near horizontal parallelism in Figure 4.104 (d) to Figure 4.107 (d).

For the cuboidal computer generated designs (OPTEX procedure and GAs), the results are similar. The D_w values are robust to change in p_1 and p_2, and slightly less robust as N increase. However, the D_w values are sensitive to change in p_l and p_q. The G_w values are slightly sensitive (weakly robust) to change in p_1, p_2 and p_q, but p_q are sensitive as $N \geq 11$. However, the G_w values are very sensitive to change in p_l.

The boxplots of weighted D-optimality criterion values for the spherical region for OPTEX designs (light shade) only slightly increase, but the boxplots for the GA designs (dark shade) are almost equal when N increases. The D_w value of OPTEX designs decrease slightly at $N = 11$ but slightly increase after $N = 11$. The D_w values of GA designs are higher than for the OPTEX designs, and the range of D_w values for GA designs are larger than for OPTEX designs. This is shown in Figure 4.95.

D_w for OPTEX procedure designs in the spherical region

The presence of near parallel lines in Figure 4.108 and 4.109 indicates no interaction between pairs of p_i probabilities on D_w values. Similar patterns occur in all plots as N increases, and weighted D-optimality criterion values decrease. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The decrease in D_w when increasing p_q is almost equal for both levels of p_l, p_1, and p_2, and D_w slightly decreases as N increases. This is shown in Figure 4.108 (a), (e), (f) and Figure 4.109 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in D_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.108 (b), (c) and Figure 4.109 (b), (c). There is only a decrease in D_w when increasing p_1 or p_2. The decreases are similar which is reflected by the clear parallelism in Figure 4.108 (d) and Figure 4.109 (d). Thus D_w is quite robust to changes in p_1 or p_2.

D_w for GA designs in the spherical region

The presence of near parallel lines in Figure 4.110 and 4.111 indicates no interaction between pairs of p_i probabilities on D_w values. Similar patterns occur in all plots as N increases, and weighted D-optimality criterion values slightly decrease. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The decrease in D_w when increasing p_q is larger when p_l increases than when p_1 or p_2 increase. This is shown in Figure 4.110 (a), (e), (f) and Figure 4.111 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in D_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.110 (b), (c) and Figure 4.111 (b), (c). the decrease in D_w when increasing p_1 is larger when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the

Figure 4.110 (d) and Figure 4.111 (d).

The boxplots of weighted G-optimality criterion values for the spherical region for OPTEX designs (light shade) are similar for $N \leq 13$ but with an increase for $N = 14$, and the G_w values have the largest range at $N = 12$. The G_w value for GA designs (dark shade) slightly decrease at $N = 11$ and then slightly increase after $N = 11$. The G_w values of GA designs are higher than for OPTEX designs. This is shown in Figure 4.96.

G_w for OPTEX procedure designs in the spherical region

The G_w values increase across each combination of p_l, p_1, and p_2 when p_q increases. The increase in G_w when increasing p_l or p_2 is smaller when p_q is at its low level ($p_1 = 0.35$) than when it is at its high level ($p_1 = 0.95$). This is reflected in the lack-of-parallelism, and similar patterns occur in all plots as N increases. This is shown in Figure 4.112 (a), (e), (f) and Figure 4.113 (a), (e), (f). However, the increase in G_w when increasing p_l is smaller when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the increase in G_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.112 (b), (c) and Figure 4.113 (b), (c). There is a little increase in G_w when increasing p_1 or p_2. The increases are similar which is reflected by the clear parallelism in Figure 4.112 (d), (e), (f) and Figure 4.113 (d), (e), (f). Thus G_w is quite robust to changes in p_1 or p_2.

G_w for GA designs in the spherical region

The increase in G_w values when increasing p_l is smaller when p_q is at its low level ($p_q = 0.35$) than when it is at its high level ($p_q = 0.95$). This is shown in Figure 4.114 (a), Figure 4.115 (a), and Figure 4.116 (a).

The rate of change in G_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the rate of change in G_w when increasing p_l is smaller when p_2 is at its low level

($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.114 (b), (c), Figure 4.115 (b), (c) and Figure 4.116 (b), (c).

For the combinations of p_1 and p_q, the G_w values slight increase and the increases are similar as N increases. This is reflected by the clear parallelism in Figure 4.114 (e), Figure 4.115 (e) and Figure 4.116 (e). For $N = 10$, the G_w values increase when p_2 increases and p_q is at its low level ($p_q = 0.35$), but decrease when p_2 increases and p_q is at its high level ($p_q = 0.95$). For $N > 10$ the increase in G_w values when increasing p_2 is smaller when p_q is at its low level ($p_q = 0.35$) than when it is at its high level ($p_q = 0.95$). This is reflected in the lack-of-parallelism, but the G_w values increase and then gradually become similar for both levels of p_q as N increases. This is shown in Figure 4.114 (f), Figure 4.115 (f) and Figure 4.116 (f). The rate of change in G_w when increasing p_1 is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism and Similar patterns occur in all plots as N increases. This is shown in Figure 4.114 (d) to Figure 4.116 (d).

From the results for the spherical computer generated designs (OPTEX designs and GAs), the result are similar. The D_w values are robust to change in p_q and are weak robust in p_1 and p_2. However, the D_w values are sensitive to change in p_l. For the OPTEX procedure designs, the G_w values are slightly sensitive (weakly robust) to change in p_1, p_2 and p_l. However, the G_w values are very sensitive to change in p_q. In case of GA designs, the G_w values are slightly sensitive (weakly robust) to change in p_1. However, the G_w values are very sensitive to change in p_l, p_q and p_2.

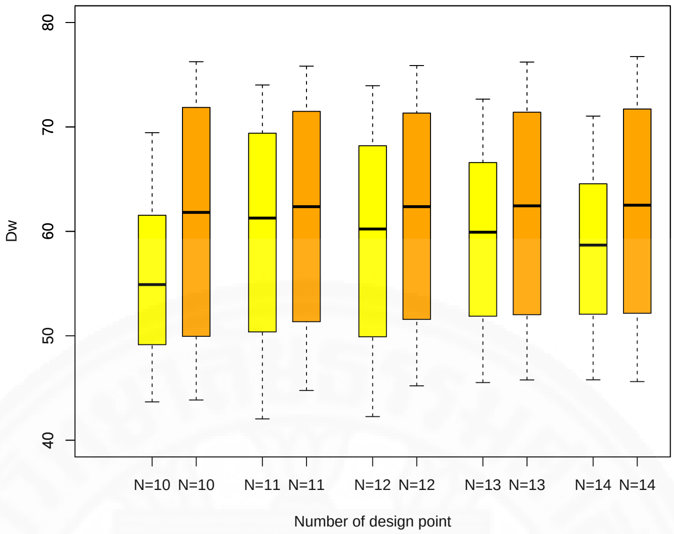


Figure 4.93: Boxplot for the weighted D-optimality criterion values in cuboidal region for OPTEX designs (light shade) and GA designs (dark shade), $k = 3$

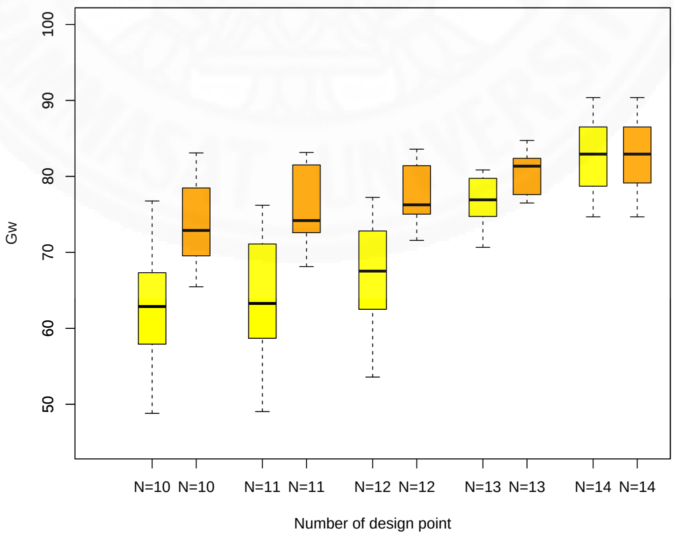


Figure 4.94: Boxplot for the weighted G-optimality criterion values in cuboidal region for OPTEX designs (light shade) and GA designs (dark shade), $k = 3$

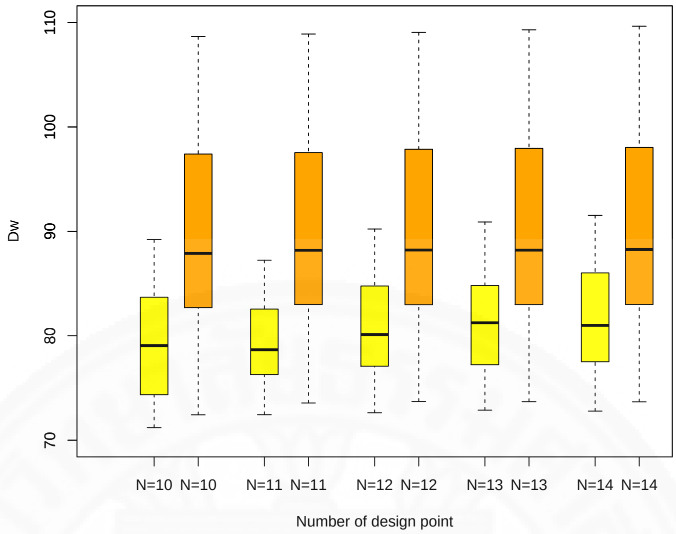


Figure 4.95: Boxplot for the weighted D-optimality criterion values in spherical region for OPTEX designs (light shade) and GA designs (dark shade), $k = 3$

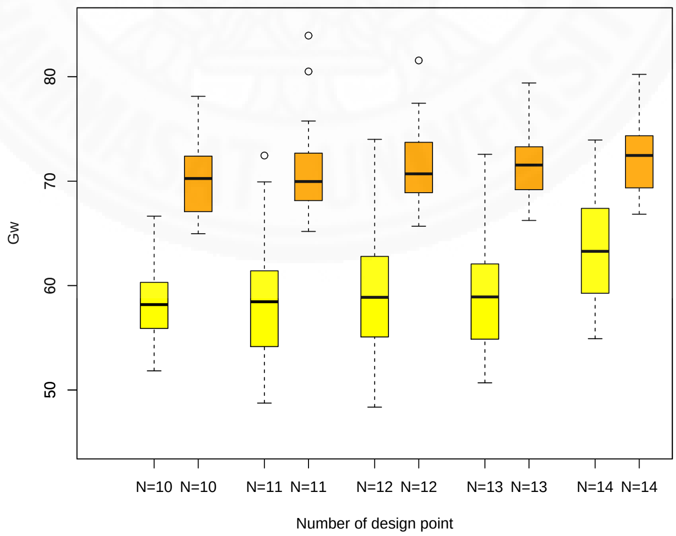


Figure 4.96: Boxplot for the weighted G-optimality criterion values in spherical region for OPTEX designs (light shade) and GA designs (dark shade), $k = 3$

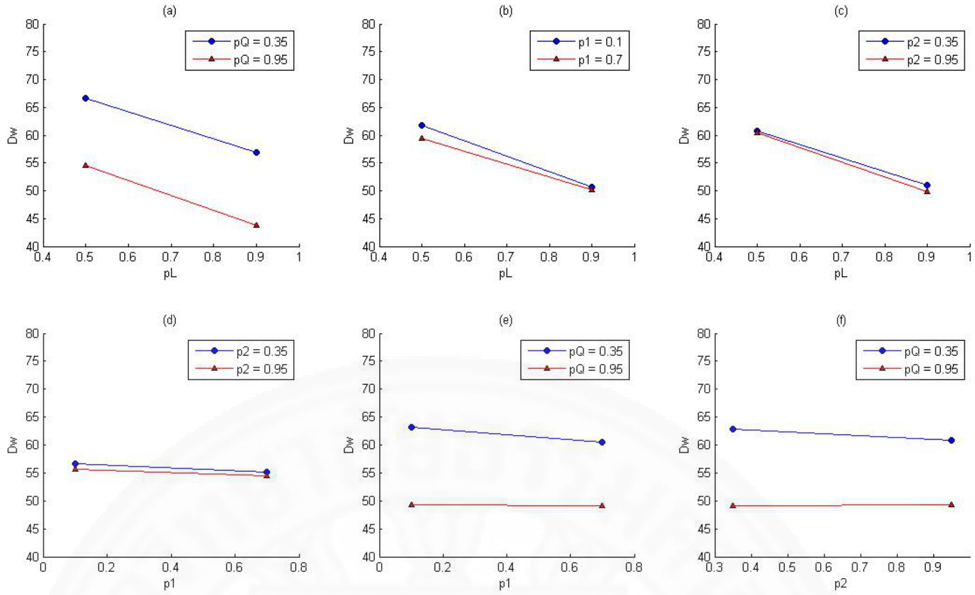


Figure 4.97: The weighted D-optimality criterion values in cuboidal region for OPTEX procedure designs for $k = 3$ and $N = 10$

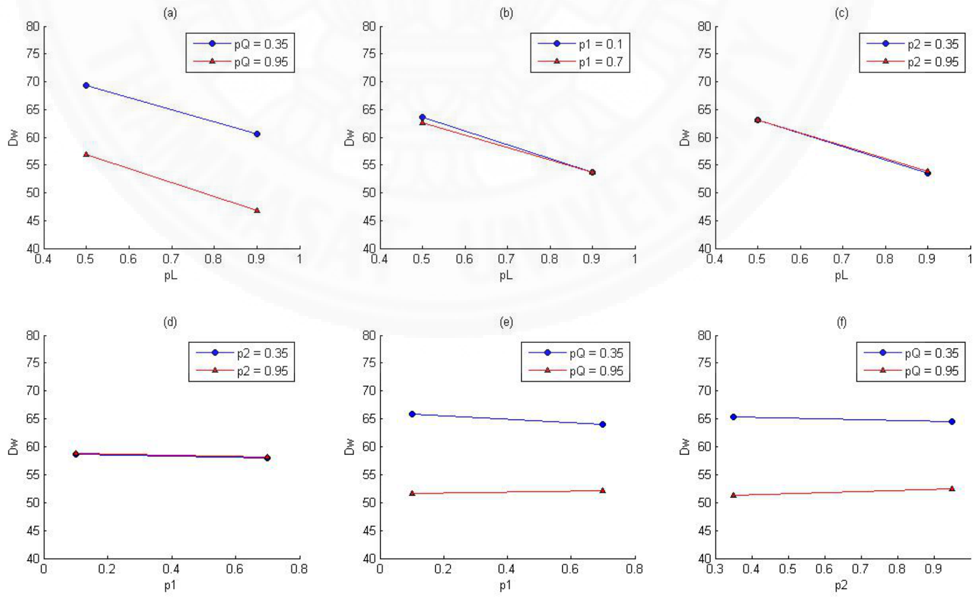


Figure 4.98: The weighted D-optimality criterion values in cuboidal region for OPTEX procedure designs for $k = 3$ and $N = 14$

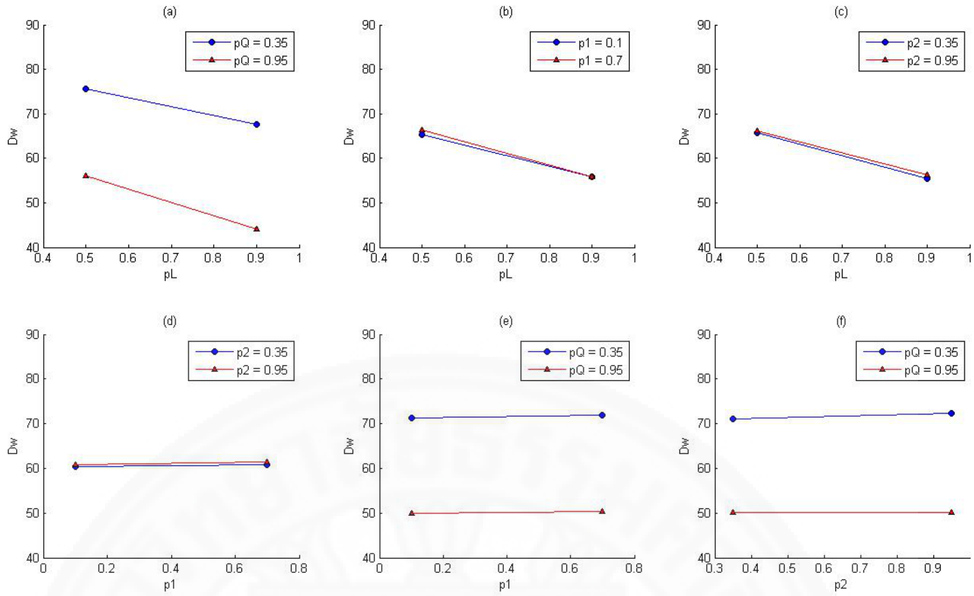


Figure 4.99: The weighted D-optimality criterion values in cuboidal region for GA designs for $k = 3$ and $N = 10$

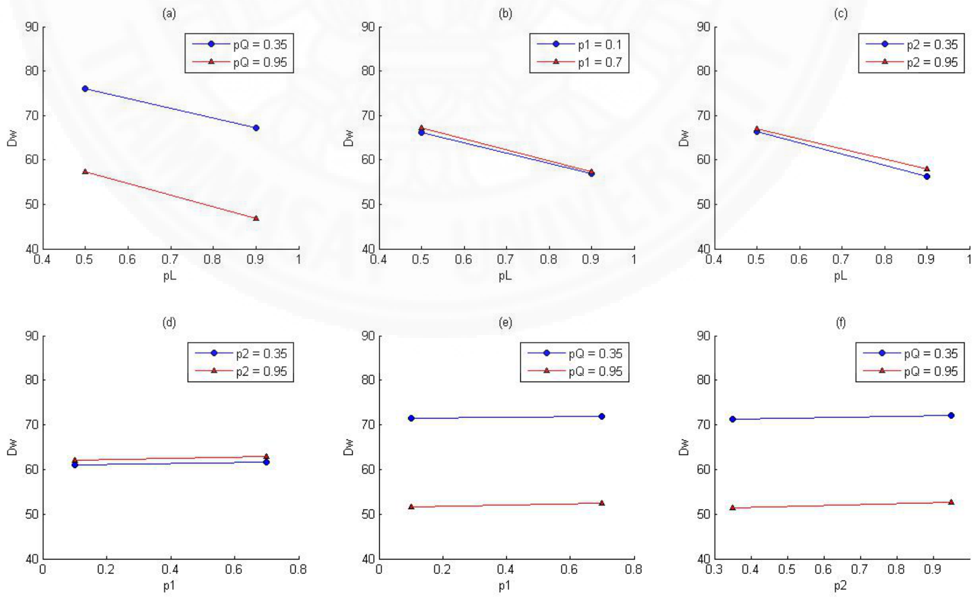


Figure 4.100: The weighted D-optimality criterion values in cuboidal region for GA designs for $k = 3$ and $N = 14$

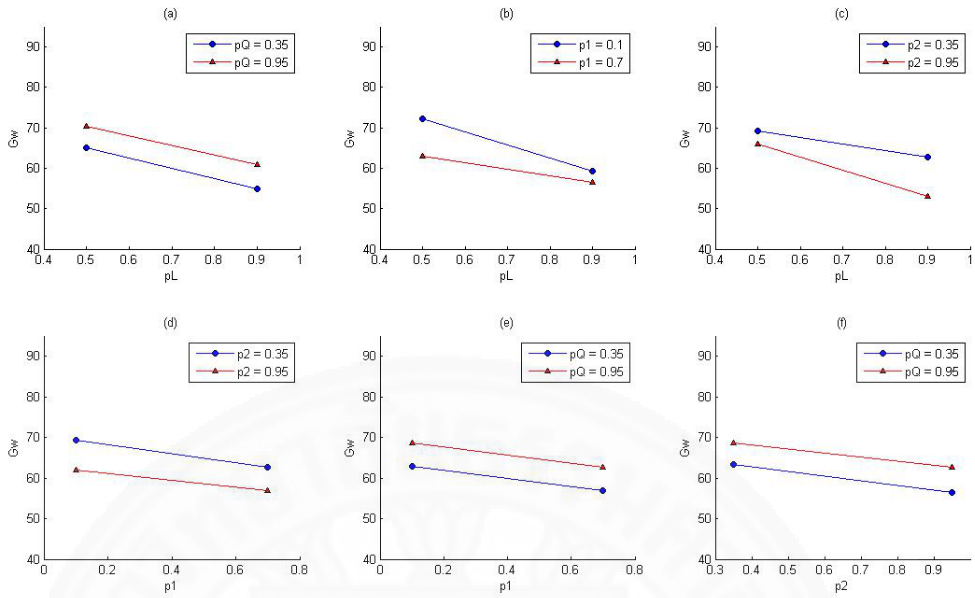


Figure 4.101: The weighted G-optimality criterion values in cuboidal region for OPTEX procedure designs for $k = 3$ and $N = 10$

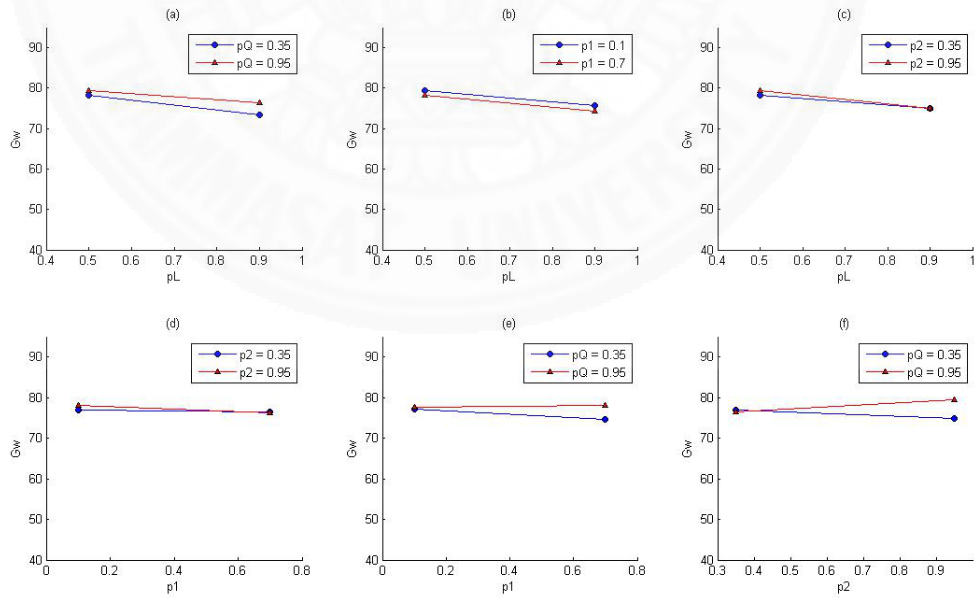


Figure 4.102: The weighted G-optimality criterion values in cuboidal region for OPTEX procedure designs for $k = 3$ and $N = 13$

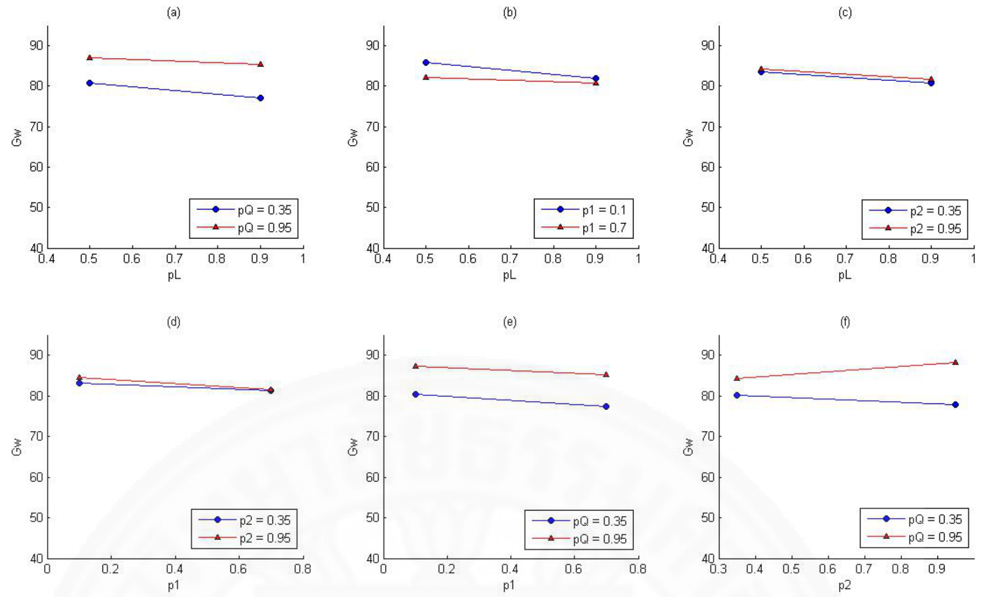


Figure 4.103: The weighted G-optimality criterion values in cuboidal region for OPTEX procedure designs for $k = 3$ and $N = 14$

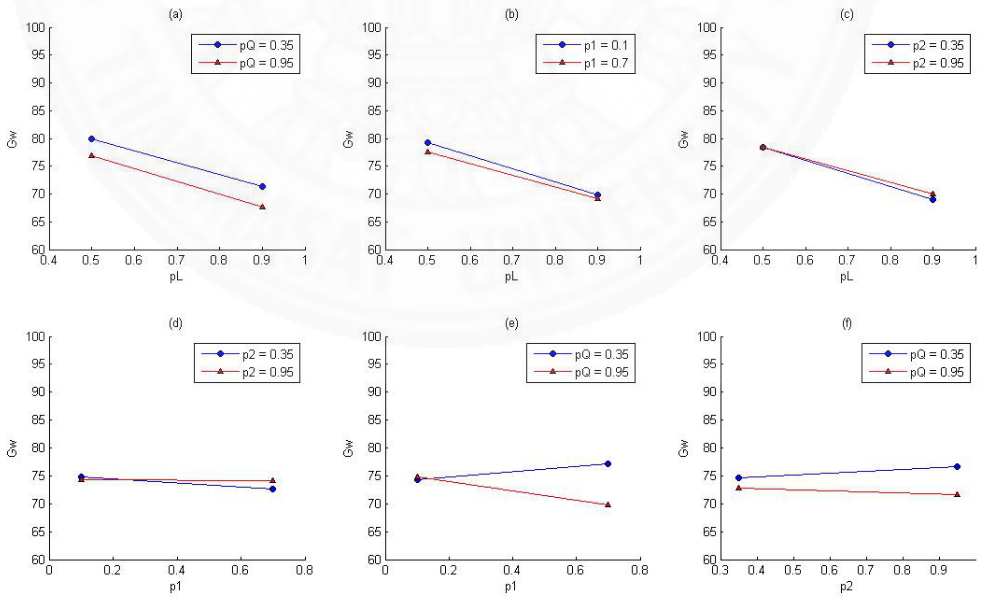


Figure 4.104: The weighted G-optimality criterion values in cuboidal region for GA designs for $k = 3$ and $N = 10$

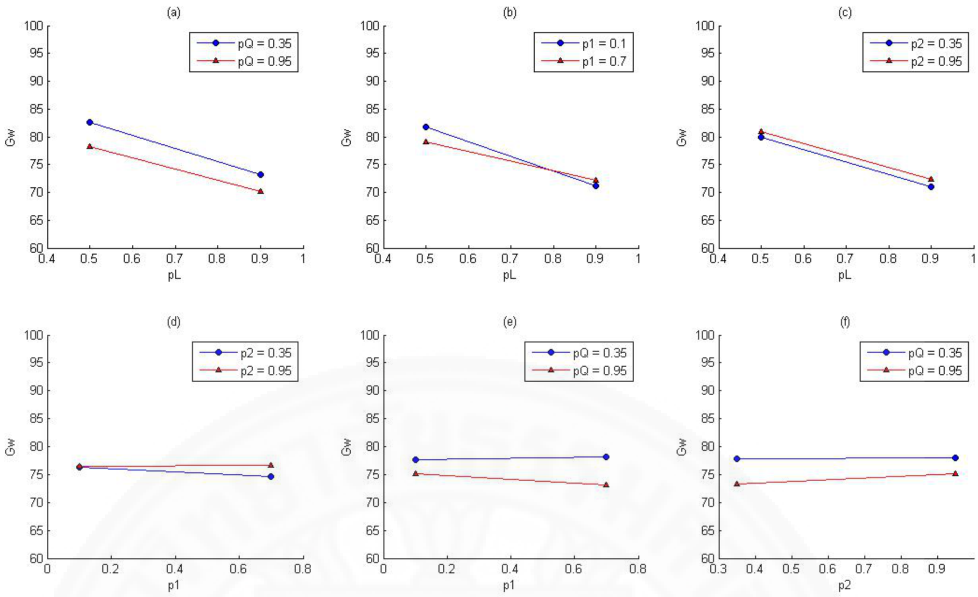


Figure 4.105: The weighted G-optimality criterion values in cuboidal region for GA designs for $k = 3$ and $N = 11$

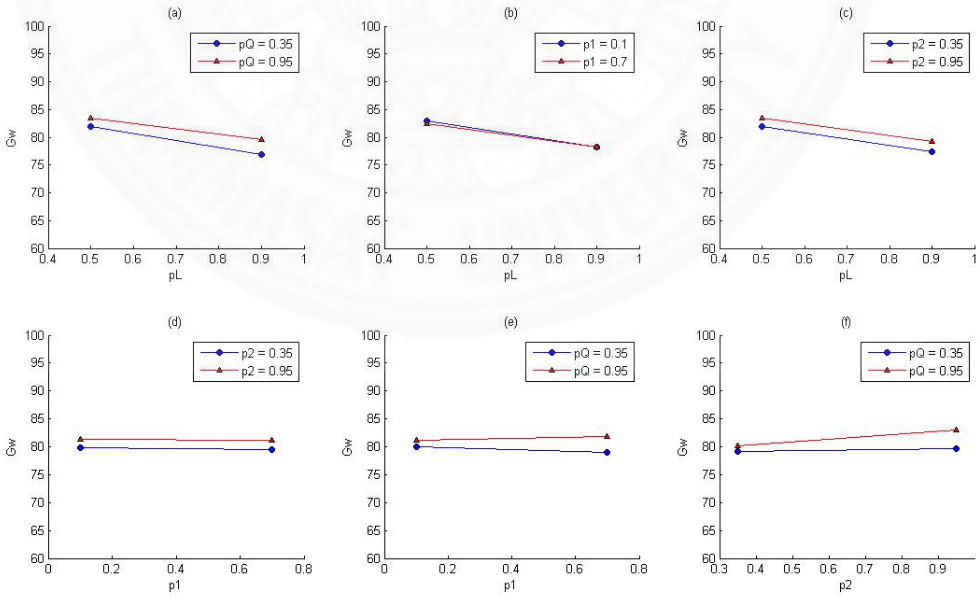


Figure 4.106: The weighted G-optimality criterion values in cuboidal region for GA designs for $k = 3$ and $N = 13$

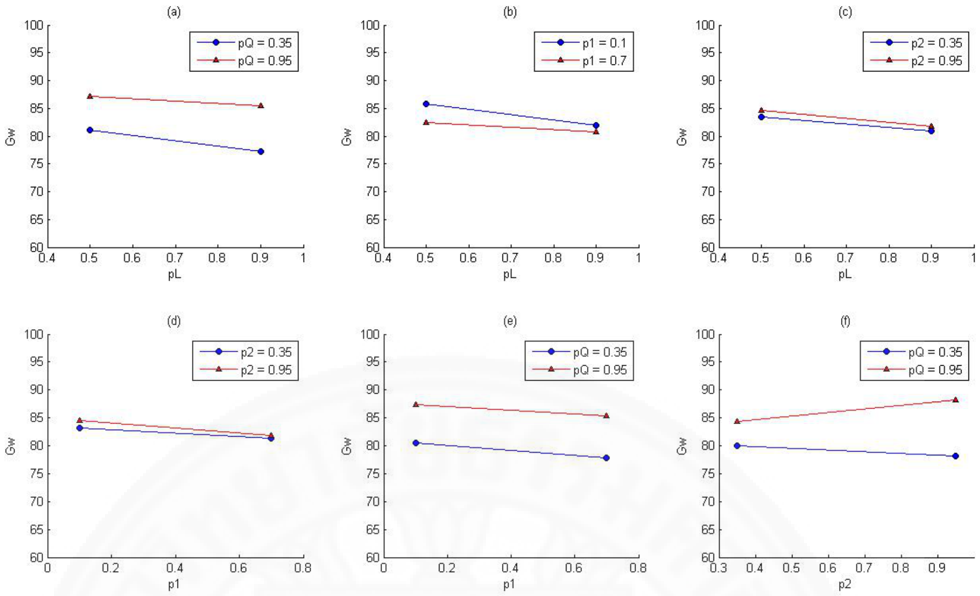


Figure 4.107: The weighted G-optimality criterion values in cuboidal region for GA design for $k = 3$ and $N = 14$

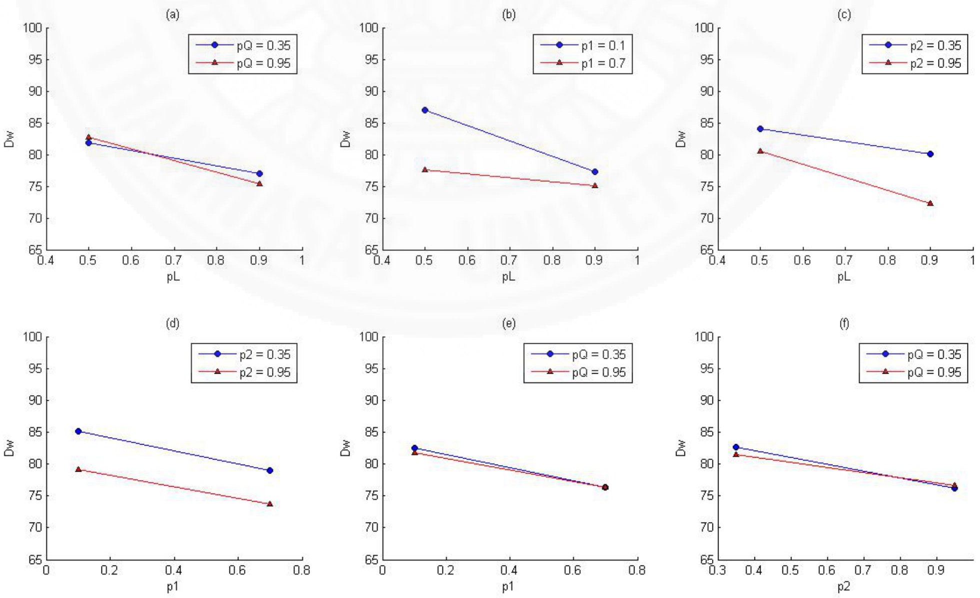


Figure 4.108: The weighted D-optimality criterion values in spherical region for OPTEX procedure designs for $k = 3$ and $N = 10$

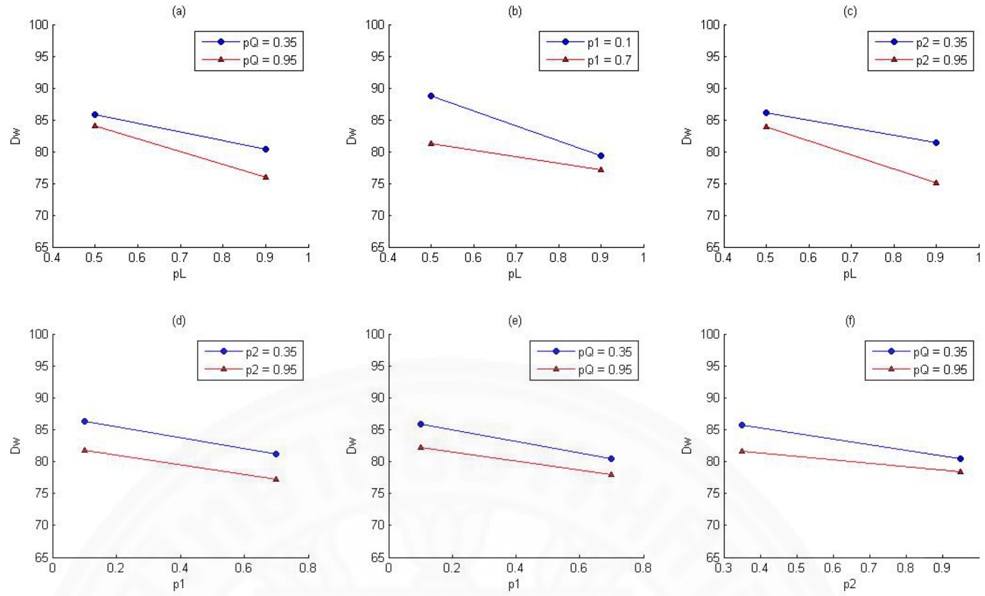


Figure 4.109: The weighted D-optimality criterion values in spherical region for OPTEX procedure designs for $k = 3$ and $N = 14$

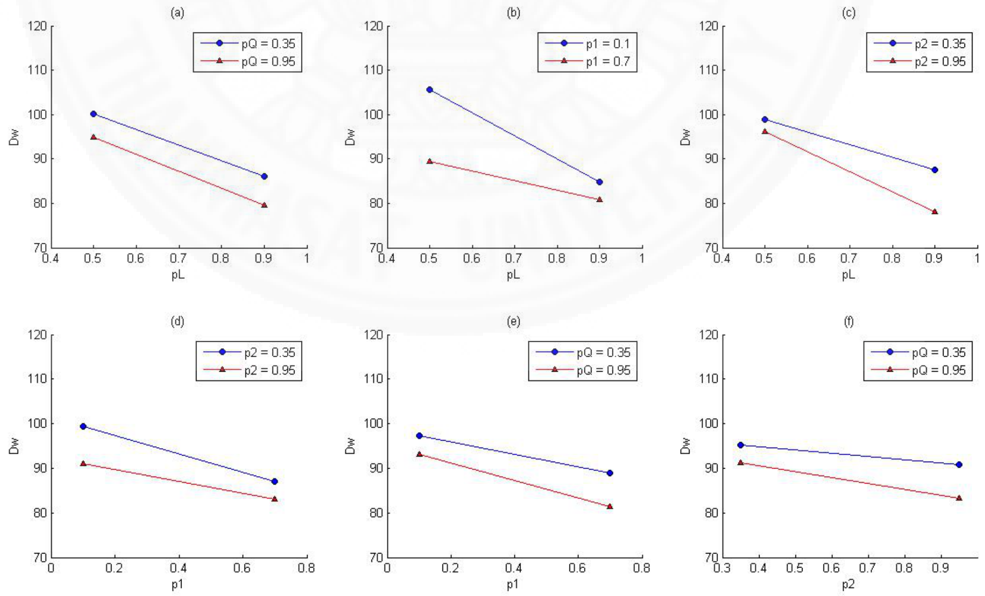


Figure 4.110: The weighted D-optimality criterion values in spherical region for GA designs for $k = 3$ and $N = 10$

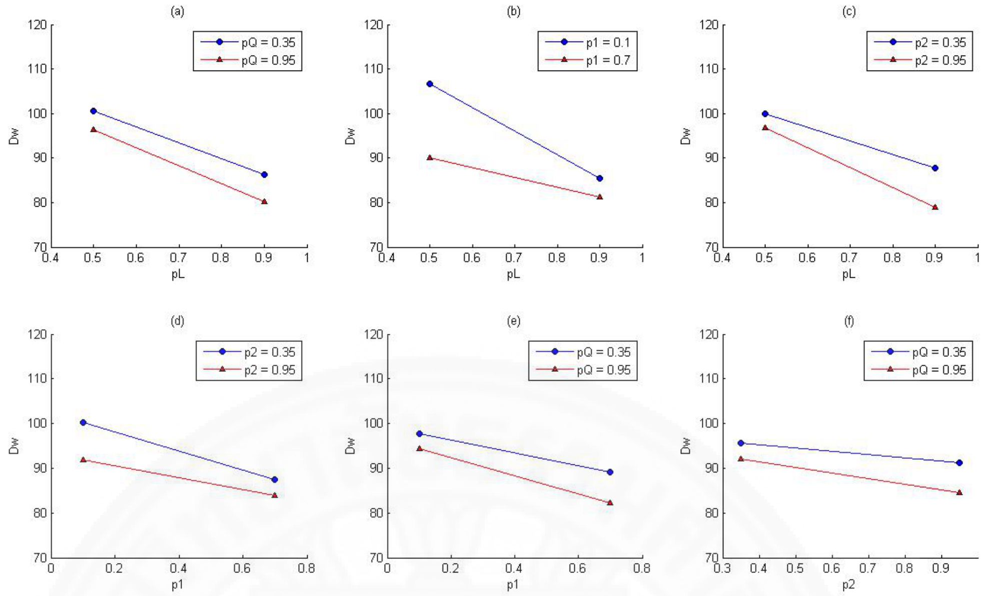


Figure 4.111: The weighted D-optimality criterion values in spherical region for GA designs for $k = 3$ and $N = 14$

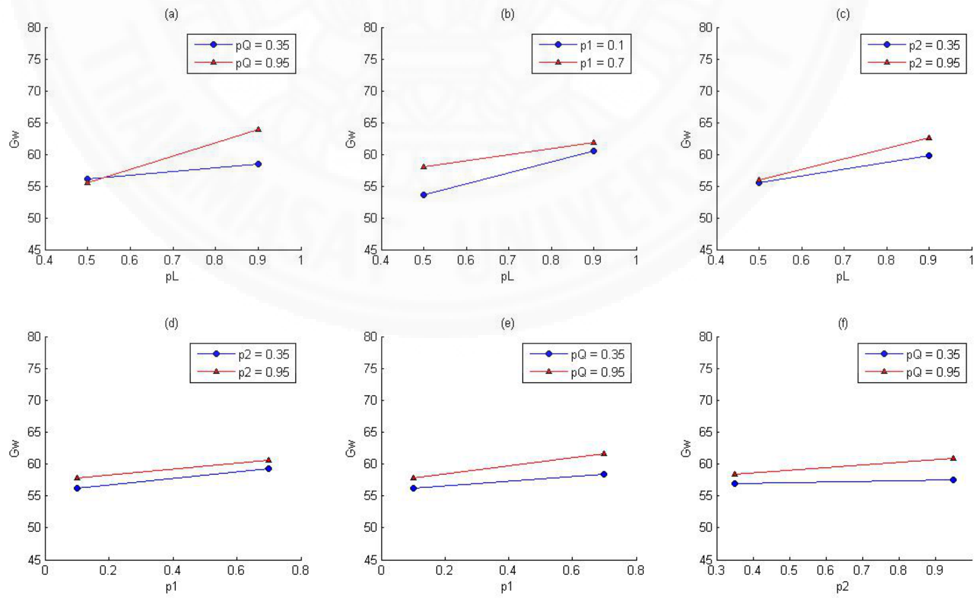


Figure 4.112: The weighted D-optimality criterion values in spherical region for GA designs for $k = 3$ and $N = 10$

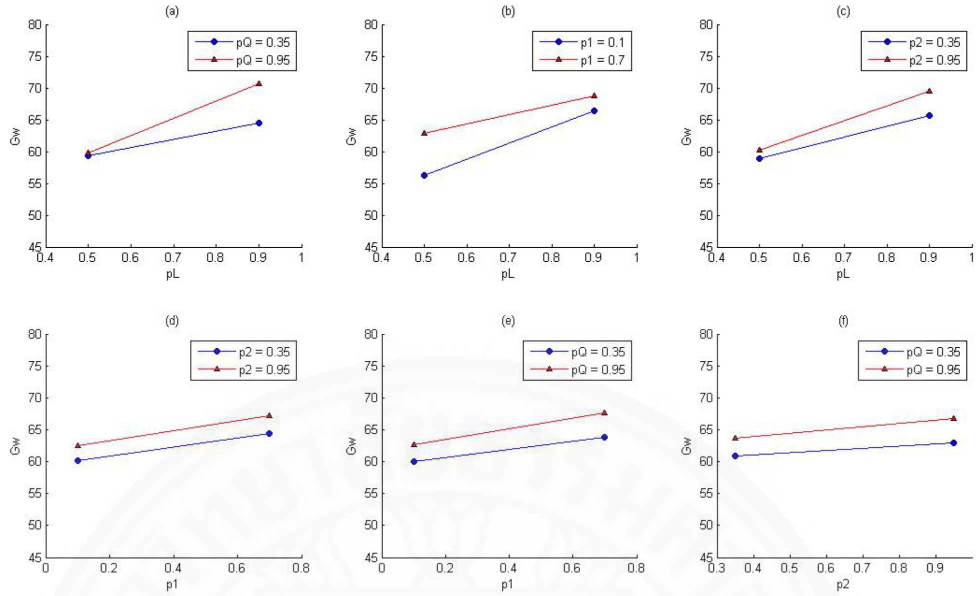


Figure 4.113: The weighted G-optimality criterion values in spherical region for OPTEX procedure designs for $k = 3$ and $N = 14$

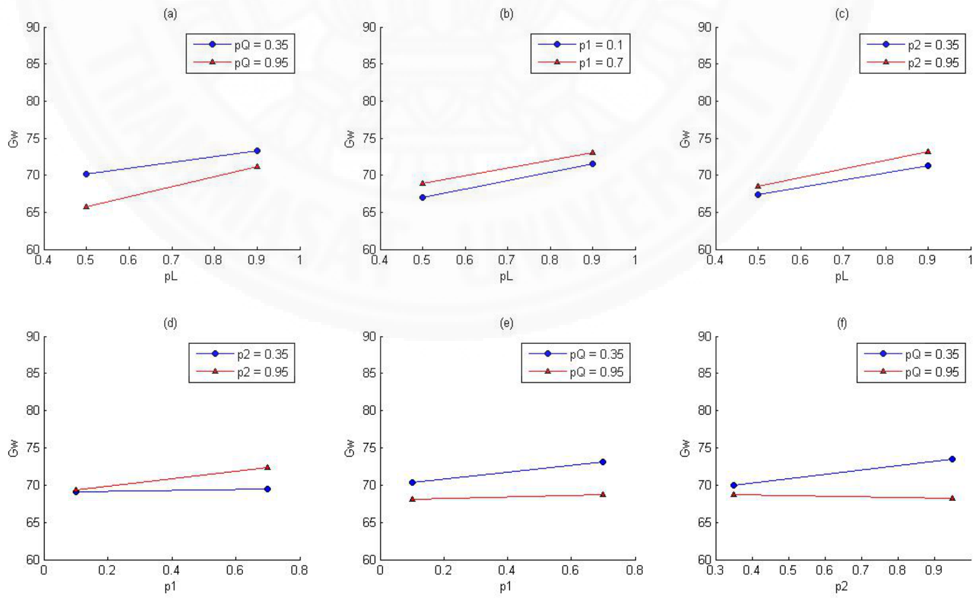


Figure 4.114: The weighted G-optimality criterion values in spherical region for GA designs for $k = 3$ and $N = 10$

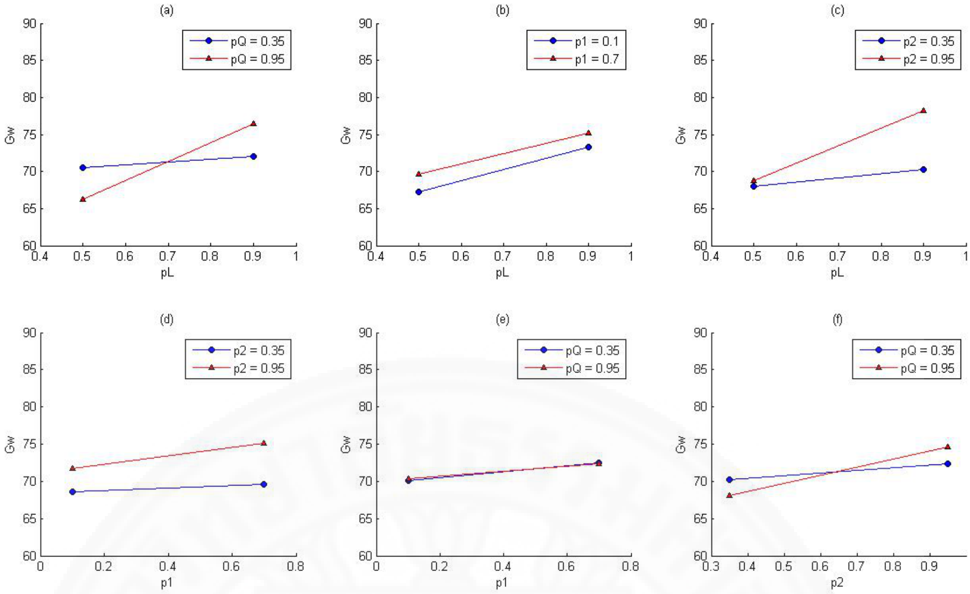


Figure 4.115: The weighted G-optimality criterion values in spherical region for GA designs for $k = 3$ and $N = 11$

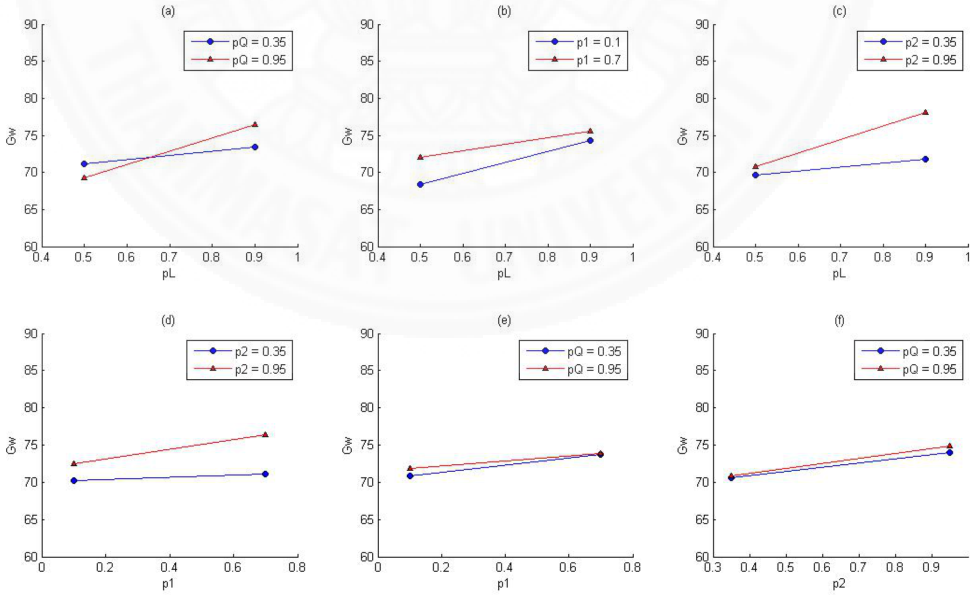


Figure 4.116: The weighted G-optimality criterion values in spherical region for GA designs for $k = 3$ and $N = 14$

4.3.3 for $k = 4$ factors

The boxplots of weighted D-optimality criterion values for the cuboidal region for OPTEX designs (light shade) has a small wave shape, but the D_w values for GA designs (dark shade) are nearly constant and only very slightly increase when N increases. The D_w values of GA designs are higher than for OPTEX designs. This is shown in Figure 4.117.

D_w for OPTEX procedure designs in the cuboidal region

The presence of parallel lines in Figure 4.121 and 4.122 indicates no interaction between pairs of p_i probabilities on D_w values. There are similar patterns in all plots as N increases, and weighted D-optimality criterion values decrease. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The decrease in D_w when increasing p_q is larger than increasing p_1 and p_2. This is shown in Figure 4.121 (a), (e), (f) and Figure 4.122 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger than increasing p_1 or p_2. This is reflected by the nearly parallel lines in Figure 4.121 (b), (c) and Figure 4.122 (b), (c). There are small decreases in D_w when increasing p_1 or p_2. The decreases are similar which is reflected by the clear horizontal parallelism in Figure 4.121 (d), (e), (f) and Figure 4.122 (d), (e), (f). Thus D_w is quite robust to changes in p_1 or p_2.

D_w for GA designs in the cuboidal region

The presence of parallel lines in Figure 4.123 and 4.124 indicates no interaction between pairs of p_i probabilities on D_w values. There are similar patterns in all plots as N increases, and weighted D-optimality criterion values decrease. The D_w values decrease across combination of p_l, but it increase across each combination of p_1 and p_2 when p_q increases. The increase in D_w when increasing p_q is larger than increasing p_1 and p_2. This is shown in Figure 4.123 (a), (e), (f) and Figure 4.124 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger than increasing p_1 or p_2, and the decrease in D_w when increasing p_l is smaller when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in D_w when increasing p_l is larger when p_2 is at its low level ($p_1 = 0.35$) than when it is at its high level ($p_1 = 0.95$). This is reflected in the lack-of-parallelism in Figure 4.123 (b), (c) and Figure 4.124 (b), (c). There are small increases in D_w when increasing p_1 or p_2. The increases are similar which is reflected by the clear parallelism in Figure 4.123 (d), (e), (f) and Figure 4.124 (d), (e), (f). Thus D_w is quite robust to changes in p_1 or p_2.

The boxplots of weighted G-optimality criterion values for the cuboidal region for the OPTEX designs (light shade) has a wave shape, and the G_w values are lowest at $N = 16$ and $N = 18$. The G_w values for GA designs (dark shade) increase very slightly as N increases. The G_w values of GA designs are higher than for OPTEX designs. This is shown in Figure 4.118.

G_w for OPTEX procedure designs in the cuboidal region

For $N = 16$, the decrease in G_w when increasing p_l is larger when p_q is at its low level ($p_q = 0.35$) than when it is at its high level ($p_q = 0.95$). The G_w values when increasing p_1 or p_2 decrease when p_q is at its low level ($p_q = 0.35$), but increase when p_q is at its high level ($p_q = 0.95$). For $N > 16$, the decrease in G_w when increasing p_1 is larger when p_q is at its low level ($p_q = 0.35$) than when it is at its high level ($p_q = 0.95$). This is shown in Figure 4.125 (a), (e), (f), Figure 4.126 (a), (e), (f) and Figure 4.127 (a), (e), (f). However, the decrease in G_w when increasing p_l are similar for both levels of p_1 for $N = 16$, but it is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$) for $N > 16$. This is reflected in the lack-of-parallelism. Also, the decrease in G_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$) for $N = 16$, but it are similar for both levels of p_2 as N increase. This is reflected in the near parallelism. This is shown in Figure 4.125 (b), (c), Figure 4.126 (b), (c) and Figure 4.127 (b), (c). This is little change in G_w when increasing p_1 or p_2. This is reflected by the parallelism in Figure 4.125 (d), Figure 4.126 (d) and Figure 4.127 (d). Thus G_w is quite robust to changes in

p_1 or p_2.

G_w for GA designs in the cuboidal region

The G_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The G_w values when increasing p_l, p_1 or p_2 are smaller when p_q is at its low level ($p_q = 0.35$) than when it is at its high level ($p_q = 0.95$). Similar patterns occur in all plots as N increases, and this is reflected in the lack-of-parallelism in Figure 4.128 (a), (e), (f) and Figure 4.129 (a), (e), (f). However, the decrease in G_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in G_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.128 (b), (c) and Figure 4.129 (b), (c). The decrease in G_w when increasing p_1 is larger when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in Figure 4.128 (d) to Figure 4.129 (d).

For the cuboidal computer generated designs (OPTEX procedure and GA), the results are similar. The D_w values are robust to change in p_1 and p_2. However, the D_w values are sensitive to change in p_l and p_q. The G_w values are slightly sensitive (weakly robust) to change in p_1, p_2 and p_q. However, the G_w values are very sensitive to change in p_l.

The boxplots of weighted D-optimality criterion values for the spherical region for OPTEX designs (light shade) are similar with a slight wave shape, but the D_w values of GA designs (dark shade) are almost equal when N increases. The D_w values of OPTEX designs decrease at $N = 17$ and are similar after $N = 17$. The D_w values of GA designs are higher than for OPTEX designs, and the ranges of GA designs are larger than for OPTEX designs. This is shown in Figure 4.119.

D_w for OPTEX procedure designs in the spherical region

The presence of near parallel lines in Figure 4.130 and 4.131 indicates no interaction between pairs of p_i probabilities on D_w values. Similar patterns occur in all plots as N increases, and weighted D-optimality criterion values decrease. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The decrease in D_w when increasing p_q is almost equal for both levels of p_l, p_1, and p_2. This is shown in Figure 4.130 (a), (e), (f) and Figure 4.131 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in D_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.130 (b), (c) and Figure 4.131 (b), (c). There is only a decrease in D_w when increasing p_1 or p_2. The decreases are similar which is reflected by the clear parallelism in Figure 4.130 (d) and Figure 4.131 (d). Thus D_w is quite robust to changes in p_1 or p_2.

D_w for GA designs in the spherical region

The presence of near parallel lines in Figure 4.132 and 4.133 indicates no interaction between pairs of p_i probabilities on D_w values. Similar patterns occur in all plots as N increases. The D_w values decrease across each combination of p_l, p_1, and p_2 when p_q increases. The decrease in D_w when increasing p_q is larger when p_l increases than when p_1 or p_2 increase. This is shown in Figure 4.132 (a), (e), (f) and Figure 4.133 (a), (e), (f). However, the decrease in D_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the decrease in D_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.132 (b), (c) and Figure 4.133 (b), (c). The decrease in D_w when increasing p_1 is larger when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.132 (d) and Figure 4.133 (d).

The boxplots of weighted G-optimality criterion values for the spherical region for OPTEX designs (light shade) are almost equal for $N \leq 18$, but the G_w values increase at $N = 19$. The G_w values for GA designs (dark shade) are similar for all N. The G_w values for GA designs are higher than for OPTEX designs, but with smaller ranges than for OPTEX designs. This is shown in Figure 4.120.

G_w for OPTEX procedure designs in the spherical region

The presence of near parallel lines in Figure 4.134 and 4.135 indicates no interaction between pairs of p_i probabilities on G_w values. Similar patterns occur in all plots as N increases. The G_w values increase across combination of p_l when p_q increases. The increase in G_w when increasing p_l is smaller when p_q is at its low level ($p_1 = 0.35$) than when it is at its high level ($p_1 = 0.95$). This is reflected in the lack-of-parallelism, and similar patterns occur in all plots as N increases. This is shown in Figure 4.134 (a), (e), (f) and Figure 4.135 (a), (e), (f). However, the increase in G_w when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the increase in G_w when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.134 (b), (c) and Figure 4.135 (b), (c). There is increase in G_w when increasing p_1 or p_2. The increases are similar which is reflected by the clear parallelism in Figure 4.134 (d), (e), (f) and Figure 4.135 (d), (e), (f). Thus G_w is quite robust to changes in p_1 or p_2.

G_w for GA designs in the spherical region

The increase in G_w values when increasing p_l is slightly smaller when p_q is at its low level ($p_q = 0.35$) than it is at its high level ($p_q = 0.95$). This is shown in Figure 4.136 (a), Figure **??** (a), and Figure4.138 (a).

The increase in G_w values when increasing p_l is larger when p_1 is at its low level ($p_1 = 0.1$) than when it is at its high level ($p_1 = 0.7$). Also, the increase in G_w values when increasing p_l is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism in the Figure 4.136 (b), (c), Figure 4.137 (b), (c), and Figure 4.138 (b), (c).

For the combination of p_1 and p_q, the G_w values slightly increase and the increases are similar as N increases. This is reflected by the clear parallelism in Figure 4.136 (e), Figure 4.137 (e), and Figure 4.138 (e). For $N = 16$, the G_w values slightly increase when increasing p_2 and p_q, and have similar values for both levels of p_q. For $N > 16$ the increase in G_w values when increasing p_2 is smaller when p_q is at its low level ($p_q = 0.35$) than when it is at its high level ($p_q = 0.95$). This is reflected in the lack-of-parallelism in Figure 4.136 (f), Figure 4.137 (f), and Figure 4.138 (f). The rate of change in G_w when increasing p_1 is smaller when p_2 is at its low level ($p_2 = 0.35$) than when it is at its high level ($p_2 = 0.95$). This is reflected in the lack-of-parallelism and Similar patterns occur in all plots as N increases. This is shown in Figure 4.136 (d) to Figure 4.138 (d).

For the spherical computer generated designs (OPTEX procedure and GAs), the results are similar. The D_w values are robust to change in p_q and are weakly robust in p_1 and p_2. However, the D_w values are sensitive to change in p_l. For the OPTEX procedure designs, the G_w values are slightly sensitive (weakly robust) to change in p_1, p_2 and p_l. However, the G_w values are very sensitive to change in p_q. For GA designs, the G_w values are robust to change in p_q and p_2, and are slightly sensitive (weakly robust) to change in p_1. However, the G_w values are very sensitive to change in p_l.

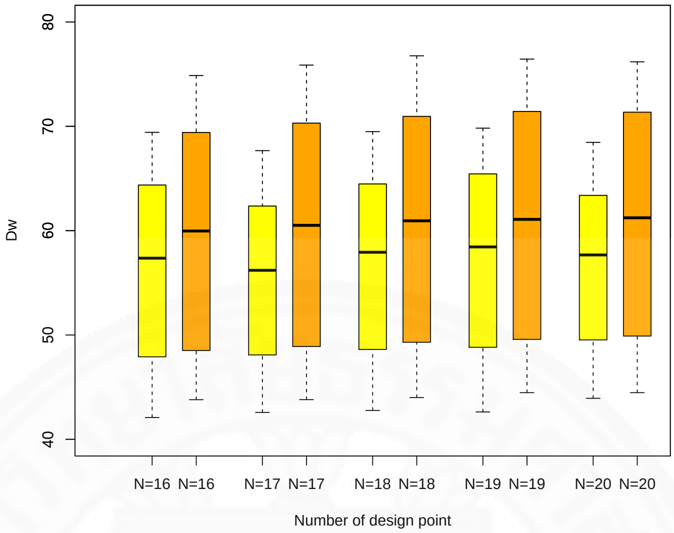


Figure 4.117: Boxplot for the weighted D-optimality criterion values in cuboidal region for OPTEX designs (light shade) and GA designs (dark shade), $k = 4$

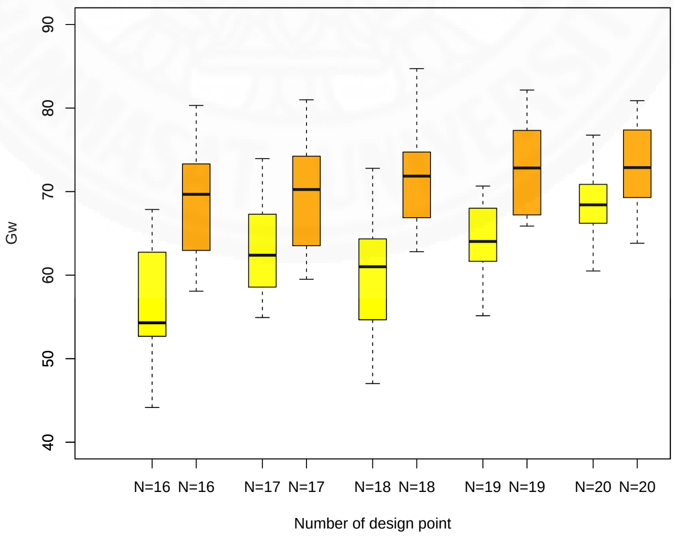


Figure 4.118: Boxplot for the weighted G-optimality criterion values in cuboidal region for OPTEX designs (light shade) and GA designs (dark shade), $k = 4$

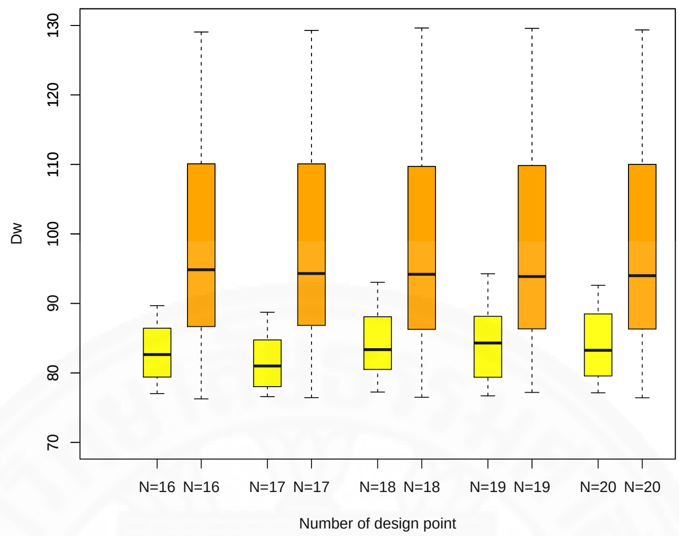


Figure 4.119: Boxplot for the weighted D-optimality criterion values in spherical region for OPTEX designs (light shade) and GA designs (dark shade), $k = 4$

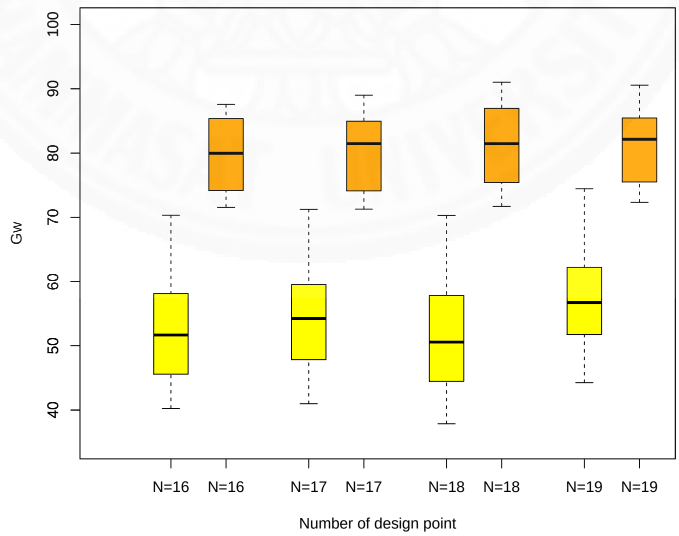


Figure 4.120: Boxplot for the weighted G-optimality criterion values in spherical region for OPTEX designs (light shade) and GA designs (dark shade), $k = 4$

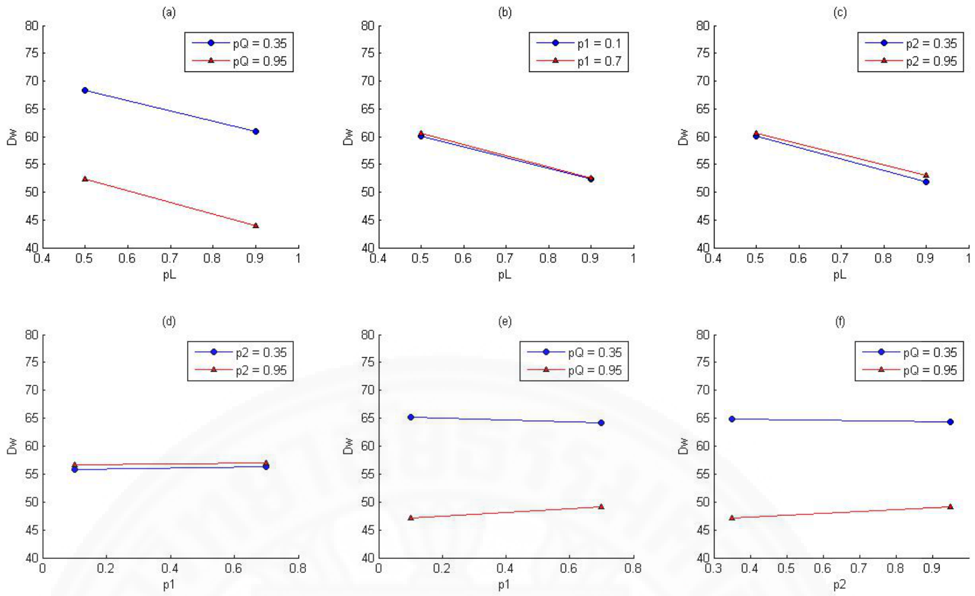


Figure 4.121: The weighted D-optimality criterion values in cuboidal region for OPTEX procedure designs for $k = 4$ and $N = 16$

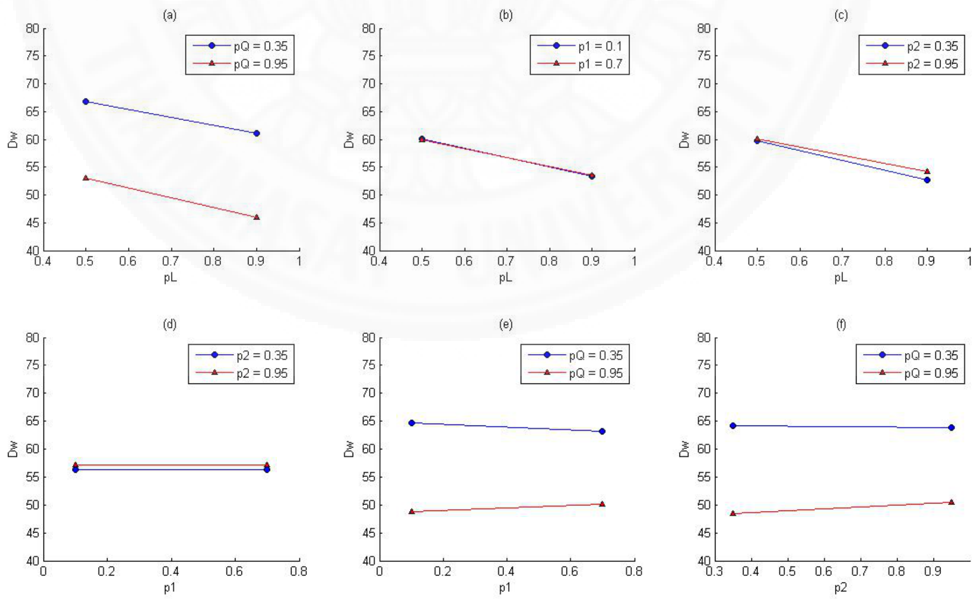


Figure 4.122: The weighted D-optimality criterion values in cuboidal region for OPTEX procedure designs for $k = 4$ and $N = 20$

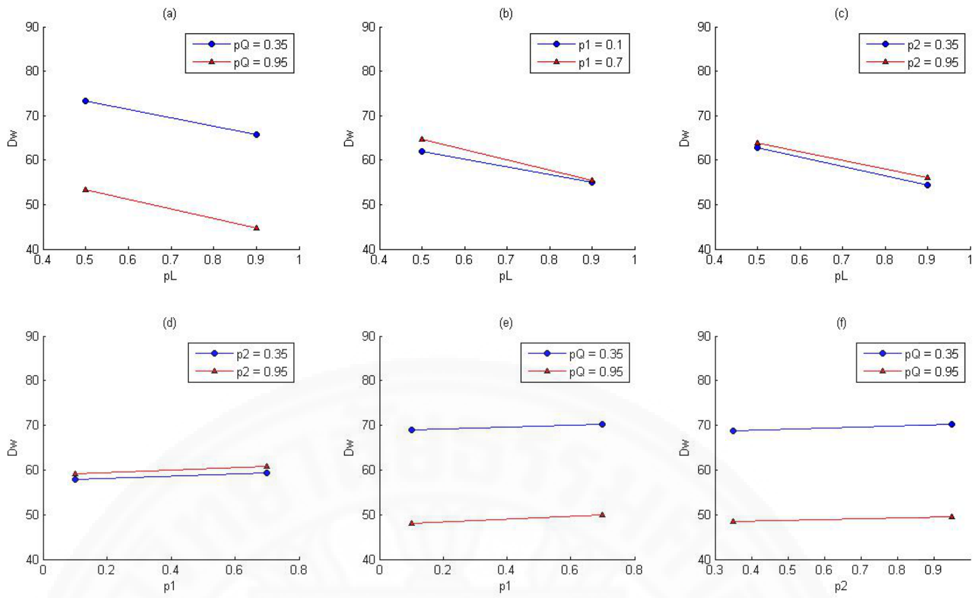


Figure 4.123: The weighted D-optimality criterion values in cuboidal region for GA designs for $k = 4$ and $N = 16$

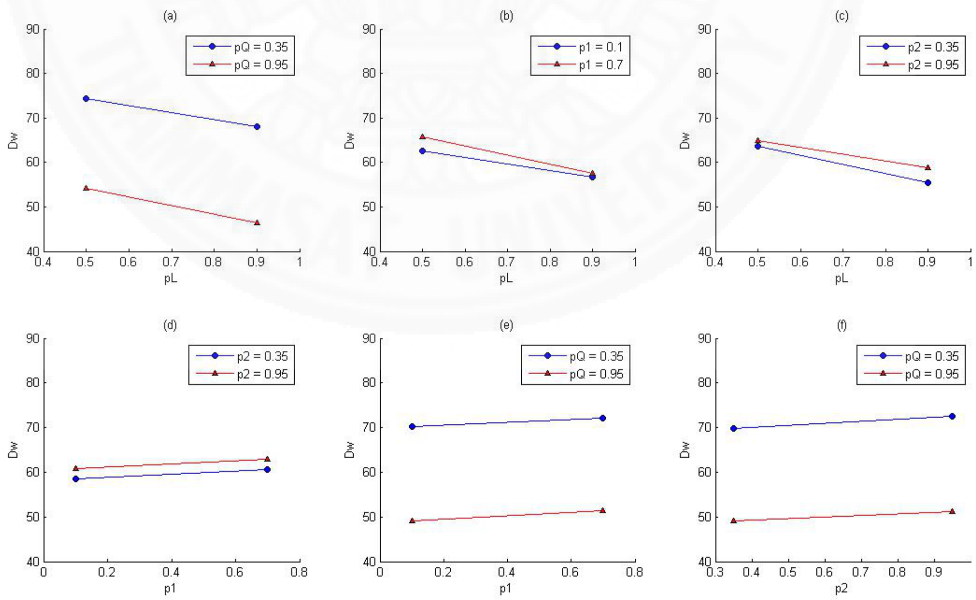


Figure 4.124: The weighted D-optimality criterion values in cuboidal region for GA designs for $k = 4$ and $N = 20$

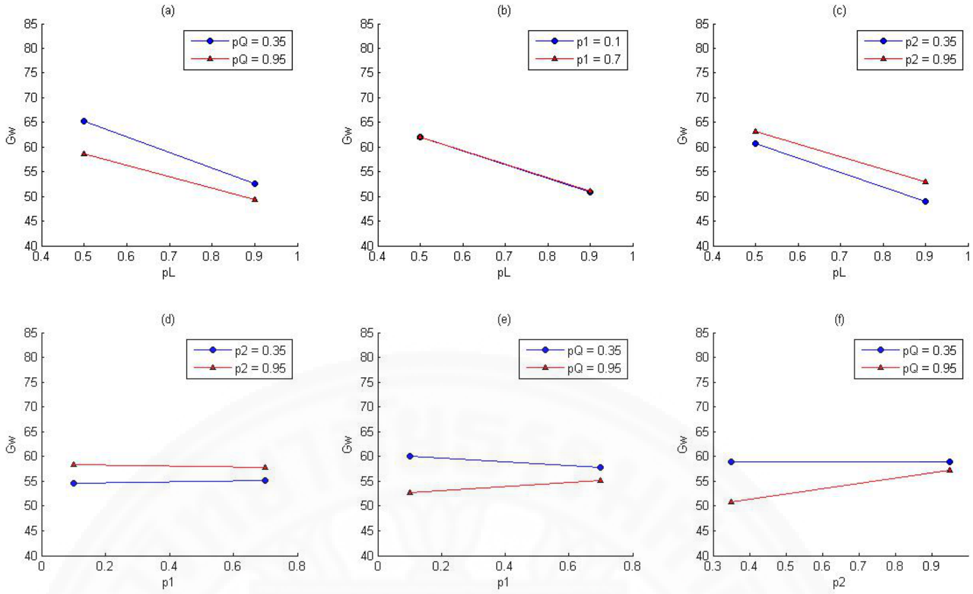


Figure 4.125: The weighted G-optimality criterion values in cuboidal region for OPTEX procedure designs for $k = 4$ and $N = 16$

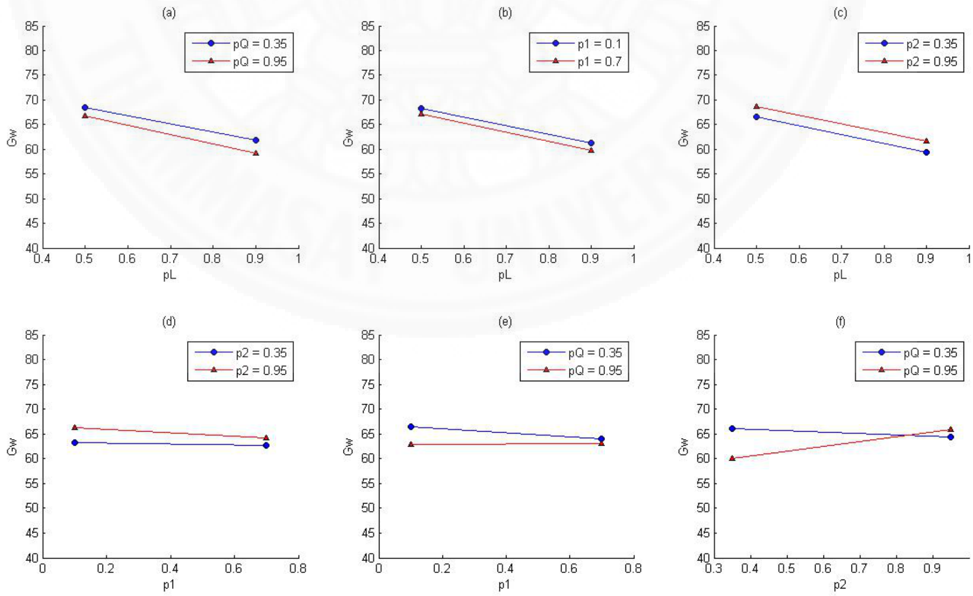


Figure 4.126: The weighted G-optimality criterion values in cuboidal region for OPTEX procedure designs for $k = 4$ and $N = 19$

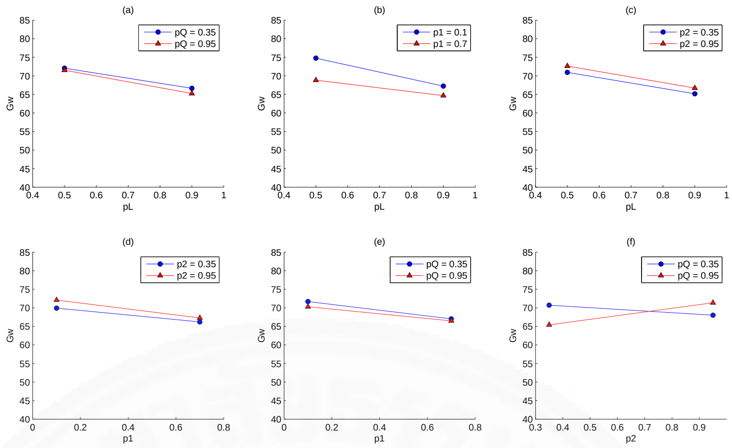


Figure 4.127: The weighted G-optimality criterion values in cuboidal region for OPTEX procedure designs for $k = 4$ and $N = 20$

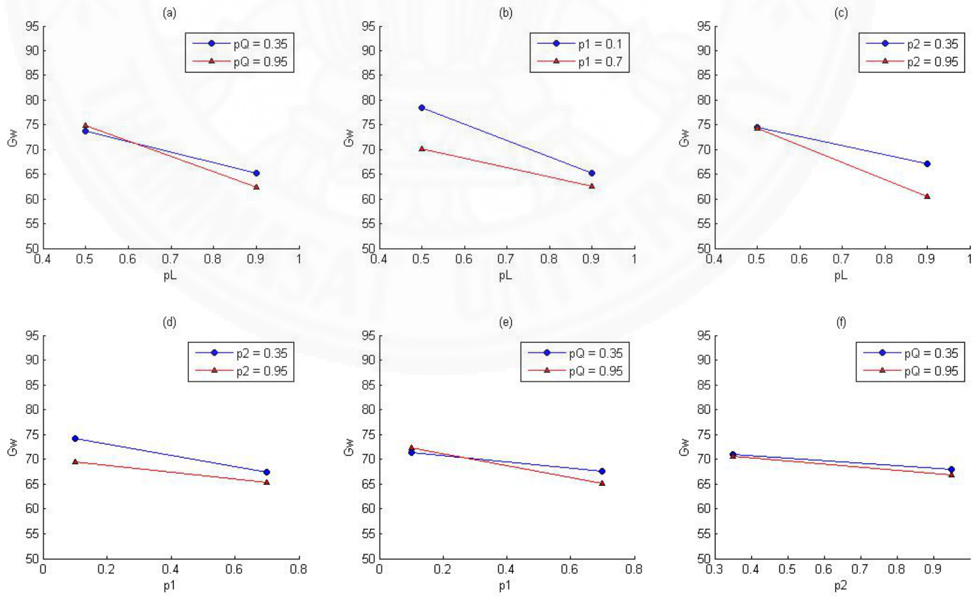


Figure 4.128: The weighted G-optimality criterion values in cuboidal region for GA designs for $k = 4$ and $N = 16$

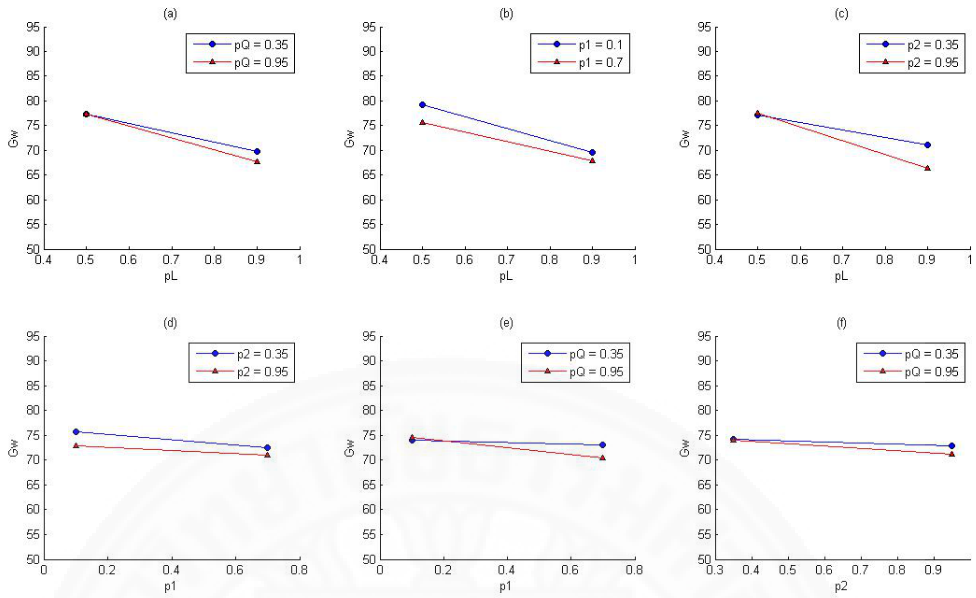


Figure 4.129: The weighted G-optimality criterion values in cuboidal region for GA designs for $k = 4$ and $N = 20$

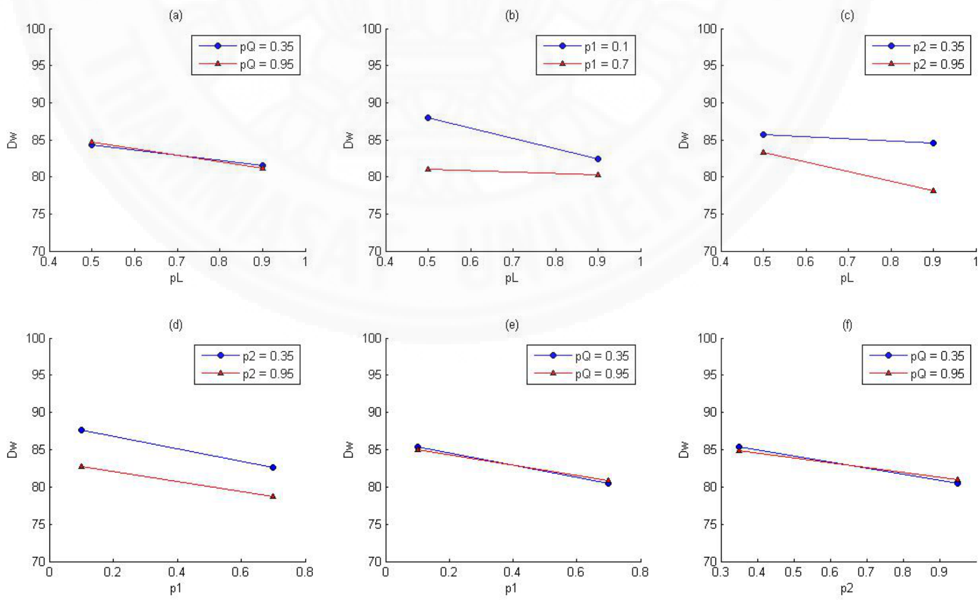


Figure 4.130: The weighted D-optimality criterion values in spherical region for OPTEX procedure designs for $k = 4$ and $N = 16$

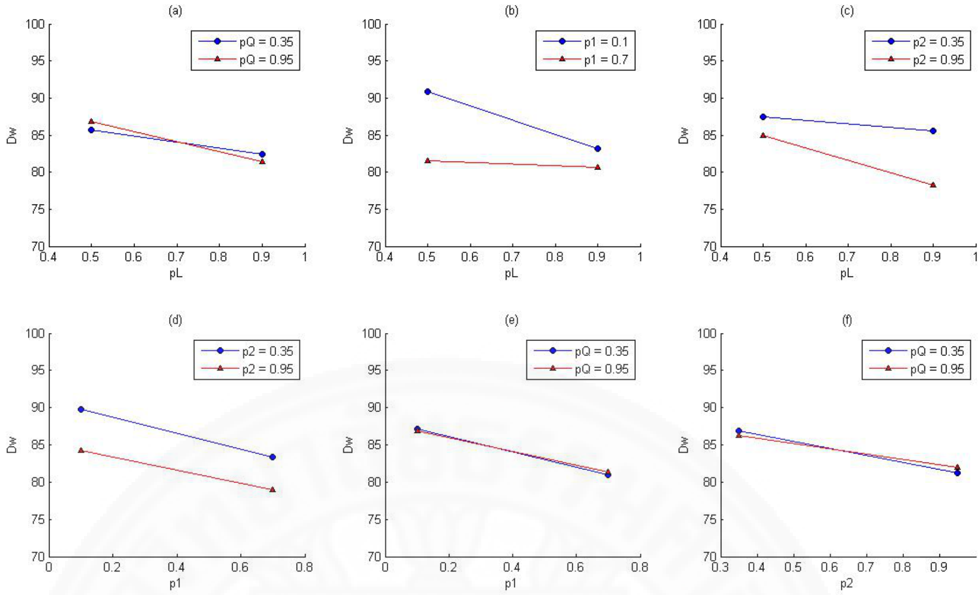


Figure 4.131: The weighted D-optimality criterion values in spherical region for OPTEX procedure designs for $k = 4$ and $N = 20$

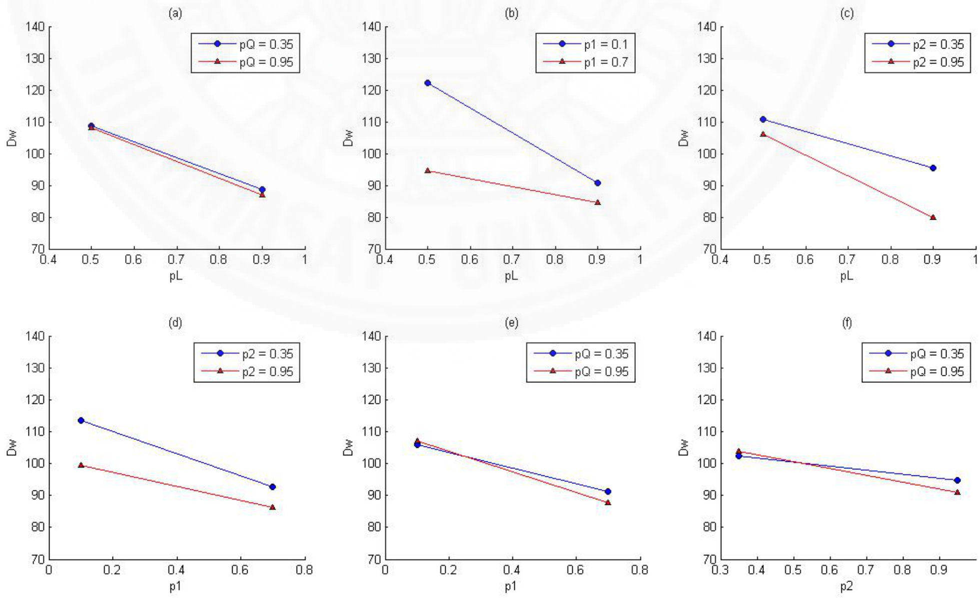


Figure 4.132: The weighted D-optimality criterion values in spherical region for GA designs for $k = 4$ and $N = 16$

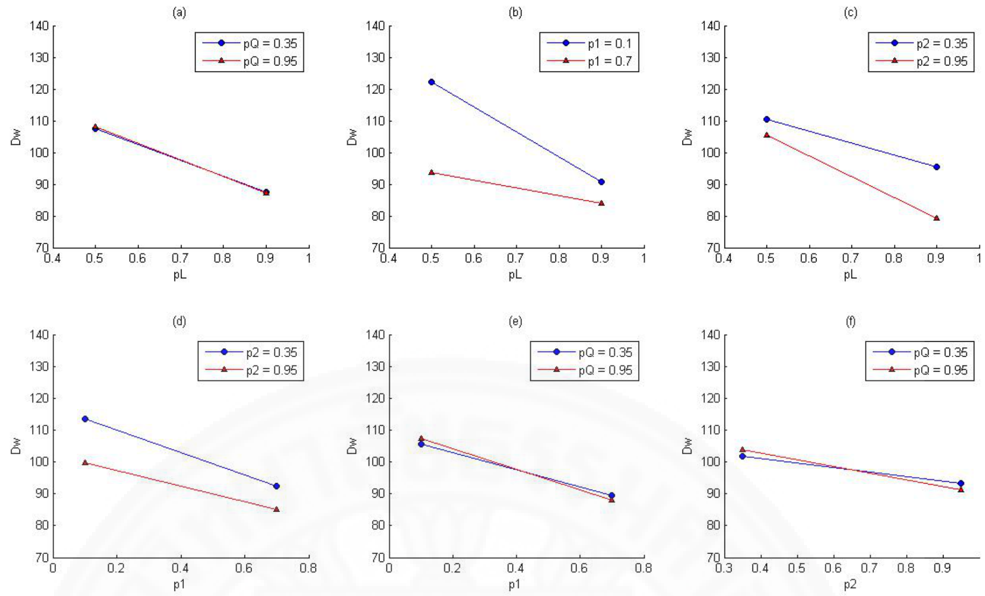


Figure 4.133: The weighted D-optimality criterion values in spherical region for GA designs for $k = 4$ and $N = 20$

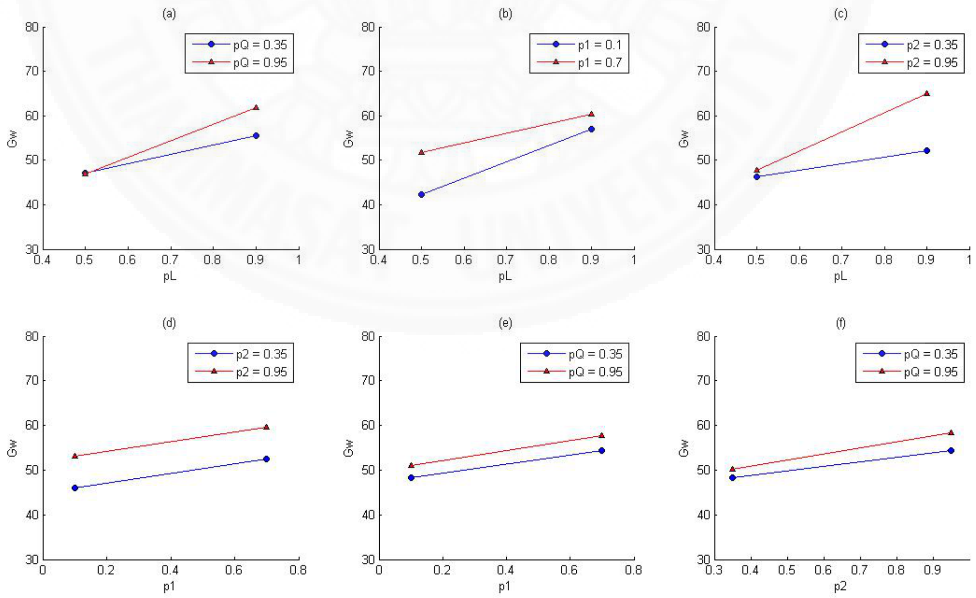


Figure 4.134: The weighted G-optimality criterion values in spherical region for OPTEX procedure designs for $k = 4$ and $N = 16$

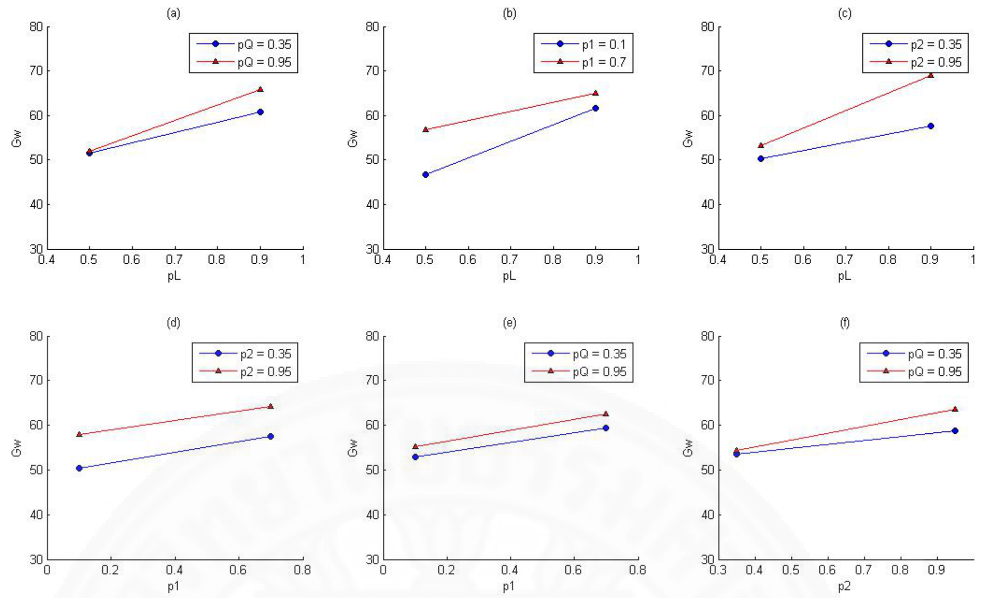


Figure 4.135: The weighted G-optimality criterion values in spherical region for OPTEX procedure designs for $k = 4$ and $N = 19$

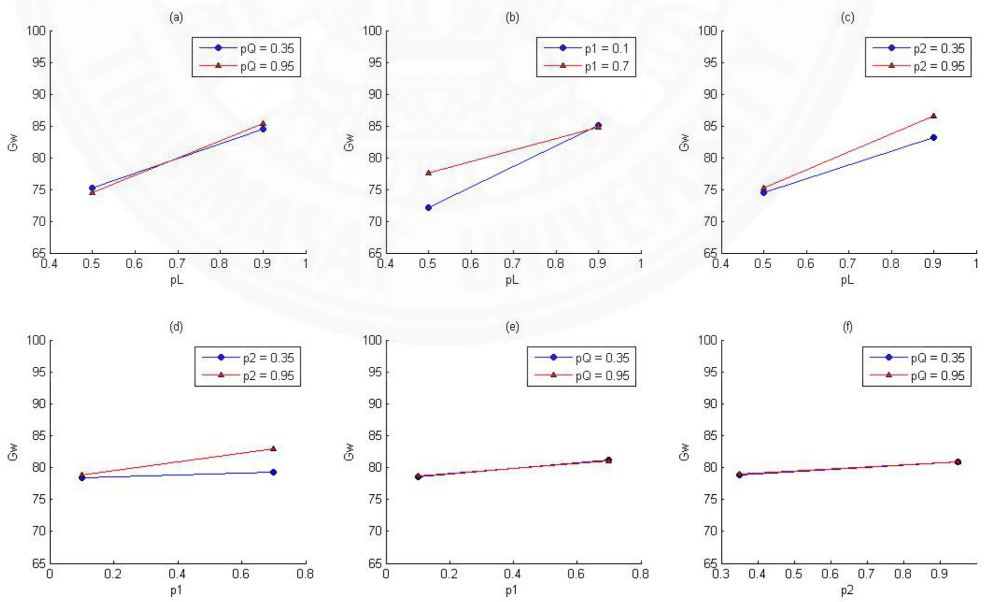


Figure 4.136: The weighted G-optimality criterion values in spherical region for GA designs for $k = 4$ and $N = 16$

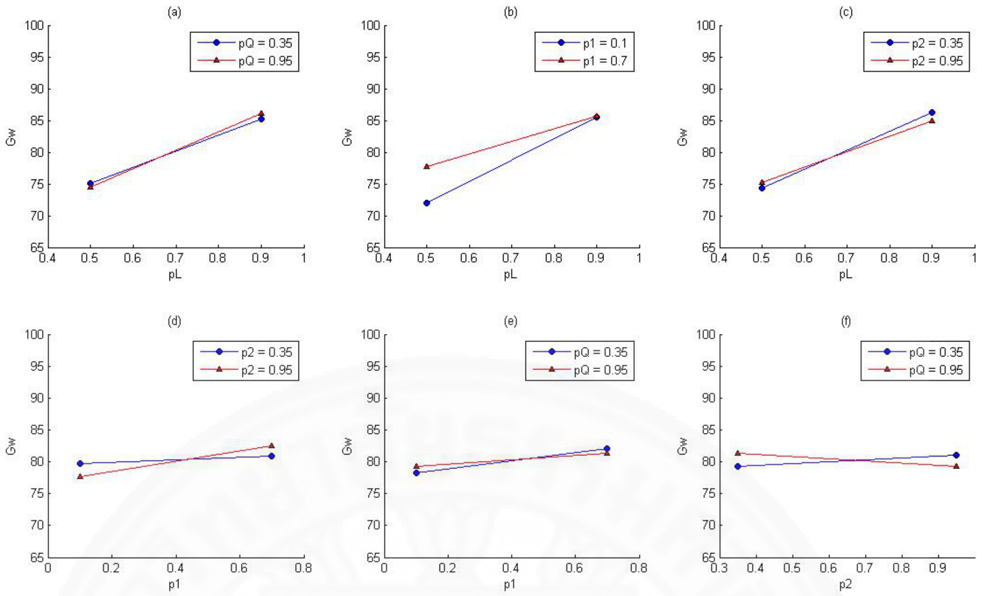


Figure 4.137: The weighted G-optimality criterion values in spherical region for GA designs for $k = 4$ and $N = 17$

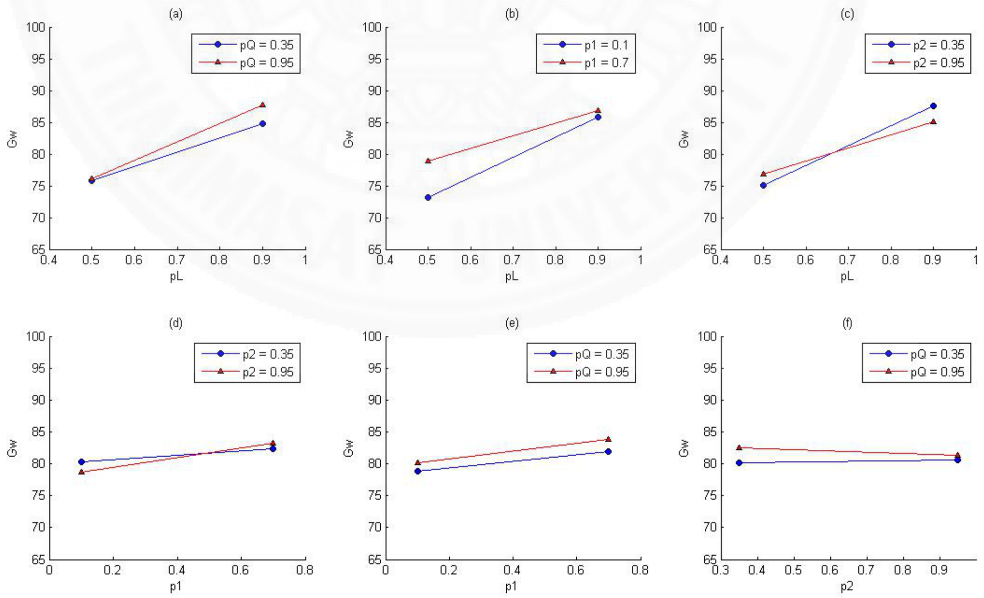


Figure 4.138: The weighted G-optimality criterion values in spherical region for GA designs for $k = 4$ and $N = 19$

4.4 Using the Catalog of Designs

In this section, the goal is to briefly describe how to use the catalog of designs that appear in the Appendix. That is, if the researcher has a specific N, k, design region (cuboidal or spherical), combination of prior probabilities (p_l, p_1, p_2, and p_q), and a criterion (D_w or G_w), show how to use the catalog to create a design using uncoded variables used in the experiment.

Example 1 : The relationship between the response variable yield (y) in a chemical precess and the two process variables reaction temperature (ξ_1) and reaction concentrate (ξ_2). The low and high levels of the process variables are 200°C and 250°C for temperature, 15% and 25% for concentrate.

From equation 2.3, Table 4.1 shows the levels used for ξ_1 and ξ_2 in the natural units of measurements.

Table 4.1: The levels of ξ_1 and ξ_2 for Chemical Process Example

Temperature (°C) ξ_1	Concentration (%) ξ_2	Coded Variable x
189.65	12.93	$-\sqrt{2}$
200.00	15.00	-1
225.00	20.00	0
250.00	25.00	1
260.35	27.07	$\sqrt{2}$

conversion for ξ_1:

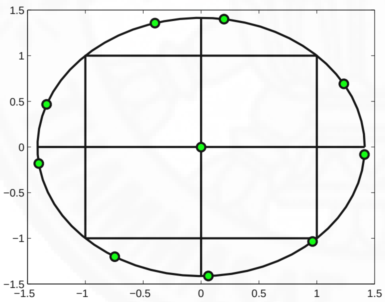
$$\begin{aligned} x_1 &= \frac{2\xi_1 - (200+250)}{250-200} \\ &= \frac{2\xi_1 - 450}{50} \\ &= \frac{\xi - 225}{25} \\ \xi_1 &= 25x_1 + 225 \end{aligned} \tag{4.1}$$

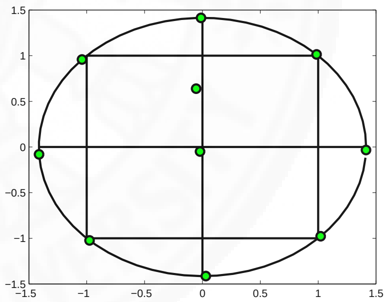
conversion for ξ_2:

$$
\begin{aligned}
x_2 &= \frac{2\xi_2 - (15+25)}{25-15} \\
&= \frac{2\xi_2 - 40}{10} \\
&= \frac{\xi - 20}{5} \\
\xi_2 &= 5x_2 + 20 \qquad (4.2)
\end{aligned}
$$

In case of the best design for spherical region, $k = 2$, $N = 10$, $p_l = 0.5$, $p_1 = 0.7$, $p_2 = 0.35$, and $p_q = 0.95$. The best design for D_w and G_w are as follow :

$$
D_w = \begin{bmatrix} 0.0632 & -1.4128 \\ -0.3991 & 1.3567 \\ -1.3351 & 0.4663 \\ -1.4026 & -0.1811 \\ 1.2339 & 0.6910 \\ -0.7457 & -1.2016 \\ 1.4118 & -0.0827 \\ 0.1985 & 1.4002 \\ 0.0000 & -0.0022 \\ 0.9636 & -1.0351 \end{bmatrix}
\qquad
G_w = \begin{bmatrix} 1.4138 & -0.0330 \\ 0.9866 & 1.0132 \\ -1.0406 & 0.9577 \\ -0.0125 & 1.4142 \\ -0.0209 & -0.0494 \\ -1.4119 & -0.0809 \\ -0.9760 & -1.0234 \\ 0.0290 & -1.4139 \\ 1.0215 & -0.9780 \\ -0.0547 & 0.6384 \end{bmatrix}
$$





From the best designs, there are conversed by Equation (4.1) and (4.2).

	D_w				G_w			
Point	x_1	x_2	ξ_1	ξ_2	x_1	x_2	ξ_1	ξ_2
1	0.0632	-1.4128	226.58	12.94	1.4138	-0.033	260.35	19.84
2	-0.3991	1.3567	215.02	26.78	0.9866	1.0132	249.67	25.07
3	-1.3351	0.4663	191.62	22.33	-1.0406	0.9577	198.99	24.79
4	-1.4026	-0.1811	189.94	19.09	-0.0125	1.4142	224.69	27.07
5	1.2339	0.6910	255.85	23.46	-0.0209	-0.0494	224.48	19.75
6	-0.7457	-1.2016	206.36	13.99	-1.4119	-0.0809	189.70	19.60
7	1.4118	-0.0827	260.30	19.59	-0.9760	-1.0234	200.60	14.88
8	0.1985	1.4002	229.96	27.00	0.0290	-1.4139	225.73	12.93
9	0.0000	-0.0022	225.00	19.99	1.0215	-0.978	250.54	15.11
10	0.9636	-1.0351	249.09	14.82	-0.0547	0.6384	223.63	23.19

Example 2 : The relationship between the number of cycles to failure of worsted gain (y) and three factors defined as follows:

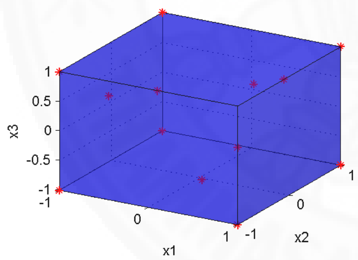
$$\text{Length of test specimen (mm):} \quad x_1 = \frac{\xi_1 - 300}{50}$$

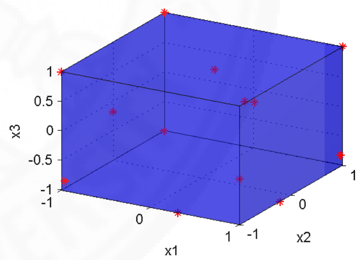
$$\text{Amplitude of load cycle (MM):} \quad x_2 = \xi_2 - 9$$

$$\text{Load (grams):} \quad x_3 = \frac{\xi_3 - 45}{5}$$

In case of the best design for cuboidal region, $k = 3$, $N = 13$, $p_l = 0.9$, $p_1 = 0.7$, $p_2 = 0.95$, and $p_q = 0.95$. The best design for D_w and G_w are as follow :

$$D_w = \begin{bmatrix} 1.0000 & 1.0000 & -1.0000 \\ 1.0000 & 1.0000 & 1.0000 \\ 0.1000 & -1.0000 & 1.0000 \\ -1.0000 & 1.0000 & 1.0000 \\ 0.0225 & 1.0000 & 0.0887 \\ 0.0477 & -0.0458 & -1.0000 \\ -1.0000 & -0.0427 & 0.1172 \\ -1.0000 & -1.0000 & 1.0000 \\ -1.0000 & -1.0000 & -1.0000 \\ 1.0000 & -0.1013 & 1.0000 \\ 1.0000 & -1.0000 & 0.2964 \\ -1.0000 & 1.0000 & -1.0000 \\ 1.0000 & -1.0000 & -1.0000 \end{bmatrix} \qquad G_w = \begin{bmatrix} 0.3031 & -1.0000 & -1.0000 \\ 1.0000 & -0.2230 & -1.0000 \\ 1.0000 & -1.0000 & -0.2331 \\ -1.0000 & 0.0071 & -0.1754 \\ 0.0114 & 1.0000 & -0.2239 \\ 0.2555 & -0.1961 & 1.0000 \\ 0.9762 & -0.8556 & 1.0000 \\ 0.9741 & 1.0000 & -0.8412 \\ 1.0000 & 1.0000 & 1.0000 \\ -1.0000 & -0.9303 & -0.8779 \\ -1.0000 & -1.0000 & 1.0000 \\ -1.0000 & 0.9941 & -1.0000 \\ -1.0000 & 1.0000 & 1.0000 \end{bmatrix}$$





From the best designs, there are conversed by Equation (4.1) and (4.2).

Point	D_w x_1	x_2	x_3	ξ_1	ξ_2	ξ_3	G_w x_1	x_2	x_3	ξ_1	ξ_2	ξ_3
1	1.0000	1.0000	-1.0000	350	10	40	0.3031	-1.0000	-1.0000	315.1550	8	40
2	1.0000	1.0000	1.0000	350	10	50	1.0000	-0.2230	-1.0000	350	8.7770	40
3	0.1000	-1.0000	1.0000	305	8	50	1.0000	-1.0000	-0.2331	350	8	43.8345
4	-1.0000	1.0000	1.0000	250	10	50	-1.0000	0.0071	-0.1754	250	9.0071	44.1230
5	0.0225	1.0000	0.0887	301.1250	10	45.4435	0.0114	1.0000	-0.2239	300.5700	10	43.8805
6	0.0477	-0.0458	-1.0000	302.3850	8.9542	40	0.2555	-0.1961	1.0000	312.7750	8.8039	50
7	-1.0000	-0.0427	0.1172	250	8.9573	45.586	0.9762	-0.8556	1.0000	348.8100	8	50
8	-1.0000	-1.0000	1.0000	250	8	50	0.9741	1.0000	-0.8412	348.7050	10	40.7940
9	-1.0000	-1.0000	-1.0000	250	8	40	1.0000	1.0000	1.0000	350	10	50
10	1.0000	-0.1013	1.0000	350	8.8987	50	-1.0000	-0.9303	-0.8779	250	8.0697	40.6105
11	1.0000	-1.0000	0.2964	350	8	46.482	-1.0000	-1.0000	1.0000	250	8	50
12	-1.0000	1.0000	-1.0000	250	10	40	-1.0000	0.9941	-1.0000	250	9.9941	40
13	1.0000	-1.0000	-1.0000	350	8	40	-1.0000	1.0000	1.0000	250	10	50

4.5 Summarize

The results can be summarized in two parts. The first part summarizes the main effect of p_i probabilities. The second part summarizes the interaction effects across each combination of p_i.
The meaning of the symbols in Table 4.1 to Table 4.6 are :

Symbol	Meaning
$+$	increasing in main effect
$-$	decreasing in main effect
0	no or very little change in main effect
I_1	interaction effect that increasing in one effect
I_2	interaction effect that increasing in two effects
D_1	interaction effect that decreasing in one effect
D_2	interaction effect that decreasing in two effects
I_D	interaction effect that one decreas and one increase effect
P_I	interaction effect that parallel increasing
P_D	interaction effect that parallel decreasing
P_H	interaction effect that parallel horizontal

The summary results are shown in Table 4.2 to 4.7. For the details of D_w and G_w values for CCDs, BBDs, and computer-generated desings evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2 are shown in Appendix A. The catalog of computer-generated designs from the Genetic Algorithms for cuboidal and spherical designs are shown in Appendix B and Appendix C, respectively.

Table 4.2: The results of the effect of weighted probabilities for the cuboidal designs in $k = 2$

The effect	D_w			G_w		
	CCD	OPTEX	GA	CCD	OPTEX	GA
p_l	$-$	$-$	$-$	$-$	$-$	$-$
p_1	0	0	0	$-$	$-$	0
p_2	0	0	0	$-$	$-$	0
p_q	$-$	$-$	$-$	$+$	$+$	$+$
p_lp_q	P_D	P_D	P_D	D_2	P_D	D_2
p_lp_1	D_2	P_D	P_D	D_2	D_2	D_2
p_lp_2	D_2	P_D	P_D	D_2	D_2	P_D
p_1p_2	P_H	P_H	P_H	P_D	P_D	P_H
p_1p_q	P_H	P_H	P_H	P_D	P_D	D_1
p_2p_q	P_H	P_H	P_H	P_D	P_D	P_H

Table 4.3: The results of the effect of weighted probabilities for the spherical designs in $k = 2$

The effect	D_w			G_w		
	CCD	OPTEX	GA	CCD	OPTEX	GA
p_l	$-$	$-$	$-$	$-$	$+$	$+$
p_1	$-$	$-$	$-$	$+$	0	$+$
p_2	$-$	$-$	$-$	$+$	0	$+$
p_q	$-$	$-$	$-$	$+$	$+$	$+$
p_lp_q	P_D	D_2	D_2	D_1	I_1	I_D
p_lp_1	D_2	D_2	D_2	D_2	I_2	I_2
p_lp_2	D_2	D_2	D_2	D_2	I_2	I_1
p_1p_2	P_D	P_D	P_D	P_I	P_I	P_I
p_1p_q	P_D	P_D	P_D	P_I	P_I	P_I
p_2p_q	P_D	P_D	P_D	I_1	I_1	I_1

Table 4.4: The results of the effect of weighted probabilities for the cuboidal designs in $k = 3$

The effect	D_w				G_w			
	CCD	BBD	OPTEX	GA	CCD	BBD	OPTEX	GA
p_l	$-$	$-$	$-$	$-$	$-$	$-$	0	0
p_1	0	$-$	0	0	0	$-$	0	0
p_2	0	$-$	0	0	0	$-$	0	0
p_q	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$
p_lp_q	P_D	P_D	P_D	P_D	P_D	P_D	P_H	P_H
p_lp_1	D_2	D_2	P_D	P_D	D_2	D_2	D_1	D_1
p_lp_2	D_2	D_2	P_D	P_D	D_2	D_2	P_H	P_H
p_1p_2	P_H	P_D	P_H	P_H	P_H	P_D	P_H	P_H
p_1p_q	P_H	P_D	P_H	P_H	P_H	P_D	P_H	P_H
p_2p_q	P_H	P_D	P_H	P_H	I_1	P_D	I_1	I_1

Table 4.5: The results of the effect of weighted probabilities for the spherical designs in $k = 3$

The effect	D_w				G_w			
	CCD	BBD	OPTEX	GA	CCD	BBD	OPTEX	GA
p_l	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$
p_1	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$
p_2	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$
p_q	$-$	$-$	$-$	$-$	$+$	$-$	$+$	0
p_lp_q	P_D	P_D	D_2	P_D	I_2	I_1	I_2	I_2
p_lp_1	D_2	D_2	D_2	D_2	I_2	I_1	I_2	I_2
p_lp_2	D_2	D_2	D_2	D_2	I_2	I_1	I_2	I_1
p_1p_2	P_D	P_D	P_D	P_D	P_I	P_I	P_I	I_1
p_1p_q	P_D	P_D	P_D	P_D	P_I	P_I	P_I	P_I
p_2p_q	P_D	P_D	D_2	P_D	P_I	P_I	P_I	P_I

Table 4.6: The results of the effect of weighted probabilities for the cuboidal designs in $k = 4$

The effect	D_w				G_w			
	CCD	BBD	OPTEX	GA	CCD	BBD	OPTEX	GA
p_l	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
p_1	0	$-$	0	0	0	$-$	$-$	0
p_2	0	$-$	0	0	$+$	$-$	0	0
p_q	$-$	$-$	$-$	$-$	$-$	$+$	0	0
p_lp_q	P_D	P_D	P_D	P_D	P_D	P_D	P_D	D_2
p_lp_1	P_D	D_2	D_2	D_2	D_2	D_2	D_1	D_2
p_lp_2	D_2	D_2	D_2	D_2	D_2	D_2	P_D	D_2
p_1p_2	P_H	P_D	P_D	P_H	P_H	P_D	P_D	P_H
p_1p_q	P_H	D_2	P_D	P_H	D_1	P_D	P_D	P_H
p_2p_q	P_H	D_2	P_D	P_H	I_1	P_D	I_D	P_H

Table 4.7: The results of the effect of weighted probabilities for the spherical designs in $k = 4$

The effect	D_w				G_w			
	CCD	BBD	OPTEX	GA	CCD	BBD	OPTEX	GA
p_l	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$
p_1	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$
p_2	$-$	$-$	$-$	$-$	$+$	$+$	$+$	0
p_q	$+$	$+$	0	0	$+$	$+$	0	0
p_lp_q	P_D	P_D	P_D	P_D	D_2	I_2	I_2	I_2
p_lp_1	D_1	D_1	D_1	D_2	D_2	I_2	I_2	I_2
p_lp_2	D_1	D_1	D_1	D_2	D_2	I_2	I_2	I_2
p_1p_2	P_D	P_D	P_D	D_2	P_I	P_I	P_I	P_H
p_1p_q	P_D	P_D	P_D	P_D	P_I	P_I	P_I	P_H
p_2p_q	P_D	P_D	P_D	P_D	P_I	P_I	P_I	P_H

CHAPTER 5

CONCLUSIONS AND RECOMMENDATION

In this study, weighted D and G efficiencies were found for commonly-used response surface designs. The designs have two different design regions, the k-dimensional hypercube and the k-dimensional hypersphere, and the number of design variables (k) is 2, 3, or 4. The conclusions regarding design robustness can be classified in six parts as follows:

PART 1 : The 2-dimensional hypercube region

PART 2 : The 2-dimensional hypersphere region

PART 3 : The 3-dimensional hypercube region

PART 4 : The 3-dimensional hypersphere region

PART 5 : The 4-dimensional hypercube region

PART 6 : The 4-dimensional hypersphere region

5.1 PART 1 : The 2-dimensional hypercube region

For all designs (CCDs, OPTEX designs, and GA designs), the results are similar. For the weighted D-optimality criterion, the D_w values are robust to change in p_1 and p_2 and slightly less robust as N increases. However, the D_w values are sensitive to change in p_l and p_q. In this situation, we can select any of the CCDs, OPTEX designs, or GA designs.

For weighted G-optimality criterion for CCDs and OPTEX designs, the results are similar. That is the G_w values are slightly sensitive (weakly robust) to change in p_1 and p_2. However, the G_w values are very sensitive to change in p_l and p_q. But for GA designs, the G_w values are robust to change in p_1 and p_2. Then, a GA design is the best design for this situation.

5.2 PART 2 : The 2-dimensional hypersphere region

For all designs (CCDs, OPTEX design, and GA design) the results are similar. For weighted D-optimality criterion, the D_w value are robust to change in p_1 and p_2 and slightly less robust as N increases. However, the D_w values are sensitive to change in p_l and p_q. In this situation, we can select any of the CCDs, OPTEX designs, or GA designs.

For weighted G-optimality criterion for CCDs and GA designs, the results are similar. That is the G_w values are slightly sensitive (weakly robust) to change in p_1 and p_2. However, the G_w values are very sensitive to change in p_l and p_q. But for OPTEX designs, the G_w values are slightly sensitive to change in p_1, p_2 and p_q. However, the G_w values are very sensitive to change in p_l. Then, an OPTEX design is the best design for this situation.

5.3 PART 3 : The 3-dimensional hypercube region

For all designs (CCDs, BBDs, OPTEX designs, and GA designs), the results are similar. For weighted D-optimality criterion, the D_w value are robust to change in p_1 and p_2 and slightly less robust as N increases. However, the D_w values are sensitive to change in p_l and p_q, while the D_w values for CCDs, OPTEX designs and GA designs change very little to changes in p_1 and p_2. In this situation, we can select any of the CCDs, OPTEX designs, or GA designs.

For the weighted G-optimality criterion, for CCDs the G_w values are slightly sensitive (weakly robust) to change in p_1 and p_2. However, the G_w values are robust to change in p_l and very sensitive to change in p_q. But for BBDs, OPTEX designs and GA design, the G_w values are slightly sensitive to change in

p_1, p_2 and p_q. However, the G_w values are very sensitive to change in p_l. While, the G_w values decrease for an increase in a main effect p_l probability for BBDs, there is very little change for OPTEX designs and GA designs. Thus, CCDs are the best designs for this situation.

5.4 PART 4 : The 3-dimensional hypersphere region

The D_w values are weakly robust to change in p_1 and p_2 for all designs. While, the D_w values for CCDs are weakly robust to change in p_l, they are very sensitive for BBDs, OPTEX designs, and GA designs. The D_w values are robust to change in p_q for CCDs, OPTEX designs, and GA designs but are sensitive for BBDs. Thus, CCDs are the best designs for this situation.

The G_w values are slightly sensitive to change in p_l, p_1 and p_2 for CCDs, BBDs and OPTEX designs. However, the G_w values are robust to change in p_q for CCDs and BBDs, but are very sensitive for OPTEX designs. While, the G_w values are slightly sensitive to change in p_1, and are very sensitive to change in p_l, p_q and p_2 for GA designs. Then, CCDs and BBDs are the best designs for this situation.

5.5 PART 5 : The 4-dimensional hypercube region

For all designs (CCDs, OPTEX designs, and GA designs), the results are similar. For weighted D-optimality criterion, the D_w value are robust to change in p_1 and p_2, and slightly less robust as N increases. However, the D_w values are sensitive to change in p_l and p_q. But for BBDs, the D_w values are robust to change in p_1, p_2 and p_q, and slightly less robust as N increases. However, the D_w values are sensitive to change in p_l. Thus, BBDs are the best designs for this situation.

For OPTEX designs and GA designs, the results are similar. The G_w values are slightly sensitive to change in p_1, p_2, and p_q. However, the G_w values are very sensitive to change in p_l. For CCDs, the G_w values are robust to change in p_1 and p_q. While for BBDs, the G_w values are very sensitive to change in p_1. Thus, CCDs are the best designs for this situation.

5.6 PART 6 : The 4-dimensional hypersphere region

For OPTEX designs and GA designs, the results are similar. The D_w values are robust to change in p_q, and are weakly robust in p_1 and p_2. However, the D_w values are sensitive to change in p_l. For CCDs, the D_w values are weakly robust to change in p_l, and are robust to change in p_q. While for BBDs, the D_w values are robust to change in p_l and p_q, and are slightly sensitive to change in p_1 and p_2. Thus, CCDs are the best designs for this situation.

For CCDs, the G_w values are slightly sensitive to change in p_l, p_1, and p_2. However, the G_w values are sensitive to change in p_2. While for BBDs, the G_w values are slightly sensitive to change in p_q, and are very sensitive to change in p_l, p_1, and p_2. For the OPTEX designs, the G_w values are slightly sensitive to change in p_l, p_1, and p_2. However, the G_w values are very sensitive to change in p_q. For GA designs, the G_w values are robust to change in p_q and p_2, and are slightly sensitive to change in p_1. However, the G_w values are very sensitive to change in p_l. Thus, the GA designs are the best designs for this situation.

From the conclusions in PART 1 to PART 6, and from the actual values of the D_w and G_w efficiencies, we summarize the results for design robustness and design efficiency in Table 5.1:

For $k = 2$ design variables:

From Table 5.1, the D_w values for GA cuboidal designs increase as N increases, and are higher than D_w values for CCDs and OPTEX designs. Thus, the GA designs are the best for D_w values in a cuboidal design region because they are both robust and have the highest D_w efficiencies.

The G_w values in a cuboidal design region for CCDs, OPTEX designs, and GA designs are similar. Thus, the GA designs are the best for G_w values in cuboidal regions because they are both robust and have high G_w efficiencies.

The D_w values in a spherical design region for OPTEX designs and GA designs are similar, and are higher than the D_w values of CCDs. Thus, the OPTEX designs and GA designs are the best for D_w values in a spherical region because they are both robust and have high D_w efficiencies.

Table 5.1: Conclusions regarding the most robust and the most efficient designs for each combination of weighted optimality criteria, design region, and number of design variables.

k		2		3		4	
		Robustness	Efficient	Robustness	Efficient	Robustness	Efficient
cube	D_w	CCD OPTEX GA	GA	CCD OPTEX GA	GA	BBD	GA
	G_w	GA	CCD OPTEX GA	CCD	CCD	CCD	GA
sphere	D_w	CCD OPTEX GA	OPTEX GA	CCD	GA	CCD	GA
	G_w	OPTEX	GA	BBD	GA	CCD GA	GA

The G_w values in a spherical design region for GA designs are higher than the G_w values for CCDs and OPTEX designs, but the OPTEX designs are the most robust.

For $k = 3$ design variables:

The D_w values for GA cuboidal designs and $k = 3$ are almost equal as N increases, and are higher than the D_w values for CCDs, BBDs and OPTEX designs. Thus, the GA designs are the best for D_w values in a cuboidal region because they are robust and have the highest D_w efficiencies.

The G_w values in a cuboidal design region for CCDs are higher than the G_w values for BBDs, OPTEX designs, and GA designs. Thus, the CCDs are the best for G_w values in a cuboidal region because they are robust and have high G_w efficiencies.

The D_w and G_w values in a spherical design region for GA designs are higher than the D_w and G_w values for CCDs, BBDs and OPTEX designs, but the BBDs are the most robust.

For $k = 4$ design variables:

The D_w values for GA cuboidal designs are almost equal as N increases, and are higher than the D_w values for CCDs, BBDs and OPTEX designs. However, in a cuboidal design region, the BBDs are the most robust.

The G_w values in a cuboidal design region slightly increase for GA designs, but slightly decrease for CCDs, as N increases. The G_w values for GA designs are higher than the G_w values for CCDs, BBDs and OPTEX designs as N increases, but the CCDs are the most robust.

The D_w values in a spherical design region for GA designs are higher than the D_w values for CCDs, BBDs and OPTEX designs, but the CCDs are the most robust.

The G_w values in a spherical design region for GA designs are higher than the G_w values for CCDs, BBDs and OPTEX designs. Thus, the GA designs are the best for D_w values in spherical designs and because there are robust and have the highest D_w efficiencies.

5.7 Recommendation for Future Research

In this dissertation, the research was focused on CCDs, BBDs, and computer-generated designs (OPTEX procedure designs and GA designs) in two different regions (hypercube and hypersphere), and each design having 2, 3, or 4 design variables. The weighted D_w and G_w optimality criteria were calculated for each reduced model assuming weak heredity (WH) for the 16 combination of model probabilities. The recommendations for future research are as follows:

1. The proposed research will consider two additional criteria for evaluating and comparing designs. The research can be extended to evaluating weighted A and IV-optimality criteria.

2. The results for the reduced models assuming weak heredity (WH) in this

research can be compared to results for the reduced models assuming strong heredity (SH).

3. The proposed research will be extended to design regions other the hypersphere or hypercube. For example, a design region that is a hypercylinder or a simplex.

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APPENDIXES

APPENDIX A

D_W AND G_W VALUES FOR CCDS, BBDS, AND COMPUTER GENERATED DESINGS EVALUATED AT 16 FACTORIAL COMBINATIONS OF P_L, P_Q, P_1, AND P_2

Table A.1: D_w and G_w values for CCDs, $k = 2$, $N = 9, 10, \ldots, 14$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2

N	p_l	p_1	p_2	p_q	Cuboidal D_w	Cuboidal G_w	Spherical D_w	Spherical G_w
9	0.5	0.1	0.35	0.35	75.6720	88.0203	88.5595	75.1583
	0.5	0.1	0.35	0.95	63.8690	96.9489	80.8196	68.8006
	0.5	0.1	0.95	0.35	74.8802	86.3088	86.8722	75.1924
	0.5	0.1	0.95	0.95	63.7798	94.7116	80.0728	70.5079
	0.5	0.7	0.35	0.35	72.5132	83.2914	83.3922	78.2120
	0.5	0.7	0.35	0.95	63.0008	90.8327	77.5450	74.0348
	0.5	0.7	0.95	0.35	71.7215	81.5799	81.7049	78.2461
	0.5	0.7	0.95	0.95	62.9116	88.5953	76.7982	75.7420
	0.9	0.1	0.35	0.35	63.6009	82.1647	81.6268	76.4823
	0.9	0.1	0.35	0.95	49.0880	94.4893	69.3168	61.0521
	0.9	0.1	0.95	0.35	61.0357	76.6196	76.1599	76.5926
	0.9	0.1	0.95	0.95	48.7991	87.2403	66.8971	66.5835
	0.9	0.7	0.35	0.35	62.4638	80.4623	79.7666	77.5816
	0.9	0.7	0.35	0.95	48.7755	92.2875	68.1380	62.9364
	0.9	0.7	0.95	0.35	59.8986	74.9172	74.2997	77.6919
	0.9	0.7	0.95	0.95	48.4865	85.0385	65.7183	68.4678
10	0.5	0.1	0.35	0.35	73.3822	83.2398	85.6084	72.4015
	0.5	0.1	0.35	0.95	63.1896	91.2642	80.0215	72.6533
	0.5	0.1	0.95	0.35	72.5407	81.6270	83.8891	72.4824
	0.5	0.1	0.95	0.95	63.0020	89.6215	79.0473	74.9616
	0.5	0.7	0.35	0.35	70.0361	78.6036	80.3022	74.8088
	0.5	0.7	0.35	0.95	61.9838	86.0390	76.4414	77.1910
	0.5	0.7	0.95	0.35	69.1946	76.9907	78.5829	74.8897
	0.5	0.7	0.95	0.95	61.7962	84.3963	75.4672	79.4993
	0.9	0.1	0.35	0.35	60.4451	75.9332	77.6356	73.3247
	0.9	0.1	0.35	0.95	48.0326	88.8376	69.8730	79.2722
	0.9	0.1	0.95	0.35	57.7186	70.7076	72.0651	73.5867
	0.9	0.1	0.95	0.95	47.4248	83.5154	66.7166	86.7510
	0.9	0.7	0.35	0.35	59.2405	74.2641	75.7254	74.1913
	0.9	0.7	0.35	0.95	47.5985	86.9565	68.5842	80.9058
	0.9	0.7	0.95	0.35	56.5140	69.0385	70.1549	74.4534
	0.9	0.7	0.95	0.95	46.9906	81.6344	65.4277	88.3846
11	0.5	0.1	0.35	0.35	71.2342	79.3508	82.8454	68.6479
	0.5	0.1	0.35	0.95	62.1089	85.2421	78.4205	68.3494
	0.5	0.1	0.95	0.35	70.3648	78.0115	81.1247	68.5715
	0.5	0.1	0.95	0.95	61.8607	83.0436	77.3417	70.3906
	0.5	0.7	0.35	0.35	67.7597	75.3648	77.4597	70.9986
	0.5	0.7	0.35	0.95	60.6680	81.6356	74.6494	72.5592
	0.5	0.7	0.95	0.35	66.8903	74.0256	75.7390	70.9222
	0.5	0.7	0.95	0.95	60.4198	79.4371	73.5706	74.6004
	0.9	0.1	0.35	0.35	57.4685	71.3709	73.7797	67.9377
	0.9	0.1	0.35	0.95	46.3884	80.2746	68.2308	72.1527
	0.9	0.1	0.95	0.35	54.6517	67.0318	68.2045	67.6901
	0.9	0.1	0.95	0.95	45.5842	73.1514	64.7356	78.7662
	0.9	0.7	0.35	0.35	56.2176	69.9360	71.8408	68.7839
	0.9	0.7	0.35	0.95	45.8697	78.9762	66.8732	73.6682
	0.9	0.7	0.95	0.35	53.4008	65.5969	66.2656	68.5363
	0.9	0.7	0.95	0.95	45.0655	71.8530	63.3780	80.2817
12	0.5	0.1	0.35	0.35	69.2648	75.0086	80.3236	65.5841
	0.5	0.1	0.35	0.95	60.9009	80.5942	76.6092	65.2447
	0.5	0.1	0.95	0.35	68.3806	73.2040	78.6162	65.4801
	0.5	0.1	0.95	0.95	60.6133	78.0319	75.4795	67.0797
	0.5	0.7	0.35	0.35	65.7013	70.5117	74.8968	67.5045
	0.5	0.7	0.35	0.95	59.2921	76.0690	72.7196	68.8613
	0.5	0.7	0.95	0.35	64.8172	68.7071	73.1893	67.4005
	0.5	0.7	0.95	0.95	59.0044	73.5067	71.5899	70.6964
	0.9	0.1	0.35	0.35	54.7430	64.9074	70.2354	62.5762
	0.9	0.1	0.35	0.95	44.6002	75.7644	65.9602	66.4498
	0.9	0.1	0.95	0.35	51.8783	59.0604	64.7031	62.2394
	0.9	0.1	0.95	0.95	43.6681	67.4626	62.2999	72.3952
	0.9	0.7	0.35	0.35	53.4602	63.2885	68.2817	63.2675
	0.9	0.7	0.35	0.95	44.0210	74.1353	64.5599	67.7518
	0.9	0.7	0.95	0.35	50.5955	57.4415	62.7495	62.9307
	0.9	0.7	0.95	0.95	43.0889	65.8335	60.8996	73.6972

Table A.1 D_w and G_w values for CCDs, $k = 2$, $N = 9, 10, \ldots, 14$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2 (Continued)

					Cuboidal		**Spherical**	
N	p_l	p_1	p_2	p_q	D_w	G_w	D_w	G_w
13	0.5	0.1	0.35	0.35	67.4712	71.9978	78.0353	62.7974
	0.5	0.1	0.35	0.95	59.6782	76.9911	74.7821	61.7118
	0.5	0.1	0.95	0.35	66.5804	70.2016	76.3488	62.5602
	0.5	0.1	0.95	0.95	59.3641	74.8317	73.6296	63.4192
	0.5	0.7	0.35	0.35	63.8465	67.8312	72.5929	64.7080
	0.5	0.7	0.35	0.95	57.9472	73.2638	70.8217	65.5584
	0.5	0.7	0.95	0.35	62.9558	66.0349	70.9064	64.4708
	0.5	0.7	0.95	0.95	57.6331	71.1045	69.6692	67.2657
	0.9	0.1	0.35	0.35	52.2697	61.0858	67.0176	58.5441
	0.9	0.1	0.35	0.95	42.8293	71.6977	63.5372	61.4954
	0.9	0.1	0.95	0.35	49.3837	55.2659	61.5534	57.7755
	0.9	0.1	0.95	0.95	41.8117	64.7014	59.8033	67.0272
	0.9	0.7	0.35	0.35	50.9648	59.5858	65.0584	59.2319
	0.9	0.7	0.35	0.95	42.2061	70.3558	62.1115	62.8802
	0.9	0.7	0.95	0.35	48.0788	53.7659	59.5942	58.4633
	0.9	0.7	0.95	0.95	41.1885	63.3596	58.3776	68.4120
14	0.5	0.1	0.35	0.35	65.8389	68.9725	75.9584	60.5079
	0.5	0.1	0.35	0.95	58.4899	73.2937	73.0154	59.2575
	0.5	0.1	0.95	0.35	64.9471	67.6419	74.2971	60.3273
	0.5	0.1	0.95	0.95	58.1580	71.0979	71.8565	60.6528
	0.5	0.7	0.35	0.35	62.1730	64.8541	70.5181	62.2928
	0.5	0.7	0.35	0.95	56.6691	69.6349	69.0167	62.7412
	0.5	0.7	0.95	0.35	61.2811	63.5236	68.8568	62.1122
	0.5	0.7	0.95	0.95	56.3371	67.4390	67.8578	64.1365
	0.9	0.1	0.35	0.35	50.0284	56.5785	64.1029	55.1237
	0.9	0.1	0.35	0.95	41.1386	66.4450	61.1433	57.6258
	0.9	0.1	0.95	0.35	47.1387	52.2676	58.7202	54.5385
	0.9	0.1	0.95	0.95	40.0632	59.3303	57.3885	62.1467
	0.9	0.7	0.35	0.35	48.7087	55.0959	62.1444	55.7663
	0.9	0.7	0.35	0.95	40.4831	65.1278	59.7038	58.8799
	0.9	0.7	0.95	0.35	45.8189	50.7850	56.7617	55.1810
	0.9	0.7	0.95	0.95	39.4077	58.0132	55.9490	63.4009

Table A.2: D_w and G_w values for CCDs, $k = 3$, $N = 15, 16, \ldots, 20$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2

					Cuboidal		Spherical	
N	p_l	p_1	p_2	p_q	D_w	G_w	D_w	G_w
15	0.5	0.1	0.35	0.35	69.2543	85.3177	90.6935	59.1270
	0.5	0.1	0.35	0.95	55.8224	91.0471	88.1954	56.0710
	0.5	0.1	0.95	0.35	68.4789	84.6103	87.3233	59.9555
	0.5	0.1	0.95	0.95	56.2673	95.6337	85.9419	59.5764
	0.5	0.7	0.35	0.35	65.9014	82.7691	81.7555	64.9214
	0.5	0.7	0.35	0.95	56.0738	88.8296	81.1236	65.2762
	0.5	0.7	0.95	0.35	65.3933	82.5420	79.3653	66.9521
	0.5	0.7	0.95	0.95	56.4516	91.3570	79.4411	68.6147
	0.9	0.1	0.35	0.35	59.2701	82.4981	85.6450	66.2159
	0.9	0.1	0.35	0.95	44.7542	85.1361	79.8193	57.4825
	0.9	0.1	0.95	0.35	57.7940	82.8746	77.5633	73.4580
	0.9	0.1	0.95	0.95	46.1334	97.1013	74.8007	67.7236
	0.9	0.7	0.35	0.35	58.5089	82.1643	83.0037	68.3532
	0.9	0.7	0.35	0.95	45.0046	85.3266	77.8326	60.7103
	0.9	0.7	0.95	0.35	57.2060	82.8525	75.5570	76.3744
	0.9	0.7	0.95	0.95	46.3403	95.9574	73.1840	70.8433
16	0.5	0.1	0.35	0.35	67.4936	82.5661	88.0098	56.6860
	0.5	0.1	0.35	0.95	55.0719	86.7974	86.8746	56.4893
	0.5	0.1	0.95	0.35	66.6231	81.0457	84.6103	57.6638
	0.5	0.1	0.95	0.95	55.3955	90.8575	84.3600	60.3034
	0.5	0.7	0.35	0.35	63.8501	80.0615	78.9230	62.1764
	0.5	0.7	0.35	0.95	54.8926	84.4943	79.4403	65.2865
	0.5	0.7	0.95	0.35	63.2710	78.3712	76.5165	64.2696
	0.5	0.7	0.95	0.95	55.1727	87.5691	77.5442	68.7334
	0.9	0.1	0.35	0.35	57.0913	81.1566	82.4048	63.1650
	0.9	0.1	0.35	0.95	43.4617	80.4869	80.2100	72.3075
	0.9	0.1	0.95	0.35	55.3854	77.2887	74.2587	70.6386
	0.9	0.1	0.95	0.95	44.5529	90.7761	74.2457	84.2211
	0.9	0.7	0.35	0.35	56.2509	80.8354	79.7369	65.2643
	0.9	0.7	0.35	0.95	43.6012	80.6133	78.0965	75.4111
	0.9	0.7	0.95	0.35	54.7339	76.8573	72.2343	73.4607
	0.9	0.7	0.95	0.95	44.6642	90.2639	72.5330	87.0869
17	0.5	0.1	0.35	0.35	65.8001	78.6433	85.4810	54.2372
	0.5	0.1	0.35	0.95	54.1752	82.6046	85.1527	53.8395
	0.5	0.1	0.95	0.35	64.8562	77.0298	82.0760	55.1167
	0.5	0.1	0.95	0.95	54.4028	86.1821	82.4959	57.2964
	0.5	0.7	0.35	0.35	61.9228	76.7129	76.2953	59.3532
	0.5	0.7	0.35	0.95	53.6492	79.6818	77.4707	62.1438
	0.5	0.7	0.95	0.35	61.2898	74.6933	73.8906	61.3179
	0.5	0.7	0.95	0.95	53.8525	81.2837	75.4640	65.3988
	0.9	0.1	0.35	0.35	55.0026	75.7472	79.2811	59.5540
	0.9	0.1	0.35	0.95	42.0884	75.1849	78.9259	67.7353
	0.9	0.1	0.95	0.35	53.1228	71.5114	71.1421	66.3738
	0.9	0.1	0.95	0.95	42.9529	82.9998	72.4971	78.5804
	0.9	0.7	0.35	0.35	54.1002	75.9841	76.6018	61.5391
	0.9	0.7	0.35	0.95	42.1399	75.6474	76.7320	70.6765
	0.9	0.7	0.95	0.35	52.4219	71.4852	69.1111	69.0621
	0.9	0.7	0.95	0.95	42.9886	82.1821	70.7245	81.3907
18	0.5	0.1	0.35	0.35	64.1913	75.9774	83.1155	52.1479
	0.5	0.1	0.35	0.95	53.2150	79.7707	83.3065	51.6018
	0.5	0.1	0.95	0.35	63.1901	74.6523	79.7198	52.6812
	0.5	0.1	0.95	0.95	53.3651	82.5974	80.5682	54.7546
	0.5	0.7	0.35	0.35	60.1240	74.2966	73.8676	57.1806
	0.5	0.7	0.35	0.95	52.4054	77.1048	75.4558	59.6938
	0.5	0.7	0.95	0.35	59.4496	72.7313	71.4754	58.9386
	0.5	0.7	0.95	0.95	52.5471	79.4677	73.3890	62.9494
	0.9	0.1	0.35	0.35	53.0281	71.7581	76.3377	57.0072
	0.9	0.1	0.35	0.95	40.7149	72.9835	77.0685	63.7925
	0.9	0.1	0.95	0.35	51.0150	69.0307	68.2462	63.5928
	0.9	0.1	0.95	0.95	41.3962	78.4442	70.3919	74.6622
	0.9	0.7	0.35	0.35	52.0767	71.6731	73.6576	58.9271
	0.9	0.7	0.35	0.95	40.6951	72.7972	74.8243	66.6442
	0.9	0.7	0.95	0.35	50.2753	68.7900	66.2164	66.3063
	0.9	0.7	0.95	0.95	41.3710	77.9574	68.5829	77.5805

Table A.2 D_w and G_w values for CCDs, $k = 3$, $N = 15, 16, \ldots, 20$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2 (Continued)

N	p_l	p_1	p_2	p_q	**Cuboidal** D_w	**Cuboidal** G_w	**Spherical** D_w	**Spherical** G_w
19	0.5	0.1	0.35	0.35	62.6722	72.8839	80.9077	50.4676
	0.5	0.1	0.35	0.95	52.2364	76.5980	81.4427	50.0883
	0.5	0.1	0.95	0.35	61.6260	71.0151	77.5309	50.9319
	0.5	0.1	0.95	0.95	52.3227	79.8607	78.6595	53.1065
	0.5	0.7	0.35	0.35	58.4495	69.5534	71.6250	54.9234
	0.5	0.7	0.35	0.95	51.1922	74.1958	73.4802	57.3414
	0.5	0.7	0.95	0.35	57.7428	68.2033	69.2522	56.3567
	0.5	0.7	0.95	0.95	51.2834	76.0022	71.3827	60.1608
	0.9	0.1	0.35	0.35	51.1741	68.3220	73.5852	54.1440
	0.9	0.1	0.35	0.95	39.3791	69.4615	74.9997	60.6477
	0.9	0.1	0.95	0.35	49.0576	63.0686	65.5661	59.4465
	0.9	0.1	0.95	0.95	39.9097	75.8008	68.1960	71.0102
	0.9	0.7	0.35	0.35	50.1836	67.6391	70.9116	55.8779
	0.9	0.7	0.35	0.95	39.3008	69.7982	72.7259	63.2220
	0.9	0.7	0.95	0.35	48.2872	62.7219	63.5430	61.8083
	0.9	0.7	0.95	0.95	39.8345	75.1938	66.3665	73.4557
20	0.5	0.1	0.35	0.35	61.2420	70.1016	78.8473	48.6972
	0.5	0.1	0.35	0.95	51.2650	73.6676	79.6114	47.8360
	0.5	0.1	0.95	0.35	60.1602	68.5278	75.4960	48.9868
	0.5	0.1	0.95	0.95	51.2981	75.9591	76.8071	50.6020
	0.5	0.7	0.35	0.35	56.8912	66.5514	69.5507	52.9233
	0.5	0.7	0.35	0.95	50.0247	70.8254	71.5788	55.4100
	0.5	0.7	0.95	0.35	56.1595	65.6151	67.2018	54.5321
	0.5	0.7	0.95	0.95	50.0739	71.1264	69.4692	57.9971
	0.9	0.1	0.35	0.35	49.4382	64.2680	71.0181	51.4904
	0.9	0.1	0.35	0.95	38.0985	66.7341	72.8755	58.2403
	0.9	0.1	0.95	0.35	47.2414	63.3550	63.0867	56.5197
	0.9	0.1	0.95	0.95	38.5032	71.8892	66.0186	67.0007
	0.9	0.7	0.35	0.35	48.4163	63.4110	68.3562	53.1556
	0.9	0.7	0.35	0.95	37.9715	66.4318	70.5866	61.0087
	0.9	0.7	0.95	0.35	46.4464	62.9110	61.0743	59.0397
	0.9	0.7	0.95	0.95	38.3867	70.2970	64.1799	69.6531

Table A.3: D_w and G_w values for CCDs, $k = 4$, $N = 25, 26, \ldots, 30$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2

					Cuboidal		Spherical	
N	p_l	p_1	p_2	p_q	D_w	G_w	D_w	G_w
25	0.5	0.1	0.35	0.35	67.3586	79.3545	92.2279	45.5865
	0.5	0.1	0.35	0.95	50.1023	70.3254	92.1363	44.0935
	0.5	0.1	0.95	0.35	67.3614	79.7732	88.3778	48.7788
	0.5	0.1	0.95	0.95	51.8459	76.0857	89.1699	49.7663
	0.5	0.7	0.35	0.35	66.0211	76.3877	82.9955	56.2647
	0.5	0.7	0.35	0.95	53.4169	73.2184	84.1309	56.4912
	0.5	0.7	0.95	0.35	66.1519	76.4607	80.8509	60.8029
	0.5	0.7	0.95	0.95	54.6867	78.0313	82.3267	62.1699
	0.9	0.1	0.35	0.35	59.6037	73.4918	88.4040	56.6793
	0.9	0.1	0.35	0.95	41.0414	61.7681	85.1605	50.7011
	0.9	0.1	0.95	0.35	60.4813	75.1279	80.8119	74.2327
	0.9	0.1	0.95	0.95	45.3961	74.7234	79.9040	65.1751
	0.9	0.7	0.35	0.35	59.6442	73.0940	85.9270	60.7457
	0.9	0.7	0.35	0.95	42.2360	62.9588	83.1661	55.1832
	0.9	0.7	0.95	0.35	60.5369	74.6991	79.2785	78.8558
	0.9	0.7	0.95	0.95	46.3026	75.3794	78.5325	69.6582
26	0.5	0.1	0.35	0.35	66.1888	78.3875	90.2307	44.3805
	0.5	0.1	0.35	0.95	49.6487	70.3737	91.0643	44.4283
	0.5	0.1	0.95	0.35	66.0652	78.1009	86.3350	47.4455
	0.5	0.1	0.95	0.95	51.2609	75.8217	87.8622	50.4105
	0.5	0.7	0.35	0.35	64.5209	73.9827	80.8420	54.6919
	0.5	0.7	0.35	0.95	52.5382	71.0461	82.7005	56.4387
	0.5	0.7	0.95	0.35	64.5722	73.9784	78.6765	59.0690
	0.5	0.7	0.95	0.95	53.7124	75.6196	80.7347	62.4267
	0.9	0.1	0.35	0.35	58.2045	71.2791	86.1415	55.1934
	0.9	0.1	0.35	0.95	40.1724	59.8288	85.7546	65.1820
	0.9	0.1	0.95	0.35	58.8239	72.3938	78.4670	72.0228
	0.9	0.1	0.95	0.95	44.2609	72.0912	79.5786	84.3731
	0.9	0.7	0.35	0.35	58.1610	70.6488	83.6381	59.1268
	0.9	0.7	0.35	0.95	41.2765	60.9130	83.6259	69.5161
	0.9	0.7	0.95	0.35	58.8212	71.9158	76.9189	76.4984
	0.9	0.7	0.95	0.95	45.0953	72.6496	78.1227	88.7081
27	0.5	0.1	0.35	0.35	65.0245	77.3989	88.3106	43.2049
	0.5	0.1	0.35	0.95	49.1097	70.3218	89.7677	43.1923
	0.5	0.1	0.95	0.35	64.7917	76.5140	84.3844	46.1431
	0.5	0.1	0.95	0.95	50.6058	75.4845	86.4091	48.9701
	0.5	0.7	0.35	0.35	63.0674	71.7238	78.7966	53.1691
	0.5	0.7	0.35	0.95	51.6254	68.9594	81.1264	54.8270
	0.5	0.7	0.95	0.35	63.0506	71.6604	76.6195	57.3874
	0.5	0.7	0.95	0.95	52.7157	73.3236	79.0601	60.6104
	0.9	0.1	0.35	0.35	56.8232	69.1435	83.9292	53.4692
	0.9	0.1	0.35	0.95	39.2809	57.9359	85.1028	62.8559
	0.9	0.1	0.95	0.35	57.2219	69.8384	76.2137	69.5590
	0.9	0.1	0.95	0.95	43.1333	69.6023	78.4153	81.3297
	0.9	0.7	0.35	0.35	56.7075	68.3204	81.4087	57.2757
	0.9	0.7	0.35	0.95	40.3052	58.9263	82.8793	67.0460
	0.9	0.7	0.95	0.35	57.1695	69.3248	74.6567	73.8940
	0.9	0.7	0.95	0.95	43.9046	70.0804	76.9007	85.5207
28	0.5	0.1	0.35	0.35	63.8793	75.1771	86.4710	42.0889
	0.5	0.1	0.35	0.95	48.5185	68.2586	88.3742	42.0122
	0.5	0.1	0.95	0.35	63.5513	74.2934	82.5255	44.9098
	0.5	0.1	0.95	0.95	49.9110	73.2478	84.9057	47.5963
	0.5	0.7	0.35	0.35	61.6674	69.5628	76.8568	51.7253
	0.5	0.7	0.35	0.95	50.7026	66.9026	79.5161	53.2864
	0.5	0.7	0.95	0.35	61.5920	69.4676	74.6746	55.7955
	0.5	0.7	0.95	0.95	51.7186	71.0817	77.3829	58.8761
	0.9	0.1	0.35	0.35	55.4750	66.8594	81.7931	51.8259
	0.9	0.1	0.35	0.95	38.3892	56.0755	83.9563	60.6731
	0.9	0.1	0.95	0.35	55.6831	67.4349	74.0645	67.2457
	0.9	0.1	0.95	0.95	42.0303	67.2305	76.9487	78.4830
	0.9	0.7	0.35	0.35	55.2965	66.0476	79.2626	55.5118
	0.9	0.7	0.35	0.95	39.3425	57.0215	81.6650	64.7256
	0.9	0.7	0.95	0.35	55.5876	66.9088	72.5031	71.4472
	0.9	0.7	0.95	0.95	42.7456	67.6671	75.3928	82.5364

Table A.3 D_w and G_w values for CCDs, $k = 4$, $N = 25, 26, \ldots, 30$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2 (Continued)

N	p_l	p_1	p_2	p_q	Cuboidal		Spherical	
					D_w	G_w	D_w	G_w
29	0.5	0.1	0.35	0.35	62.7613	73.0856	84.7109	41.0269
	0.5	0.1	0.35	0.95	47.8967	66.2978	86.9412	40.8908
	0.5	0.1	0.95	0.35	62.3497	72.2051	80.7550	43.6956
	0.5	0.1	0.95	0.95	49.1961	71.1235	83.3944	46.2859
	0.5	0.7	0.35	0.35	60.3236	67.5343	75.0175	50.3534
	0.5	0.7	0.35	0.95	49.7850	64.9541	77.9134	51.8212
	0.5	0.7	0.95	0.35	60.1972	67.4144	72.8352	54.2335
	0.5	0.7	0.95	0.95	50.7346	68.9664	75.7349	57.2205
	0.9	0.1	0.35	0.35	54.1681	64.7068	79.7421	50.2436
	0.9	0.1	0.35	0.95	37.5104	54.3025	82.5746	58.6263
	0.9	0.1	0.95	0.35	54.2102	65.1857	72.0206	64.9789
	0.9	0.1	0.95	0.95	40.9607	65.0001	75.3593	75.8116
	0.9	0.7	0.35	0.35	53.9347	63.9069	77.2071	53.8148
	0.9	0.7	0.35	0.95	38.4001	55.2088	80.2347	62.5481
	0.9	0.7	0.95	0.35	54.0773	64.6525	70.4580	69.0432
	0.9	0.7	0.95	0.95	41.6261	65.4027	73.7746	79.7328
30	0.5	0.1	0.35	0.35	61.6750	71.1150	83.0282	40.0206
	0.5	0.1	0.35	0.95	47.2590	64.4391	85.5003	39.8281
	0.5	0.1	0.95	0.35	61.1897	70.2391	79.0688	42.5523
	0.5	0.1	0.95	0.95	48.4743	69.1108	81.8981	45.0460
	0.5	0.7	0.35	0.35	59.0365	65.6280	73.2728	49.0546
	0.5	0.7	0.35	0.95	48.8818	63.1116	76.3402	50.4316
	0.5	0.7	0.95	0.35	58.8653	65.4889	71.0944	52.7625
	0.5	0.7	0.95	0.95	49.7717	66.9720	74.1316	55.6526
	0.9	0.1	0.35	0.35	52.9067	62.6785	77.7784	48.7446
	0.9	0.1	0.35	0.95	36.6521	52.6191	81.0785	56.7070
	0.9	0.1	0.95	0.35	52.8032	63.0779	70.0788	62.8587
	0.9	0.1	0.95	0.95	39.9291	62.9030	73.7289	73.3111
	0.9	0.7	0.35	0.35	52.6249	61.8907	75.2434	52.2070
	0.9	0.7	0.35	0.95	37.4844	53.4895	78.7044	60.5053
	0.9	0.7	0.95	0.35	52.6375	62.5413	68.5178	66.7941
	0.9	0.7	0.95	0.95	40.5497	63.2769	72.1246	77.1076

Table A.4: D_w and G_w values for BBDs, $k = 3$, $N = 13, 14, \ldots, 17$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2

N	p_l	p_1	p_2	p_q	**Cuboidal** D_w	**Cuboidal** G_w	**Spherical** D_w	**Spherical** G_w
13	0.5	0.1	0.35	0.35	64.9749	74.3548	87.5088	53.6676
	0.5	0.1	0.35	0.95	54.0313	79.1914	78.8353	46.6161
	0.5	0.1	0.95	0.35	62.3401	69.0333	86.1766	54.8538
	0.5	0.1	0.95	0.95	53.0912	74.6464	78.5889	50.1405
	0.5	0.7	0.35	0.35	56.6811	60.2763	83.4712	61.2764
	0.5	0.7	0.35	0.95	50.2758	63.7263	77.4013	53.8788
	0.5	0.7	0.95	0.35	54.9038	57.1047	82.5233	63.3784
	0.5	0.7	0.95	0.95	49.5754	61.1407	77.2249	57.4086
	0.9	0.1	0.35	0.35	53.2211	63.6180	82.4615	56.4901
	0.9	0.1	0.35	0.95	42.0670	61.5475	71.7036	55.9368
	0.9	0.1	0.95	0.35	47.3537	52.1198	79.3885	65.0109
	0.9	0.1	0.95	0.95	40.0410	54.3643	71.3573	67.2394
	0.9	0.7	0.35	0.35	51.0194	59.5693	81.3514	59.1345
	0.9	0.7	0.35	0.95	41.1659	57.3152	71.3978	58.5302
	0.9	0.7	0.95	0.35	45.7077	49.4642	78.5276	68.2488
	0.9	0.7	0.95	0.95	39.2953	51.4016	71.0969	69.8363
14	0.5	0.1	0.35	0.35	63.0840	70.6729	84.8314	51.5393
	0.5	0.1	0.35	0.95	53.5340	76.2601	78.0633	44.7735
	0.5	0.1	0.95	0.35	60.4354	65.5866	83.3516	52.6456
	0.5	0.1	0.95	0.95	52.4183	71.0981	77.5031	48.0782
	0.5	0.7	0.35	0.35	54.7344	57.2182	80.3533	58.6960
	0.5	0.7	0.35	0.95	49.4238	61.5879	75.9349	51.6908
	0.5	0.7	0.95	0.35	52.9490	54.1826	79.2966	60.6808
	0.5	0.7	0.95	0.95	48.5868	58.3805	75.5035	55.0006
	0.9	0.1	0.35	0.35	51.0855	59.8778	79.1929	53.5742
	0.9	0.1	0.35	0.95	42.1380	64.4415	71.8397	52.2230
	0.9	0.1	0.95	0.35	45.1792	48.7831	75.7542	61.6279
	0.9	0.1	0.95	0.95	39.5955	53.4061	70.5779	62.7588
	0.9	0.7	0.35	0.35	48.8739	56.0296	77.9636	56.0676
	0.9	0.7	0.35	0.95	41.1212	60.3217	71.3143	54.6763
	0.9	0.7	0.95	0.35	43.5269	46.2638	74.7991	64.6906
	0.9	0.7	0.95	0.95	38.7593	50.5528	70.1359	65.2155
15	0.5	0.1	0.35	0.35	61.2806	67.3454	82.2577	49.5449
	0.5	0.1	0.35	0.95	52.6645	72.6809	76.6702	43.0019
	0.5	0.1	0.95	0.35	58.6400	62.5136	80.6741	50.5709
	0.5	0.1	0.95	0.95	51.4514	67.5891	75.9118	46.1038
	0.5	0.7	0.35	0.35	52.9191	54.4738	77.4381	56.2848
	0.5	0.7	0.35	0.95	48.3181	58.8136	74.0280	49.5720
	0.5	0.7	0.95	0.35	51.1420	51.5963	76.3071	58.1546
	0.5	0.7	0.95	0.95	47.4088	55.6192	73.4404	52.6789
	0.9	0.1	0.35	0.35	48.9954	56.2844	75.9698	50.8011
	0.9	0.1	0.35	0.95	41.2784	62.2205	70.3701	48.9014
	0.9	0.1	0.95	0.35	43.1131	45.7482	72.2885	58.4010
	0.9	0.1	0.95	0.95	38.4788	50.9271	68.5817	58.7569
	0.9	0.7	0.35	0.35	46.7883	52.6257	74.6527	53.1532
	0.9	0.7	0.35	0.95	40.1900	58.2736	69.6917	51.2176
	0.9	0.7	0.95	0.35	41.4655	43.3560	71.2647	61.2998
	0.9	0.7	0.95	0.95	37.5873	48.2097	68.0141	61.0763
16	0.5	0.1	0.35	0.35	59.5887	64.3666	79.8326	47.7036
	0.5	0.1	0.35	0.95	51.6569	69.3010	75.0477	41.3603
	0.5	0.1	0.95	0.35	56.9685	59.7734	78.1744	48.6543
	0.5	0.1	0.95	0.95	50.3864	64.3739	74.1534	44.2794
	0.5	0.7	0.35	0.35	51.2442	52.0341	74.7441	54.0638
	0.5	0.7	0.35	0.95	47.1506	56.1906	72.0140	47.6003
	0.5	0.7	0.95	0.35	49.4843	49.3052	73.5614	55.8258
	0.5	0.7	0.95	0.95	46.2008	53.0872	71.3218	50.5241
	0.9	0.1	0.35	0.35	47.0201	53.0400	72.9132	48.2277
	0.9	0.1	0.35	0.95	40.1185	59.4116	68.3838	45.9487
	0.9	0.1	0.95	0.35	41.1953	43.0395	69.0664	55.4037
	0.9	0.1	0.95	0.95	37.1795	48.2960	66.2554	55.2019
	0.9	0.7	0.35	0.35	44.8262	49.5563	71.5301	50.4500
	0.9	0.7	0.35	0.95	38.9850	55.6493	67.5946	48.1374
	0.9	0.7	0.95	0.35	39.5590	40.7637	67.9914	58.1517
	0.9	0.7	0.95	0.95	36.2538	45.7154	65.5973	57.3937

Table A.4 D_w and G_w values for BBDs, $k = 3$, $N = 13, 14, \ldots, 17$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2 (Continued)

					Cuboidal		**Spherical**	
N	p_l	p_1	p_2	p_q	D_w	G_w	D_w	G_w
17	0.5	0.1	0.35	0.35	58.0111	61.6923	77.5653	46.0125
	0.5	0.1	0.35	0.95	50.6059	66.2043	73.3532	39.8571
	0.5	0.1	0.95	0.35	55.4188	57.3191	75.8530	46.8943
	0.5	0.1	0.95	0.95	49.3022	61.4705	72.3623	42.6119
	0.5	0.7	0.35	0.35	49.7030	49.8575	72.2631	52.0279
	0.5	0.7	0.35	0.95	45.9923	53.7895	70.0159	45.7895
	0.5	0.7	0.95	0.35	47.9657	47.2654	71.0440	53.6907
	0.5	0.7	0.95	0.95	45.0202	50.8017	69.2508	48.5489
	0.9	0.1	0.35	0.35	45.1766	50.1254	70.0547	45.8605
	0.9	0.1	0.35	0.95	38.8641	56.5664	66.2355	43.3186
	0.9	0.1	0.95	0.35	39.4281	40.6219	66.0945	52.6465
	0.9	0.1	0.95	0.95	35.8513	45.7845	63.8767	52.0367
	0.9	0.7	0.35	0.35	43.0015	46.8019	68.6217	47.9643
	0.9	0.7	0.35	0.95	37.7027	52.9843	65.3641	45.3907
	0.9	0.7	0.95	0.35	37.8071	38.4526	64.9811	55.2564
	0.9	0.7	0.95	0.95	34.9048	43.3338	63.1517	54.1117

Table A.5: D_w and G_w values for BBDs, $k = 4$, $N = 25, 26, \ldots, 29$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2

					Cuboidal		**Spherical**	
N	p_l	p_1	p_2	p_q	D_w	G_w	D_w	G_w
25	0.5	0.1	0.35	0.35	49.4276	50.6291	92.2262	45.5853
	0.5	0.1	0.35	0.95	42.7436	51.1407	92.1343	44.0922
	0.5	0.1	0.95	0.35	44.5787	43.2307	88.3760	48.7775
	0.5	0.1	0.95	0.95	40.009	45.3517	89.1678	49.7648
	0.5	0.7	0.35	0.35	36.5848	33.3915	82.9935	56.2630
	0.5	0.7	0.35	0.95	34.6662	34.4221	84.1288	56.4894
	0.5	0.7	0.95	0.35	34.3233	30.9580	80.8488	60.8010
	0.5	0.7	0.95	0.95	33.1536	32.4808	82.3245	62.1679
	0.9	0.1	0.35	0.35	38.0209	37.8379	88.4019	56.6774
	0.9	0.1	0.35	0.95	32.1065	29.7932	85.1582	50.7001
	0.9	0.1	0.95	0.35	29.8077	27.1781	80.8097	74.2302
	0.9	0.1	0.95	0.95	27.4699	24.7848	79.9016	65.1739
	0.9	0.7	0.35	0.35	35.0639	33.7389	85.9249	60.7436
	0.9	0.7	0.35	0.95	30.3389	26.4979	83.1638	55.1821
	0.9	0.7	0.95	0.35	28.1701	25.4902	79.2762	78.8531
	0.9	0.7	0.95	0.95	26.3247	23.1195	78.5301	69.6568
26	0.5	0.1	0.35	0.35	48.4322	49.1488	90.2290	44.3794
	0.5	0.1	0.35	0.95	42.2760	50.4110	91.0622	44.4269
	0.5	0.1	0.95	0.35	43.6478	41.9648	86.3332	47.4442
	0.5	0.1	0.95	0.95	39.4714	44.3645	87.8601	50.4089
	0.5	0.7	0.35	0.35	35.7348	32.4168	80.8400	54.6902
	0.5	0.7	0.35	0.95	34.1461	33.9902	82.6984	56.4368
	0.5	0.7	0.95	0.35	33.5079	30.0531	78.6745	59.0672
	0.5	0.7	0.95	0.95	32.5881	31.8531	80.7326	62.4247
	0.9	0.1	0.35	0.35	37.0391	36.6658	86.1395	55.1915
	0.9	0.1	0.35	0.95	32.3179	33.222	85.7523	65.1798
	0.9	0.1	0.95	0.35	28.9411	26.2537	78.4649	72.0203
	0.9	0.1	0.95	0.95	27.3464	26.2823	79.5762	84.3703
	0.9	0.7	0.35	0.35	34.124	32.6875	83.6360	59.1247
	0.9	0.7	0.35	0.95	30.5143	29.7892	83.6236	69.5137
	0.9	0.7	0.95	0.35	27.3288	24.6190	76.9167	76.4958
	0.9	0.7	0.95	0.95	26.1838	24.5930	78.1203	88.7051
27	0.5	0.1	0.35	0.35	47.4782	47.7490	88.3090	43.2038
	0.5	0.1	0.35	0.95	41.7233	49.3961	89.7657	43.1910
	0.5	0.1	0.95	0.35	42.7606	40.7756	84.3827	46.1419
	0.5	0.1	0.95	0.95	38.8839	43.2519	86.4071	48.9686
	0.5	0.7	0.35	0.35	34.9280	31.4995	78.7947	53.1675
	0.5	0.7	0.35	0.95	33.5739	33.3333	81.1243	54.8252
	0.5	0.7	0.95	0.35	32.7370	29.2062	76.6175	57.3857
	0.5	0.7	0.95	0.95	31.9961	31.1067	79.0580	60.6084
	0.9	0.1	0.35	0.35	36.0830	35.4940	83.9272	53.4674
	0.9	0.1	0.35	0.95	32.0686	34.3770	85.1005	62.8537
	0.9	0.1	0.95	0.35	28.1097	25.3543	76.2116	69.5566
	0.9	0.1	0.95	0.95	26.9429	26.4009	78.4130	81.3270
	0.9	0.7	0.35	0.35	33.2107	31.6342	81.4066	57.2737
	0.9	0.7	0.35	0.95	30.2466	30.8869	82.8770	67.0437
	0.9	0.7	0.95	0.35	26.5231	23.7711	74.6546	73.8916
	0.9	0.7	0.95	0.95	25.7729	24.7153	76.8983	85.5178
28	0.5	0.1	0.35	0.35	46.5661	46.4297	86.4694	42.0879
	0.5	0.1	0.35	0.95	41.1342	48.2848	88.3722	42.0109
	0.5	0.1	0.95	0.35	41.9159	39.6598	82.5239	44.9085
	0.5	0.1	0.95	0.95	38.2800	42.1317	84.9037	47.5949
	0.5	0.7	0.35	0.35	34.1632	30.6390	76.8550	51.7238
	0.5	0.7	0.35	0.95	32.9886	32.6069	79.5141	53.2848
	0.5	0.7	0.95	0.35	32.0085	28.4147	74.6727	55.7938
	0.5	0.7	0.95	0.95	31.4044	30.3471	77.3808	58.8743
	0.9	0.1	0.35	0.35	35.1620	34.3607	81.7912	51.8241
	0.9	0.1	0.35	0.95	31.6361	34.6120	83.9541	60.6710
	0.9	0.1	0.95	0.35	27.3175	24.4996	74.0625	67.2435
	0.9	0.1	0.95	0.95	26.4380	26.0765	76.9464	78.4804
	0.9	0.7	0.35	0.35	32.3329	30.6154	79.2606	55.5099
	0.9	0.7	0.35	0.95	29.8069	31.1118	81.6627	64.7234
	0.9	0.7	0.95	0.35	25.7567	22.9656	72.5010	71.4448
	0.9	0.7	0.95	0.95	25.2670	24.4095	75.3905	82.5337

Table A.5 D_w and G_w values for BBDs, $k = 4$, $N = 25, 26, \ldots, 29$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2 (Continued)

					Cuboidal		**Spherical**	
N	p_l	p_1	p_2	p_q	D_w	G_w	D_w	G_w
29	0.5	0.1	0.35	0.35	45.6948	45.1869	84.7094	41.0259
	0.5	0.1	0.35	0.95	40.5307	47.1513	86.9392	40.8896
	0.5	0.1	0.95	0.35	41.1119	38.6122	80.7534	43.6944
	0.5	0.1	0.95	0.95	37.6748	41.0407	83.3925	46.2845
	0.5	0.7	0.35	0.35	33.4383	29.8321	75.0157	50.3519
	0.5	0.7	0.35	0.95	32.4061	31.8671	77.9114	51.8196
	0.5	0.7	0.95	0.35	31.3197	27.6743	72.8334	54.2319
	0.5	0.7	0.95	0.95	30.8236	29.6053	75.7329	57.2187
	0.9	0.1	0.35	0.35	34.2790	33.2783	79.7402	50.2420
	0.9	0.1	0.35	0.95	31.1162	34.4023	82.5724	58.6243
	0.9	0.1	0.95	0.35	26.5645	23.6927	72.0186	64.9768
	0.9	0.1	0.95	0.95	25.8921	25.5740	75.3570	75.8090
	0.9	0.7	0.35	0.35	31.4929	29.6428	77.2052	53.8129
	0.9	0.7	0.35	0.95	29.2875	30.9208	80.2325	62.5460
	0.9	0.7	0.95	0.35	25.0292	22.2056	70.4560	69.0409
	0.9	0.7	0.95	0.95	24.7245	23.9334	73.7723	79.7301

Table A.6: D_w and G_w values for computer generated designs with OPTEX Procedure, $k = 2$, $N = 6, 7, \ldots, 10$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2

					Cuboidal		**Spherical**	
N	p_l	p_1	p_2	p_q	D_w	G_w	D_w	G_w
6	0.5	0.1	0.35	0.35	75.8676	79.1409	85.7495	68.1599
	0.5	0.1	0.35	0.95	61.8022	78.6269	77.6570	62.3286
	0.5	0.1	0.95	0.35	75.3899	78.3029	84.5679	68.8848
	0.5	0.1	0.95	0.95	61.9293	78.9440	77.1860	63.9448
	0.5	0.7	0.35	0.35	73.7647	74.8192	81.8585	71.5807
	0.5	0.7	0.35	0.95	61.7284	74.5370	75.5772	66.6172
	0.5	0.7	0.95	0.35	73.2870	73.9811	80.6768	72.3056
	0.5	0.7	0.95	0.95	61.8555	74.8541	75.1062	68.2334
	0.9	0.1	0.35	0.35	63.4277	64.1766	77.909	63.7985
	0.9	0.1	0.35	0.95	45.0629	59.5751	66.8804	55.1606
	0.9	0.1	0.95	0.35	61.8798	61.4613	74.0803	66.1472
	0.9	0.1	0.95	0.95	45.4748	60.6025	65.3545	60.3972
	0.9	0.7	0.35	0.35	62.6706	62.6208	76.5082	65.0300
	0.9	0.7	0.35	0.95	45.0364	58.1027	66.1317	56.7045
	0.9	0.7	0.95	0.35	61.1228	59.9055	72.6796	67.3787
	0.9	0.7	0.95	0.95	45.4482	59.1301	64.6058	61.9411
7	0.5	0.1	0.35	0.35	76.5462	77.6653	87.0677	70.4121
	0.5	0.1	0.35	0.95	62.8571	80.5828	80.1237	69.3742
	0.5	0.1	0.95	0.35	76.2394	78.6736	85.4088	70.6221
	0.5	0.1	0.95	0.95	63.1482	82.2095	79.3107	71.4961
	0.5	0.7	0.35	0.35	74.7723	75.8416	81.9271	72.8623
	0.5	0.7	0.35	0.95	63.0966	78.3015	76.8220	73.9518
	0.5	0.7	0.95	0.35	74.4655	76.8499	80.2683	73.0724
	0.5	0.7	0.95	0.95	63.3877	79.9282	76.0091	76.0737
	0.9	0.1	0.35	0.35	64.7047	65.0512	79.5477	69.8450
	0.9	0.1	0.35	0.95	46.8850	62.2551	69.1690	71.5941
	0.9	0.1	0.95	0.35	63.7106	68.318	74.1730	70.5255
	0.9	0.1	0.95	0.95	47.8283	67.5256	66.5350	78.4690
	0.9	0.7	0.35	0.35	64.0661	64.3946	77.6971	70.7271
	0.9	0.7	0.35	0.95	46.9712	61.4338	67.9804	73.2420
	0.9	0.7	0.95	0.35	63.0720	67.6615	72.3224	71.4076
	0.9	0.7	0.95	0.95	47.9145	66.7043	65.3464	80.1169
8	0.5	0.1	0.35	0.35	75.9660	78.6579	87.5461	65.8296
	0.5	0.1	0.35	0.95	63.1168	77.6005	80.2239	65.5248
	0.5	0.1	0.95	0.35	75.4081	78.6048	85.8302	65.7978
	0.5	0.1	0.95	0.95	63.2154	79.1338	79.3915	67.1923
	0.5	0.7	0.35	0.35	73.4603	76.2638	82.3405	67.9391
	0.5	0.7	0.35	0.95	62.8148	77.4051	76.8325	69.0262
	0.5	0.7	0.95	0.35	72.9025	76.2107	80.6245	67.9074
	0.5	0.7	0.95	0.95	62.9134	78.9383	76.0001	70.6937
	0.9	0.1	0.35	0.35	63.9541	67.2433	80.1378	63.1858
	0.9	0.1	0.35	0.95	47.8796	63.0091	68.8372	67.0708
	0.9	0.1	0.95	0.35	62.1466	67.0714	74.5782	63.0829
	0.9	0.1	0.95	0.95	48.1992	67.9768	66.1400	72.4735
	0.9	0.7	0.35	0.35	63.0521	66.3815	78.2638	63.9453
	0.9	0.7	0.35	0.95	47.7709	62.9388	67.6163	68.3314
	0.9	0.7	0.95	0.35	61.2446	66.2095	72.7041	63.8424
	0.9	0.7	0.95	0.95	48.0905	67.9065	64.9191	73.734
9	0.5	0.1	0.35	0.35	75.672	87.4502	84.2056	65.5045
	0.5	0.1	0.35	0.95	63.8690	96.5603	78.9279	65.0081
	0.5	0.1	0.95	0.35	74.8802	85.4251	82.4681	65.0309
	0.5	0.1	0.95	0.95	63.7798	94.0111	77.884	65.935
	0.5	0.7	0.35	0.35	72.5132	81.666	78.8678	66.6052
	0.5	0.7	0.35	0.95	63.0008	89.522	75.2502	67.0476
	0.5	0.7	0.95	0.35	71.7215	79.6409	77.1304	66.1315
	0.5	0.7	0.95	0.95	62.9116	86.9727	74.2064	67.9745
	0.9	0.1	0.35	0.35	63.6009	80.835	75.5881	60.1184
	0.9	0.1	0.35	0.95	49.0880	93.6739	68.5331	63.7368
	0.9	0.1	0.95	0.35	61.0357	74.2738	69.9588	58.5838
	0.9	0.1	0.95	0.95	48.7991	85.4142	65.1510	66.7399
	0.9	0.7	0.35	0.35	62.4638	78.7527	73.6665	60.5146
	0.9	0.7	0.35	0.95	48.7755	91.1401	67.2091	64.471
	0.9	0.7	0.95	0.35	59.8986	72.1915	68.0372	58.9801

Table A.6 D_w and G_w values for computer generated designs with OPTEX Procedure, $k = 2$, $N = 6, 7, \ldots, 10$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2 (Continued)

N	p_l	p_1	p_2	p_q	**Cuboidal** D_w	**Cuboidal** G_w	**Spherical** D_w	**Spherical** G_w
	0.9	0.7	0.95	0.95	48.4865	82.8804	63.8271	67.4741
10	0.5	0.1	0.35	0.35	76.3438	82.1478	91.1365	72.1143
	0.5	0.1	0.35	0.95	63.5490	89.4699	85.1906	72.2393
	0.5	0.1	0.95	0.35	75.6515	80.0171	88.9914	72.0748
	0.5	0.1	0.95	0.95	63.5483	87.2684	84.0707	74.4724
	0.5	0.7	0.35	0.35	73.5601	77.3142	84.7813	74.7739
	0.5	0.7	0.35	0.95	63.0359	84.0607	80.5755	77.2056
	0.5	0.7	0.95	0.35	72.8678	75.1835	82.6363	74.7345
	0.5	0.7	0.95	0.95	63.0352	81.8592	79.4557	79.4387
	0.9	0.1	0.35	0.35	64.4093	74.4839	84.9048	73.3853
	0.9	0.1	0.35	0.95	48.5829	84.9320	74.1866	79.4147
	0.9	0.1	0.95	0.35	62.1663	67.5802	77.9548	73.2574
	0.9	0.1	0.95	0.95	48.5807	77.7991	70.5584	86.6497
	0.9	0.7	0.35	0.35	63.4072	72.7438	82.6170	74.3427
	0.9	0.7	0.35	0.95	48.3982	82.9846	72.5252	81.2026
	0.9	0.7	0.95	0.35	61.1642	65.8401	75.6669	74.2148
	0.9	0.7	0.95	0.95	48.3959	75.8518	68.8970	88.4376

Table A.7: D_w and G_w values for computer generated designs with OPTEX Procedure, $k = 3$, $N = 10, 11, \ldots, 14$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2

					Cuboidal		**Spherical**	
N	p_l	p_1	p_2	p_q	D_w	G_w	D_w	G_w
10	0.5	0.1	0.35	0.35	69.4481	71.9718	89.2132	54.5659
	0.5	0.1	0.35	0.95	54.5179	76.7627	88.8733	51.8329
	0.5	0.1	0.95	0.35	68.1731	67.2720	84.5778	54.4274
	0.5	0.1	0.95	0.95	54.6632	72.9541	85.3635	53.3009
	0.5	0.7	0.35	0.35	65.0829	61.7311	78.5310	57.7872
	0.5	0.7	0.35	0.95	54.2972	66.8901	79.5826	57.9933
	0.5	0.7	0.95	0.35	64.1805	58.9353	75.1761	57.517
	0.5	0.7	0.95	0.95	54.4037	64.7731	76.8909	58.7256
	0.9	0.1	0.35	0.35	58.8966	61.4982	82.8222	57.226
	0.9	0.1	0.35	0.95	43.6696	67.3628	79.8074	60.9242
	0.9	0.1	0.95	0.35	56.0069	50.631	73.5513	58.6352
	0.9	0.1	0.95	0.95	43.9976	57.3931	72.9753	65.1666
	0.9	0.7	0.35	0.35	57.792	58.4387	80.0358	58.3655
	0.9	0.7	0.35	0.95	43.7056	64.0057	77.5131	62.8835
	0.9	0.7	0.95	0.35	55.1438	48.8052	71.5947	59.6894
	0.9	0.7	0.95	0.95	44.0085	55.1321	71.2112	66.6492
11	0.5	0.1	0.35	0.35	73.7359	70.1281	87.2454	52.3025
	0.5	0.1	0.35	0.95	53.7320	58.9153	84.1415	48.7478
	0.5	0.1	0.95	0.35	74.0176	72.3638	84.4031	52.9719
	0.5	0.1	0.95	0.95	55.1900	62.2379	82.2124	51.9812
	0.5	0.7	0.35	0.35	73.1764	74.0382	79.8684	57.7362
	0.5	0.7	0.35	0.95	56.9928	68.9097	78.6938	56.0889
	0.5	0.7	0.95	0.35	73.4768	76.2005	77.8236	59.963
	0.5	0.7	0.95	0.95	58.2315	72.0775	77.2207	59.1645
	0.9	0.1	0.35	0.35	64.3164	58.4659	82.9058	55.3488
	0.9	0.1	0.35	0.95	42.0418	49.0383	78.621	59.4184
	0.9	0.1	0.95	0.35	65.5381	64.3108	75.6564	60.7763
	0.9	0.1	0.95	0.95	46.0413	58.0629	73.8311	69.9278
	0.9	0.7	0.35	0.35	64.3830	59.7432	80.6237	57.1779
	0.9	0.7	0.35	0.95	43.1551	51.0678	76.9406	62.0463
	0.9	0.7	0.95	0.35	65.6168	65.5406	73.8911	63.6147
	0.9	0.7	0.95	0.95	47.0125	59.992	72.4461	72.4534
12	0.5	0.1	0.35	0.35	73.7822	75.8909	90.2263	53.0566
	0.5	0.1	0.35	0.95	53.2553	66.2820	86.7771	48.3608
	0.5	0.1	0.95	0.35	73.9448	77.2357	87.3272	54.281
	0.5	0.1	0.95	0.95	54.6420	69.1580	84.8972	51.9733
	0.5	0.7	0.35	0.35	72.4040	75.1865	81.8082	58.5037
	0.5	0.7	0.35	0.95	56.1534	68.5609	80.4697	55.8782
	0.5	0.7	0.95	0.35	72.6446	74.9756	79.7625	60.9446
	0.5	0.7	0.95	0.95	57.3326	70.6574	79.0626	59.4574
	0.9	0.1	0.35	0.35	63.4023	65.2208	84.6386	56.1562
	0.9	0.1	0.35	0.95	42.2641	54.214	79.1338	59.2519
	0.9	0.1	0.95	0.35	63.9927	68.2926	76.9759	63.5578
	0.9	0.1	0.95	0.95	45.7471	60.3085	74.2554	71.2472
	0.9	0.7	0.35	0.35	63.1195	64.705	82.0016	58.0291
	0.9	0.7	0.35	0.95	43.2136	53.5775	77.2022	62.0283
	0.9	0.7	0.95	0.35	63.7604	66.7688	74.8918	66.219
	0.9	0.7	0.95	0.95	46.5622	59.1668	72.6303	74.002
13	0.5	0.1	0.35	0.35	72.6601	80.0165	90.8996	51.9804
	0.5	0.1	0.35	0.95	55.7152	76.7384	86.8073	50.6876
	0.5	0.1	0.95	0.35	72.1578	80.3232	87.8762	52.6116
	0.5	0.1	0.95	0.95	56.514	80.0817	84.9237	53.6966
	0.5	0.7	0.35	0.35	70.1177	77.0924	83.344	57.5364
	0.5	0.7	0.35	0.95	57.1711	79.4721	81.3414	58.9169
	0.5	0.7	0.95	0.35	69.793	75.9187	81.1342	58.9686
	0.5	0.7	0.95	0.95	57.8255	80.8519	79.8866	62.122
	0.9	0.1	0.35	0.35	63.3834	75.7288	84.7300	56.0422
	0.9	0.1	0.35	0.95	45.5219	74.8269	78.0944	58.9104
	0.9	0.1	0.95	0.35	62.4534	72.7154	77.7956	62.0236
	0.9	0.1	0.95	0.95	47.6064	79.1047	74.0403	69.4829
	0.9	0.7	0.35	0.35	62.8311	74.6325	82.6392	58.1204
	0.9	0.7	0.35	0.95	46.0503	74.5167	76.6561	61.8694
	0.9	0.7	0.95	0.35	62.0161	70.6598	76.2321	64.6208

Table A.7 D_w and G_w values for computer generated designs with OPTEX Procedure, $k = 3$, $N = 10, 11, \ldots, 14$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2 (Continued)

N	p_l	p_1	p_2	p_q	**Cuboidal**		**Spherical**	
					D_w	G_w	D_w	G_w
	0.9	0.7	0.95	0.95	48.0412	77.5221	72.8798	72.5690
14	0.5	0.1	0.35	0.35	71.0363	83.3646	91.5418	56.4110
	0.5	0.1	0.35	0.95	56.2649	86.6187	88.3705	54.9160
	0.5	0.1	0.95	0.35	70.3869	82.7178	88.4535	56.6887
	0.5	0.1	0.95	0.95	56.8680	90.3763	86.3811	57.0557
	0.5	0.7	0.35	0.35	68.0501	79.1112	82.6684	61.4781
	0.5	0.7	0.35	0.95	57.0622	84.9491	81.5373	62.5903
	0.5	0.7	0.95	0.35	67.6375	77.9388	80.4738	62.7903
	0.5	0.7	0.95	0.95	57.5690	86.4042	80.0457	64.4537
	0.9	0.1	0.35	0.35	61.4721	79.4574	85.6574	61.8199
	0.9	0.1	0.35	0.95	45.7836	83.1493	79.3827	67.2823
	0.9	0.1	0.95	0.35	60.3069	76.1622	77.5999	64.8837
	0.9	0.1	0.95	0.95	47.5424	89.0223	74.4336	71.6138
	0.9	0.7	0.35	0.35	60.8139	78.3109	82.9655	63.7687
	0.9	0.7	0.35	0.95	46.1776	82.6937	77.4139	69.7882
	0.9	0.7	0.95	0.35	59.8021	74.6752	75.4872	67.5029
	0.9	0.7	0.95	0.95	47.8740	87.0747	72.7873	73.9405

Table A.8: D_w and G_w values for computer generated designs with OPTEX Procedure, $k = 4$, $N = 15, 16, \ldots, 20$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2

					Cuboidal		**Spherical**	
N	p_l	p_1	p_2	p_q	D_w	G_w	D_w	G_w
16	0.5	0.1	0.35	0.35	69.4220	66.5028	89.6798	42.8004
	0.5	0.1	0.35	0.95	50.1026	54.4158	89.1802	40.2583
	0.5	0.1	0.95	0.35	69.0781	67.8589	86.3932	42.8765
	0.5	0.1	0.95	0.95	51.5013	59.1543	86.5195	43.2995
	0.5	0.7	0.35	0.35	67.5397	63.1663	81.5299	51.0778
	0.5	0.7	0.35	0.95	53.2841	58.9058	82.3744	50.6924
	0.5	0.7	0.95	0.35	67.3573	63.4925	79.7087	52.1057
	0.5	0.7	0.95	0.95	54.2142	62.3314	80.7396	53.0698
	0.9	0.1	0.35	0.35	61.3881	53.5968	86.4769	47.8863
	0.9	0.1	0.35	0.95	42.0887	44.1607	85.1926	53.1880
	0.9	0.1	0.95	0.35	60.7229	52.6235	78.7938	59.5119
	0.9	0.1	0.95	0.95	45.0171	53.4213	79.1066	66.9929
	0.9	0.7	0.35	0.35	61.1381	52.7439	83.8190	51.2560
	0.9	0.7	0.35	0.95	43.0504	45.5275	82.9169	56.7431
	0.9	0.7	0.95	0.35	60.5087	51.4382	77.0259	63.1020
	0.9	0.7	0.95	0.95	45.7143	54.1700	77.4552	70.3356
17	0.5	0.1	0.35	0.35	67.6686	72.5742	88.5818	43.8837
	0.5	0.1	0.35	0.95	50.2043	70.0451	88.7269	40.9833
	0.5	0.1	0.95	0.35	67.1225	69.7162	84.6033	44.8571
	0.5	0.1	0.95	0.95	51.5717	73.9481	85.5468	44.7873
	0.5	0.7	0.35	0.35	64.7079	62.2793	79.5524	52.9151
	0.5	0.7	0.35	0.95	52.1835	62.9952	80.7571	51.9727
	0.5	0.7	0.95	0.35	64.5528	60.8428	77.3102	53.9816
	0.5	0.7	0.95	0.95	53.1832	64.8647	78.7752	55.3533
	0.9	0.1	0.35	0.35	60.1656	62.8266	84.8961	50.8180
	0.9	0.1	0.35	0.95	42.5794	56.7872	83.1924	55.8103
	0.9	0.1	0.95	0.35	59.6598	57.0601	78.0927	59.5656
	0.9	0.1	0.95	0.95	45.5832	62.4882	77.9620	67.7836
	0.9	0.7	0.35	0.35	59.5684	60.0749	82.5603	54.5449
	0.9	0.7	0.35	0.95	43.2032	54.9234	81.2447	59.5071
	0.9	0.7	0.95	0.35	59.2114	55.3579	76.5989	62.7224
	0.9	0.7	0.95	0.95	45.9778	60.1646	76.5867	71.2562
18	0.5	0.1	0.35	0.35	69.4897	72.6275	93.0392	40.8727
	0.5	0.1	0.35	0.95	50.4291	64.2005	92.6896	37.8642
	0.5	0.1	0.95	0.35	69.3873	72.7836	88.8107	42.4436
	0.5	0.1	0.95	0.95	52.1893	68.3637	89.4495	42.0552
	0.5	0.7	0.35	0.35	67.2590	64.4734	83.1819	49.4294
	0.5	0.7	0.35	0.95	53.3298	60.3540	84.0560	47.5256
	0.5	0.7	0.95	0.35	67.3431	62.9047	80.8084	51.7208
	0.5	0.7	0.95	0.95	54.5952	62.0768	82.0454	51.3120
	0.9	0.1	0.35	0.35	61.6965	61.6515	87.3490	46.5282
	0.9	0.1	0.35	0.95	42.7682	57.7233	83.5230	52.8951
	0.9	0.1	0.95	0.35	61.6309	51.1397	80.1906	59.3204
	0.9	0.1	0.95	0.95	46.2794	54.2388	78.5966	67.0233
	0.9	0.7	0.35	0.35	61.2637	58.6015	84.8349	49.8532
	0.9	0.7	0.35	0.95	43.5744	55.0658	81.5962	56.3639
	0.9	0.7	0.95	0.35	61.2569	47.0223	78.6001	62.5849
	0.9	0.7	0.95	0.95	46.7964	49.8444	77.2499	70.2836
19	0.5	0.1	0.35	0.35	69.7636	70.0058	92.8723	46.2585
	0.5	0.1	0.35	0.95	50.2269	63.1859	94.2638	44.2624
	0.5	0.1	0.95	0.35	69.8219	70.6656	88.2627	47.4675
	0.5	0.1	0.95	0.95	52.1337	68.8578	90.4284	49.0153
	0.5	0.7	0.35	0.35	68.0260	67.1644	82.1091	55.3215
	0.5	0.7	0.35	0.95	53.5431	65.8427	84.3302	55.4562
	0.5	0.7	0.95	0.35	68.2010	66.3595	79.4726	56.8233
	0.5	0.7	0.95	0.95	54.9208	68.9554	81.9409	59.1666
	0.9	0.1	0.35	0.35	62.1948	64.0012	88.0255	54.5384
	0.9	0.1	0.35	0.95	42.6262	55.7004	86.7924	56.6140
	0.9	0.1	0.95	0.35	62.8559	61.3340	78.4653	63.7178
	0.9	0.1	0.95	0.95	46.7493	64.0585	79.2784	71.3270
	0.9	0.7	0.35	0.35	61.9607	62.8197	85.1201	58.2593
	0.9	0.7	0.35	0.95	43.5849	55.1379	84.2806	60.7510
	0.9	0.7	0.95	0.35	62.6533	59.3890	76.7013	66.7917

Table A.8 D_w and G_w values for computer generated designs with OPTEX Procedure, $k = 4$, $N = 15, 16, \ldots, 20$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2 (Continued)

N	p_l	p_1	p_2	p_q	**Cuboidal**		**Spherical**	
					D_w	G_w	D_w	G_w
	0.9	0.7	0.95	0.95	47.4104	61.9723	77.5861	74.4319
20	0.5	0.1	0.35	0.35	68.4603	76.2303	92.3758	43.4092
	0.5	0.1	0.35	0.95	51.2749	70.8564	92.6051	41.5203
	0.5	0.1	0.95	0.35	67.8259	75.2182	88.7635	44.5850
	0.5	0.1	0.95	0.95	52.5878	76.7589	89.7385	45.1083
	0.5	0.7	0.35	0.35	65.7724	69.0611	81.7556	51.3768
	0.5	0.7	0.35	0.95	53.6312	67.6178	83.2428	51.6417
	0.5	0.7	0.95	0.35	65.4767	67.7606	79.8173	53.8939
	0.5	0.7	0.95	0.95	54.5429	70.8364	81.5403	54.5008
	0.9	0.1	0.35	0.35	61.2844	69.8496	88.2128	50.8709
	0.9	0.1	0.35	0.95	43.9294	62.7070	85.9387	53.8012
	0.9	0.1	0.95	0.35	60.9858	65.4721	79.3114	61.8512
	0.9	0.1	0.95	0.95	47.1687	70.8722	79.2611	66.6384
	0.9	0.7	0.35	0.35	60.9772	67.6844	84.9854	54.3035
	0.9	0.7	0.35	0.95	44.7892	60.4980	83.2699	57.6594
	0.9	0.7	0.95	0.35	60.7992	63.5538	77.1383	65.3760
	0.9	0.7	0.95	0.95	47.7829	66.9373	77.3054	70.0332

Table A.9: D_w and G_w values for computer generated designs with GAs, $k = 2$, $N = 6, 7, \ldots, 10$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2

					Cuboidal		**Spherical**	
N	p_l	p_1	p_2	p_q	D_w	G_w	D_w	G_w
6	0.5	0.1	0.35	0.35	78.6447	85.8708	90.6706	77.4751
	0.5	0.1	0.35	0.95	62.5690	83.2238	84.4471	73.5467
	0.5	0.1	0.95	0.35	78.5482	85.3476	88.5592	77.5638
	0.5	0.1	0.95	0.95	62.1181	82.3669	81.7720	75.9603
	0.5	0.7	0.35	0.35	77.9677	83.7292	85.4364	81.2090
	0.5	0.7	0.35	0.95	62.5153	78.0040	77.1391	78.5420
	0.5	0.7	0.95	0.35	78.1347	83.2565	84.5402	81.2950
	0.5	0.7	0.95	0.95	63.0464	79.8062	76.1521	80.9585
	0.9	0.1	0.35	0.35	66.9612	73.9695	81.6791	76.0823
	0.9	0.1	0.35	0.95	45.5586	70.2339	72.7907	81.5504
	0.9	0.1	0.95	0.35	66.6372	72.5521	76.5089	76.9677
	0.9	0.1	0.95	0.95	45.5704	73.6211	67.3681	89.5858
	0.9	0.7	0.35	0.35	66.5188	73.1985	79.4820	76.2888
	0.9	0.7	0.35	0.95	45.3672	68.6895	70.2357	83.3946
	0.9	0.7	0.95	0.35	66.1950	72.3643	75.2666	78.0425
	0.9	0.7	0.95	0.95	45.9046	73.5255	65.9175	91.3664
7	0.5	0.1	0.35	0.35	79.4242	82.3116	90.8287	77.4614
	0.5	0.1	0.35	0.95	62.9590	84.9648	84.4589	72.8811
	0.5	0.1	0.95	0.35	79.2349	81.3625	88.6056	77.5825
	0.5	0.1	0.95	0.95	63.2484	85.4487	81.8812	74.7721
	0.5	0.7	0.35	0.35	78.3676	79.0259	85.4811	81.1656
	0.5	0.7	0.35	0.95	63.1728	78.5902	77.5719	77.1052
	0.5	0.7	0.95	0.35	78.5479	79.4699	84.8525	81.2504
	0.5	0.7	0.95	0.95	63.4620	80.6021	76.5183	79.3417
	0.9	0.1	0.35	0.35	67.5834	72.1632	81.8949	76.6193
	0.9	0.1	0.35	0.95	46.9284	74.3864	72.2472	74.3257
	0.9	0.1	0.95	0.35	66.9707	73.4729	76.7275	76.0482
	0.9	0.1	0.95	0.95	47.8636	76.8163	67.3986	80.1765
	0.9	0.7	0.35	0.35	67.0906	71.3936	79.6959	76.6076
	0.9	0.7	0.35	0.95	47.0084	71.6984	69.8486	76.9809
	0.9	0.7	0.95	0.35	66.4795	73.0254	75.4824	77.8233
	0.9	0.7	0.95	0.95	47.9437	77.7926	66.0179	81.9215
8	0.5	0.1	0.35	0.35	79.3161	84.3623	91.3363	77.4344
	0.5	0.1	0.35	0.95	63.3195	85.3234	85.1697	71.0760
	0.5	0.1	0.95	0.35	79.2129	84.4031	88.6642	77.2851
	0.5	0.1	0.95	0.95	63.4117	87.2049	82.2941	72.8078
	0.5	0.7	0.35	0.35	79.0919	82.2730	85.7352	81.1804
	0.5	0.7	0.35	0.95	63.1549	85.4124	77.7377	75.2634
	0.5	0.7	0.95	0.35	79.3116	82.3138	85.1333	81.1381
	0.5	0.7	0.95	0.95	63.4636	86.7112	76.7272	77.2090
	0.9	0.1	0.35	0.35	67.6914	77.6350	82.2795	75.6539
	0.9	0.1	0.35	0.95	47.9549	78.4388	72.1649	76.1000
	0.9	0.1	0.95	0.35	67.3707	76.6365	77.1926	76.5209
	0.9	0.1	0.95	0.95	48.2536	85.1537	67.2672	82.2978
	0.9	0.7	0.35	0.35	67.3671	76.6680	79.8474	77.9592
	0.9	0.7	0.35	0.95	47.8245	81.4533	69.5655	77.6382
	0.9	0.7	0.95	0.35	67.2618	76.0487	76.2761	78.2357
	0.9	0.7	0.95	0.95	48.1232	85.4235	65.9247	83.8112
9	0.5	0.1	0.35	0.35	79.8835	87.4502	91.3608	77.2996
	0.5	0.1	0.35	0.95	63.8690	96.5603	86.6106	71.1841
	0.5	0.1	0.95	0.35	80.1331	85.4251	88.7146	76.3346
	0.5	0.1	0.95	0.95	63.7798	94.0111	83.3855	72.1872
	0.5	0.7	0.35	0.35	80.2208	81.6660	85.7797	81.1936
	0.5	0.7	0.35	0.95	63.0472	89.5220	78.0187	75.8774
	0.5	0.7	0.95	0.35	80.6213	80.0802	85.1858	81.1483
	0.5	0.7	0.95	0.95	63.4152	86.9727	76.8765	77.9812
	0.9	0.1	0.35	0.35	68.5191	80.8350	82.4555	75.7816
	0.9	0.1	0.35	0.95	49.0880	93.6739	73.1565	77.5272
	0.9	0.1	0.95	0.35	69.2194	74.5793	77.3848	76.1409
	0.9	0.1	0.95	0.95	48.7991	85.4142	67.2118	84.7974
	0.9	0.7	0.35	0.35	68.2980	78.7527	79.9546	76.9298
	0.9	0.7	0.35	0.95	48.7755	91.1401	70.0326	79.3531
	0.9	0.7	0.95	0.35	69.3526	73.4248	76.4177	78.1787
	0.9	0.7	0.95	0.95	48.4865	82.9875	65.7386	86.3160

Table A.9 D_w and G_w values for computer generated designs with GAs, $k = 2$, $N = 6, 7, \ldots, 10$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2 (Continued)

					Cuboidal		**Spherical**	
N	p_l	p_1	p_2	p_q	D_w	G_w	D_w	G_w
10	0.5	0.1	0.35	0.35	79.8600	83.5299	91.3809	77.6357
	0.5	0.1	0.35	0.95	63.5833	90.2392	86.3844	73.2997
	0.5	0.1	0.95	0.35	80.0795	82.8352	88.7342	76.9589
	0.5	0.1	0.95	0.95	63.5703	89.3971	83.4490	72.1854
	0.5	0.7	0.35	0.35	79.7983	83.0316	85.7943	81.0366
	0.5	0.7	0.35	0.95	63.2115	84.5221	78.0561	76.3309
	0.5	0.7	0.95	0.35	80.1852	84.0283	85.1934	81.3702
	0.5	0.7	0.95	0.95	63.6875	84.7153	76.9523	78.4446
	0.9	0.1	0.35	0.35	69.0297	75.8442	82.5786	75.8113
	0.9	0.1	0.35	0.95	48.6099	85.3367	73.4904	79.3870
	0.9	0.1	0.95	0.35	69.8143	74.3986	77.6771	76.0175
	0.9	0.1	0.95	0.95	48.5859	83.4160	66.9739	86.6252
	0.9	0.7	0.35	0.35	68.9902	74.8347	80.2130	76.9097
	0.9	0.7	0.35	0.95	48.4147	83.0519	70.4060	80.8463
	0.9	0.7	0.95	0.35	69.7748	76.3052	76.8149	77.7884
	0.9	0.7	0.95	0.95	48.4009	83.3038	65.4343	88.2755

Table A.10: D_w and G_w values for computer generated designs with GAs, $k = 3$, $N = 10, 11, \ldots, 14$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2

					Cuboidal		**Spherical**	
N	p_l	p_1	p_2	p_q	D_w	G_w	D_w	G_w
10	0.5	0.1	0.35	0.35	74.9732	77.5430	108.6553	68.7485
	0.5	0.1	0.35	0.95	55.5080	82.3182	107.4675	64.9647
	0.5	0.1	0.95	0.35	75.5588	78.0976	104.8438	67.6618
	0.5	0.1	0.95	0.95	55.5072	78.8593	101.3215	66.5033
	0.5	0.7	0.35	0.35	75.6967	81.1240	93.4893	70.6610
	0.5	0.7	0.35	0.95	56.4242	72.9077	86.1689	64.9883
	0.5	0.7	0.95	0.35	76.2444	83.0929	93.4968	73.5734
	0.5	0.7	0.95	0.95	57.2186	73.3283	84.6302	66.2646
	0.9	0.1	0.35	0.35	66.4130	68.4701	92.3125	70.2694
	0.9	0.1	0.35	0.95	44.3917	70.6118	89.3285	72.5567
	0.9	0.1	0.95	0.35	68.3647	72.8242	83.3046	74.6188
	0.9	0.1	0.95	0.95	44.1816	67.4764	74.6074	68.5264
	0.9	0.7	0.35	0.35	66.8239	71.3778	86.4814	70.2360
	0.9	0.7	0.35	0.95	44.1651	65.4613	82.0665	72.2314
	0.9	0.7	0.95	0.35	68.7578	72.8670	81.9777	78.1218
	0.9	0.7	0.95	0.95	43.8504	67.1435	72.4303	71.3588
11	0.5	0.1	0.35	0.35	74.8328	82.4165	108.8970	69.4319
	0.5	0.1	0.35	0.95	55.6860	80.5137	108.1150	65.1836
	0.5	0.1	0.95	0.35	75.2772	83.1504	105.1665	68.1733
	0.5	0.1	0.95	0.95	56.0656	80.7010	101.5624	65.9796
	0.5	0.7	0.35	0.35	75.3383	82.5805	93.5053	71.8066
	0.5	0.7	0.35	0.95	57.0989	74.2638	86.6017	65.7265
	0.5	0.7	0.95	0.35	75.8175	82.3199	93.5026	72.8253
	0.5	0.7	0.95	0.95	58.2425	77.3620	85.0027	68.1071
	0.9	0.1	0.35	0.35	66.4856	72.5698	92.2963	70.2241
	0.9	0.1	0.35	0.95	44.7646	70.0002	89.7323	69.7031
	0.9	0.1	0.95	0.35	67.9058	72.6234	83.6405	72.5370
	0.9	0.1	0.95	0.95	46.0475	69.5470	75.8745	80.5020
	0.9	0.7	0.35	0.35	66.7076	73.5763	86.6669	69.5010
	0.9	0.7	0.35	0.95	44.8308	68.1245	82.3555	71.5985
	0.9	0.7	0.95	0.35	68.1483	74.1053	81.9784	75.7594
	0.9	0.7	0.95	0.95	47.0181	72.9458	73.5613	83.9362
12	0.5	0.1	0.35	0.35	74.9437	83.1606	109.0488	69.1999
	0.5	0.1	0.35	0.95	55.7390	83.5853	108.5702	65.8846
	0.5	0.1	0.95	0.35	75.4484	82.0355	105.2335	68.5901
	0.5	0.1	0.95	0.95	56.4075	81.7832	102.0983	65.6839
	0.5	0.7	0.35	0.35	75.3801	80.0698	93.6184	71.6134
	0.5	0.7	0.35	0.95	57.5841	75.1366	86.9748	67.5332
	0.5	0.7	0.95	0.35	75.8774	81.0367	93.6321	73.2443
	0.5	0.7	0.95	0.95	58.5786	78.8248	85.4783	70.7505
	0.9	0.1	0.35	0.35	66.1539	74.0561	92.3117	70.5298
	0.9	0.1	0.35	0.95	45.2081	71.9333	89.4526	70.8674
	0.9	0.1	0.95	0.35	67.4497	75.5707	83.6754	74.1923
	0.9	0.1	0.95	0.95	46.6412	75.8619	76.0323	77.3328
	0.9	0.7	0.35	0.35	66.4216	74.9267	86.7022	70.6480
	0.9	0.7	0.35	0.95	45.4441	71.5770	82.2549	70.5873
	0.9	0.7	0.95	0.35	67.7127	76.1555	82.2243	77.4595
	0.9	0.7	0.95	0.95	47.4044	76.3567	73.7143	81.5575
13	0.5	0.1	0.35	0.35	75.0666	81.8046	109.1701	69.7288
	0.5	0.1	0.35	0.95	55.9554	82.0079	109.3016	67.2765
	0.5	0.1	0.95	0.35	75.6035	83.2258	105.4174	68.6407
	0.5	0.1	0.95	0.95	56.7183	84.5046	102.2533	66.2366
	0.5	0.7	0.35	0.35	75.6395	81.4154	93.6340	71.7707
	0.5	0.7	0.35	0.95	57.8121	82.3712	87.2220	68.4756
	0.5	0.7	0.95	0.35	76.2104	81.2830	93.6137	73.1209
	0.5	0.7	0.95	0.95	58.6375	84.7316	85.8030	71.8350
	0.9	0.1	0.35	0.35	66.2474	77.2006	92.2765	71.0044
	0.9	0.1	0.35	0.95	45.7728	78.2096	89.1924	71.8564
	0.9	0.1	0.95	0.35	67.4255	77.4752	83.7848	73.4583
	0.9	0.1	0.95	0.95	47.6648	80.2542	76.0103	76.8268
	0.9	0.7	0.35	0.35	66.5322	76.4982	86.7269	69.8675
	0.9	0.7	0.35	0.95	46.2219	77.7462	82.1750	71.3118
	0.9	0.7	0.95	0.35	67.7513	76.5767	82.1262	77.8638

Table A.10 D_w and G_w values for computer generated designs with GAs, $k = 3$, $N = 10, 11, \ldots, 14$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2 (Continued)

N	p_l	p_1	p_2	p_q	**Cuboidal** D_w	**Cuboidal** G_w	**Spherical** D_w	**Spherical** G_w
	0.9	0.7	0.95	0.95	48.0906	82.3840	73.6955	79.4022
14	0.5	0.1	0.35	0.35	75.3722	83.3646	109.2611	69.7490
	0.5	0.1	0.35	0.95	56.2649	86.6187	109.6404	67.9969
	0.5	0.1	0.95	0.35	76.0634	82.7178	105.4726	68.9638
	0.5	0.1	0.95	0.95	56.8680	90.3763	102.3651	66.8372
	0.5	0.7	0.35	0.35	76.1054	79.1112	93.6924	71.8872
	0.5	0.7	0.35	0.95	57.6242	84.9491	87.3278	68.9493
	0.5	0.7	0.95	0.35	76.7365	79.1641	93.6265	74.1381
	0.5	0.7	0.95	0.95	58.4857	86.4042	86.0193	73.0323
	0.9	0.1	0.35	0.35	66.5292	79.4574	92.4028	70.2506
	0.9	0.1	0.35	0.95	45.6163	83.1493	89.2324	73.0875
	0.9	0.1	0.95	0.35	67.7936	76.1622	83.8043	74.5554
	0.9	0.1	0.95	0.95	47.5424	89.0223	76.0212	79.3571
	0.9	0.7	0.35	0.35	66.8571	78.3109	86.8473	70.4585
	0.9	0.7	0.35	0.95	45.9633	82.6937	82.1826	73.1577
	0.9	0.7	0.95	0.35	68.0625	74.6752	82.2080	78.3409
	0.9	0.7	0.95	0.95	48.0521	87.0747	73.6723	80.2246

Table A.11: D_w and G_w values for computer generated designs with GAs, $k = 4$, $N = 16, 17, \ldots, 20$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2

					Cuboidal		**Spherical**	
N	p_l	p_1	p_2	p_q	D_w	G_w	D_w	G_w
16	0.5	0.1	0.35	0.35	71.7257	77.7003	123.2305	71.7857
	0.5	0.1	0.35	0.95	51.3167	80.0334	129.0541	73.5044
	0.5	0.1	0.95	0.35	72.8829	75.8281	118.0529	71.5467
	0.5	0.1	0.95	0.95	52.1981	80.3106	118.1071	71.8412
	0.5	0.7	0.35	0.35	73.9757	70.5160	96.7535	78.0024
	0.5	0.7	0.35	0.95	54.4464	69.3357	93.4059	74.7775
	0.5	0.7	0.95	0.35	74.8643	70.8003	96.2877	79.5370
	0.5	0.7	0.95	0.95	55.3529	70.0844	91.2792	77.8979
	0.9	0.1	0.35	0.35	64.5773	68.9791	99.4047	84.9457
	0.9	0.1	0.35	0.95	43.7985	70.0087	102.1243	83.4272
	0.9	0.1	0.95	0.35	66.4913	63.0663	83.1523	85.7585
	0.9	0.1	0.95	0.95	45.0898	58.6739	78.5404	86.1413
	0.9	0.7	0.35	0.35	65.1816	66.7614	90.1932	80.4117
	0.9	0.7	0.35	0.95	43.9705	62.8468	90.2092	84.0658
	0.9	0.7	0.95	0.35	67.0912	62.2932	81.3923	86.9980
	0.9	0.7	0.95	0.95	45.7043	58.0804	76.2655	87.5619
17	0.5	0.1	0.35	0.35	71.8839	77.5611	123.1104	71.8113
	0.5	0.1	0.35	0.95	51.3427	80.9866	129.2854	73.2292
	0.5	0.1	0.95	0.35	73.2780	75.4240	118.0697	71.2794
	0.5	0.1	0.95	0.95	52.3095	78.6135	118.4544	71.5343
	0.5	0.7	0.35	0.35	74.7020	72.8862	96.1299	77.4916
	0.5	0.7	0.35	0.95	54.8768	69.8387	93.3467	74.9876
	0.5	0.7	0.95	0.35	75.8651	71.9090	95.2635	79.9353
	0.5	0.7	0.95	0.95	55.9795	73.0435	91.2067	78.3584
	0.9	0.1	0.35	0.35	65.0414	70.3673	99.0283	84.8924
	0.9	0.1	0.35	0.95	43.8042	70.1295	102.0863	89.0013
	0.9	0.1	0.95	0.35	67.8859	63.8372	83.2976	85.0301
	0.9	0.1	0.95	0.95	45.4145	59.5064	79.1057	83.0822
	0.9	0.7	0.35	0.35	65.8421	66.6037	90.3793	82.9668
	0.9	0.7	0.35	0.95	44.3587	63.2131	90.6019	88.0755
	0.9	0.7	0.95	0.35	68.7315	62.0575	80.0579	87.7648
	0.9	0.7	0.95	0.95	46.4530	61.4594	76.4380	84.0364
18	0.5	0.1	0.35	0.35	72.1174	76.8127	123.1841	72.3634
	0.5	0.1	0.35	0.95	51.4099	84.7152	129.6309	73.2662
	0.5	0.1	0.95	0.35	73.7634	75.4795	117.7931	71.6956
	0.5	0.1	0.95	0.95	52.5223	80.3761	118.4978	73.8108
	0.5	0.7	0.35	0.35	75.3705	73.9900	95.5074	78.1872
	0.5	0.7	0.35	0.95	55.0653	72.3197	93.5098	76.9653
	0.5	0.7	0.95	0.35	76.7578	73.4302	94.8764	81.2236
	0.5	0.7	0.95	0.95	56.4212	72.1232	91.3877	79.1551
	0.9	0.1	0.35	0.35	65.4639	70.7331	98.9007	84.9274
	0.9	0.1	0.35	0.95	44.0010	71.5816	101.6320	91.0165
	0.9	0.1	0.95	0.35	68.9599	65.9858	82.2216	88.5453
	0.9	0.1	0.95	0.95	46.1860	62.8045	78.9883	81.6727
	0.9	0.7	0.35	0.35	66.4578	69.0173	90.3102	82.9914
	0.9	0.7	0.35	0.95	44.2072	67.7675	90.5088	87.1001
	0.9	0.7	0.95	0.35	69.7867	65.1606	79.9932	89.3774
	0.9	0.7	0.95	0.95	47.1955	63.0479	76.4914	86.7780
19	0.5	0.1	0.35	0.35	72.4863	77.6213	123.4330	72.3354
	0.5	0.1	0.35	0.95	51.5042	82.1568	129.5828	73.5230
	0.5	0.1	0.95	0.35	74.0335	77.4308	117.6157	72.6041
	0.5	0.1	0.95	0.95	52.7545	78.4107	118.1946	74.1520
	0.5	0.7	0.35	0.35	75.2255	74.9322	95.2652	77.7960
	0.5	0.7	0.35	0.95	55.2125	73.1311	93.3562	76.8476
	0.5	0.7	0.95	0.35	76.4403	77.2130	94.3890	80.6270
	0.5	0.7	0.95	0.95	56.5257	75.0702	91.4243	80.0294
	0.9	0.1	0.35	0.35	65.6398	70.4486	98.9566	86.1840
	0.9	0.1	0.35	0.95	44.4743	72.5191	102.0766	89.2948
	0.9	0.1	0.95	0.35	69.4913	67.3156	82.5467	84.1377
	0.9	0.1	0.95	0.95	46.9690	66.4046	79.5021	83.6693
	0.9	0.7	0.35	0.35	66.4196	70.0610	90.1355	84.2307
	0.9	0.7	0.35	0.95	44.7332	66.1354	90.8757	90.5610
	0.9	0.7	0.95	0.35	70.3649	67.1021	79.9554	84.7334

Table A.11 D_w and G_w values for computer generated designs with GAs, $k = 4$, $N = 16, 17, \ldots, 20$ evaluated at 16 factorial combinations of p_l, p_q, p_1, and p_2 (Continued)

N	p_l	p_1	p_2	p_q	**Cuboidal** D_w	**Cuboidal** G_w	**Spherical** D_w	**Spherical** G_w
	0.9	0.7	0.95	0.95	47.6539	65.8735	77.1936	87.6441
20	0.5	0.1	0.35	0.35	72.2809	78.5155	123.2284	-
	0.5	0.1	0.35	0.95	51.6726	80.5525	129.3574	-
	0.5	0.1	0.95	0.35	73.7801	76.5543	117.9031	-
	0.5	0.1	0.95	0.95	52.7985	80.8801	118.3427	-
	0.5	0.7	0.35	0.35	75.0227	75.9612	95.0614	-
	0.5	0.7	0.35	0.95	55.2216	73.7487	93.6776	-
	0.5	0.7	0.95	0.35	76.1827	78.1975	94.3068	-
	0.5	0.7	0.95	0.95	56.8061	74.3753	91.1522	-
	0.9	0.1	0.35	0.35	65.6558	71.9953	98.7898	-
	0.9	0.1	0.35	0.95	44.4752	71.7817	102.1082	-
	0.9	0.1	0.95	0.35	69.5886	68.8785	82.7066	-
	0.9	0.1	0.95	0.95	47.3368	65.3731	79.5523	-
	0.9	0.7	0.35	0.35	66.6089	70.6336	89.9295	-
	0.9	0.7	0.35	0.95	45.2828	69.6960	90.6027	-
	0.9	0.7	0.95	0.35	70.4250	67.6392	78.8037	-
	0.9	0.7	0.95	0.95	48.1196	63.8202	76.4228	-

APPENDIX B

CATALOG OF COMPUTER GENERATED DESIGNS FROM GENETIC ALGORITHM FOR CUBOIDAL DESIGN

Table B.1: Computer generated design from Genetic Algorithm for weighted D-optimality criterion in cuboidal design, $k = 2$

Combination	**N**	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	**Other**
1	6	1	1	1	1	(0.1175, 1), (-1, -0.1175)
	7	2	1	1	1	(-0.0336, 1), (1, -0.0336)
	8	2	2	1	1	(0.1015, 1), (1, -0.0788)
	9	2	1	2	2	(-1, 0.0494), (-0.0494, 1)
	10	2	2	2	2	(0.0373, 1), (-1, -0.0373)
2	6	1	1	0	1	(-0.0801, 0.0777), (1, -0.1310), (0.1360, -1.0000)
	7	1	1	1	1	(-0.0333, 0.0333), (1, -0.0226), (0.0226, -1.0000)
	8	1	1	1	1	(0,-1), (0, 1), (1, 0), (-1, 0)
	9	1	1	1	1	(-1, 0), (1, 0), (0, 0), (0, -1), (0, 1)
	10	1	1	2	1	(0.0035, 1), (-1, -0.0035), (0.0064, -0.0064), (-0.0298, -1), (1, 0.0298)
3	6	1	1	1	1	(-1, -0.1280), (0.1280, 1)
	7	1	1	1	2	(-1, 0.0321), (0.0321, -1)
	8	2	2	1	1	(1, 0.0983), (0.1406, -1)
	9	2	2	2	1	(-0.0039, -0.0012), (0.9669, 1)
	10	2	2	2	2	(0.0384, -1), (-1, 0.0384)
4	6	1	1	1	1	(0, -1), (0, 0.0032)
	7	1	1	1	1	(-0.0220, 1), (-1, 0.0220), (0.0316, -0.0316)
	8	1	1	1	1	(-1, 0), (0, -1), (1, 0), (0, 1)
	9	1	1	1	1	(-1, 0), (0, -1), (1, 0), (0, 1), (0, 0)
	10	1	2	1	1	(0.0411, 1), (-1, -0.0409), (0, -1), (1, -0.0037), (-0.0065, 0.0067)
5	6	1	1	1	1	(-0.9802, -1), (0.0917, 0.0853)
	7	2	1	1	1	(0.9810, 1), (-.0031, 0)
	8	1	2	1	2	(0.0750, -0.0675), (-0.9780, -1)
	9	2	2	1	2	(-0.0026, 0), (0.9789, -1)
	10	2	2	2	2	(-1, -0.9598), (0.0307, 0.0345)
6	6	1	1	1	1	(1, 0), (-0.0032, 0)
	7	1	1	1	1	(-1, -0.0287), (-0.0287, -1), (0.0362, 0.0362)
	8	2	1	1	1	(1, 0.0253), (0.0253, 1), (-0.0275, -0.0275)
	9	2	2	1	1	(1, -0.0210), (0.0565, -1), (-0.0313, 0.0239)
	10	2	2	2	1	(0.0477, 1), (1, 0.0477), (-0.0273, -0.0273)
7	6	1	1	1	1	(-0.9789, 1), (0.0864, -0.0794)
	7	1	1	1	2	(-0.9799, -1), (0.0034, 0)
	8	1	2	1	2	(0.9782, -1), (-0.0767, -0.0689)
	9	2	2	1	2	(1, -0.9782), (-0.0012, 0.0026)
	10	2	2	2	2	(-0.9583, -1), (0.0328, 0.0288)
8	6	1	1	1	1	(1, 0), (-0.0030, 0)
	7	1	1	1	1	(0.0285, -1), (1, -0.0285), (-0.0344, 0.0344)
	8	1	2	1	1	(0.0282, -1), (1, -0.0282), (-0.0267, 0.0267)

Remark :$F_{p_1} = (-1,-1), F_{p_2} = (-1,1), F_{p_3} = (1,-1), F_{p_4} = (1,1)$

Table B.1 Computer generated design from Genetic Algorithm for weighted D-optimality criterion in cuboidal design, $k = 2$ (Continued)

Combination	**N**	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	**Other**
	9	2	1	2	1	(-0.0271, 1), (-1, 0.0627), (0.0250, -0.0298)
	10	1	2	2	2	(-0.0570, -1), (-1, -0.0570), (0.0277, 0.0277)
9	6	1	1	1	1	(-1, -0.0723), (0.0723, 1)
	7	1	1	1	2	(-1, 0.0558), (0.0558, -1)
	8	2	2	1	1	(0.0997, 1), (1, -0.0667)
	9	2	2	1	2	(0.0778, -1), (1, -0.0778)
	10	2	2	2	2	(-1, 0), (0.0524, 0)
10	6	1	1	0	1	(1, -0.2360), (0.2361, -1), (-0.1423, 0.1423)
	7	1	1	1	1	(-1, 0.0518), (-0.0518, 1), (0.0780, -0.0780)
	8	1	1	1	1	(0, 1), (0, -1), (1, 0), (-1, 0)
	9	1	1	1	1	(0, 1), (0, -1), (1, 0), (-1, 0), (0, 0)
	10	1	1	1	2	(-0.0523, 1), (1, -0.0523), (-1, 0.0095), (0.0095, -1), (0.0174, 0.0174)
11	6	1	1	1	1	(-1, -0.0808), (0.0807, 1)
	7	2	1	1	1	(-0.0606, 1), (1, -0.0606)
	8	1	1	2	2	(-1, 0.1131), (-0.1845, -1)
	9	2	2	1	2	(0.8815, -1), (-0.0127, 0.0125)
	10	2	2	2	2	(-1, 0), (0.0482, 0)
12	6	1	1	1	1	(1, 0), (-0.0043, 0)
	7	1	1	1	1	(1, 0.0547), (0.0547, 1), (-0.0764, -0.0764)
	8	1	1	1	1	(0, -1), (-1, 0), (0, 1), (1, 0)
	9	1	1	1	1	(0, -1), (-1, 0), (0, 1), (1, 0), (0, 0)
	10	2	1	1	1	(-1, 0.0882), (0.0882, -1), (1, 0.0114), (0.0114, 1), (-0.0193, -0.0193)
13	6	1	1	1	1	(-0.0731, 1), (1, -0.0731)
	7	1	2	1	1	(1, 0.0673), (-0.0673, -1)
	8	2	1	2	1	(-1, 0.1181), (0.0753, 1)
	9	1	2	2	2	(-0.0950, -1), (-1, -0.0950)
	10	2	2	2	2	(0, -1), (0, 0.0512)
14	6	1	1	1	0	(0.2652, 1), (1, 0.2652), (-0.1407, -0.1407)
	7	1	1	1	1	(-1, 0.0543), (-0.0543, 1), (0.0799, -0.0799)
	8	1	1	1	1	(0, -1), (-1, 0), (0, 1), (1, 0)
	9	1	1	1	1	(0, -1), (-1, 0), (0, 1), (1, 0), (0, 0)
	10	1	1	2	1	(-0.0642, -1), (1, 0.0642), (-1, -0.0097), (0.0097, 1), (0.0187, -0.0187)
15	6	1	1	1	1	(-0.0828, 1), (1, -0.0827)
	7	1	2	1	1	(-0.0774, -1), (1, 0.0774)
	8	1	2	2	1	(-1, -0.8906), (0.0732, 0.0858)
	9	2	2	1	2	(0.8987, -1), (-0.0112, 0.0109)
	10	2	2	2	2	(1, 0), (-0.0469, 0)
16	6	1	1	1	1	(0, 1), (0, -0.0043)
	7	1	1	1	1	(-1, -0.0578), (-0.0578, -1), (0.0783, 0.0783)
	8	1	1	1	1	(0, -1), (-1, 0), (0, 1), (1, 0)

Remark :$F_{p_1} = (-1,-1), F_{p_2} = (-1,1), F_{p_3} = (1,-1), F_{p_4} = (1,1)$

Table B.1 Computer generated design from Genetic Algorithm for weighted D-optimality criterion in cuboidal design, $k = 2$ (Continued)

Combination	**N**	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	**Other**
	9	1	1	1	1	(0, -1), (-1, 0), (0, 1), (1, 0), (0, 0)
	10	1	1	2	1	(1, 0.1079), (-0.1079,-1), (-1, 0.0148), (-0.0148, 1), (0.0213, 0.0213)

Remark :$F_{p_1} = (-1, -1), F_{p_2} = (-1, 1), F_{p_3} = (1, -1), F_{p_4} = (1, 1)$

Table B.2: Computer generated design from Genetic Algorithm for weighted G-optimality criterion in cuboidal design, $k = 2$

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	Other
1	6	1	1	1	1	(0,0), (0,0)
	7	0	1	1	1	(-0.5332, -1), (-1, -0.5332), (0.4519, 0.0813), (0.0813, 0.4519)
	8	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1)
	9	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1), (0, 0)
	10	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1), (0, -0.4895), (0, 04895)
2	6	1	0	0	0	(0.1224, 1), (1, 0.1117), (-0.0714, -0.0774), (-1, 0.9861), (0.9848, -1)
	7	0	1	1	0	(-1, -0.2873), (0.4294, -0.0533), (-0.0572, 1), (-0.5586, -1), (1, 0.7296)
	8	1	1	1	1	(-1, 0), (1, 0), (0, -0.7321), (0, 0.7321)
	9	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1), (0, 0)
	10	1	1	1	1	(-1, 0.3645), (-1, -0.3645), (0.0051, 0), (1, 0), (0, 1), (0, -1)
3	6	1	1	1	1	(0,0), (0,0)
	7	1	1	1	1	(-0.2805, 0.4885), (-0.2805, -0.4885), (1, 0)
	8	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1)
	9	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1), (0, 0)
	10	1	1	1	1	(0, -1), (1, -0.3744), (-1, -0.3744), (0.5640, 1), (-0.5640, 1), (0, 0)
4	6	1	1	0	0	(-0.0942, -0.0243), (1, 0.0298), (0.7384, 1), (0.7528, -1)
	7	0	1	1	0	(1, 0.8773), (0.1048, -0.1278), (0, 1), (-1, 0.0436), (-0.8928, -1)
	8	1	1	1	1	(-1, 0), (1, 0), (0, -0.7321), (0, 0.7321)
	9	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1), (0, 0)
	10	1	1	1	1	(0, 1), (1, 0.1831), (-1, 0.1830), (0.0007, 0), (0.4896, -0.8295), (-0.4894, -0.8295)
5	6	1	1	1	1	(0,0), (0,0)
	7	1	1	1	1	(-0.4885, -0.3336), (0.4885, -0.3336), (0, 1)
	8	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1)
	9	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1), (0, 0)
	10	2	2	2	2	(0, 0), (0, 0)
6	6	0	0	0	1	(-0.7142, -1), (0.0849, 0.1409), (0.9871, -1), (-1, -0.1179), (-0.7845, 1)
	7	1	0	1	1	(-1, -0.0372), (-0.1350, -1), (-0.8522, 1), (0.2790, 0.0439)
	8	1	1	1	1	(-0.7918, 0), (0, -1), (0.7918, 0), (0, 1)
	9	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1), (0, 0)
	10	1	2	1	1	(1, -0.0400), (0.0390, -1), (-0.0240, 0.0234), (-1, -0.2581), (0.2587, 1)
7	6	1	1	1	1	(0, 0.4087), (0, -0.4087)
	7	0	1	1	1	(-0.6177, -1), (-1, -0.6468), (0.7626, -0.1000), (-0.1449, 0.7468)
	8	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1)
	9	1	1	1	1	(1, -0.4202), (-1, 0.4202), (0.4202, 1), (-0.4202, -1), (0, 0)
	10	2	2	2	2	(0, 0), (0, 0)
8	6	0	0	1	1	(-0.7480, 1), (-0.7480, -1), (-1, 0), (0.0692, 0)
	7	0	0	0	1	(0.9307, -1), (-1, 0.9307), (-0.6193, -1), (-1, -0.6193), (0.7751, -0.0360), (-0.0360, 0.7751)
	8	1	1	1	1	(1, 0), (-1, 0), (0, -0.8246), (0, 0.8246)

Remark :$F_{p_1} = (-1,-1), F_{p_2} = (-1,1), F_{p_3} = (1,-1), F_{p_4} = (1,1)$

Table B.2 Computer generated design from Genetic Algorithm for weighted G-optimality criterion in cuboidal design, $k = 2$ (Continued)

Combination	**N**	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	**Other**
	9	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1), (0, 0)
	10	1	1	1	2	(-0.3500, -1), (1, -0.6864), (0.1475, 0.1175), (-1, -0.0823), (-0.4880, 1)
9	6	1	1	1	1	(0,0), (0,0)
	7	0	1	1	1	(-0.5836, -1), (-1, -0.5836), (0.7125, -0.0902), (-0.0902, 0.7125)
	8	1	1	1	1	(-0.7918, 0), (0.7918, 0), (0, 1), (0, -1)
	9	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1), (0, 0)
	10	1	1	1	1	(0.0003, -1), (1, -0.3118), (-1, -0.3122), (0.5639, 1), (-0.5641, 1), (-0.0002, -0.1639)
10	6	1	0	0	0	(0.4363, 1), (0.8242, -1), (1, 0.4395), (-1, 0.8182), (-0.2143, -0.2217)
	7	0	1	1	0	(-1, -0.0760), (1, 0.7170), (-0.0763, 1), (-0.6826, -1), (0.3144, -0.2835)
	8	1	1	1	1	(1, -0.0522), (-1, 0.0522), (-0.0633, -0.8246), (0.0633, 0.8246)
	9	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1), (0, 0)
	10	1	1	1	1	(0, 1), (-1, 0), (1, 0), (1, 0), (0, -1), (0.0160, 0)
11	6	1	1	1	1	(-0.4087, 0), (0.4087, 0)
	7	0	0	0	1	(0.9901, -1), (-1, 0.9901), (-0.6231, -1), (-1, -0.6231), (0.7739, -0.1408), (-0.1408, 0.7739)
	8	1	1	1	1	(1, -0.0522), (-1, 0.0522), (-0.0633, -0.8246), (0.0633, 0.8246)
	9	1	1	1	1	(1, -0.4202), (-1, 0.4202), (0.4202, 1), (-0.4202, -1), (0, 0)
	10	1	1	1	1	(0, -1), (1, -0.4300), (-1, -0.4300), (0.5640, 1), (-0.5640, 1), (0, 0.1766)
12	6	1	0	0	0	(-0.1819, -0.1819), (1, 0.5230), (0.5230, 1), (0.8621, -1), (-1.0000, 0.8621)
	7	0	0	0	1	(0.9382, -1), (-1, 0.9382), (-0.6033, -1), (-1, -0.6033), (0.7730, -0.0398), (-0.0398, 0.7730)
	8	1	1	1	1	(-1, 0), (1, 0), (0, 0.7918), (0, -0.7918)
	9	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1), (0, 0)
	10	1	1	1	1	(0, -1), (1, -0.4301), (-1, -0.4301), (0.5640, 1), (-0.5640, 1), (0, 0.1754)
13	6	1	1	1	1	(0,0), (0,0)
	7	0	1	1	1	(-0.5842, -1), (-1, -0.5842), (0.7131, -0.0928), (-0.0928, 0.7131)
	8	1	1	1	1	(0, -1), (0, 1), (-0.7918, 0), (0.7918, 0)
	9	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1), (0, 0)
	10	1	1	1	1	(0, -1), (1, -0.4419), (-1, -0.4419), (0.7041, 1), (-0.7041, 1), (0, 0)
14	6	1	0	0	0	(0.4555, 1), (-0.2164, -0.1495), (1, 0.4257), (0.7935, -1), (-1, 0.8433)
	7	0	0	0	1	(0.9375, -1), (-1, 0.9375), (-0.6009, -1), (-1, -0.6009), (0.7758, -0.0231), (-0.0231, 0.7758)
	8	1	1	1	1	(-0.8425, 0), (0.8425, 0), (0, 0.9346), (0, -0.9345)
	9	1	1	1	1	(-1, 0), (1, 0), (0, 1), (0, -1), (0, 0)
	10	1	1	1	1	(-1, 0), (-1, 0), (0, 1), (0, -1), (-0.0204, 0), (1, 0)
15	6	1	1	1	1	(0, -0.4087), (0, 0.4087)
	7	0	0	0	1	(0.9901, -1), (-1, 0.9901), (-0.6231, -1), (-1, -0.6231), (0.7739, -0.1408), (-0.1408, 0.7739)
	8	1	1	1	1	(1, 0), (-1, 0), (0, -0.8282), (0, 0.8282)
	9	1	1	1	1	(1, -0.4202), (-1, 0.4202), (0.4202, 1), (-0.4202, -1), (0, 0)
	10	1	1	1	1	(0, -1), (1, -0.5064), (-1, -0.5064), (0.7140, 1), (-0.7140, 1), (0, 0)
16	6	0	0	1	0	(-0.2082, -0.2082), (1, 0.5222), (0.5222, 1), (0.8640, -1), (-1, 0.8640)
	7	0	0	0	1	(0.9378, -1), (-1, 0.9378), (-0.6031, -1), (-1, -0.6031), (0.7730, -0.0509), (-0.0509, 0.7730)
	8	1	1	1	1	(0.8281, 0), (-0.8281, 0), (0, -1), (0, 1)

Remark :$F_{p_1} = (-1,-1), F_{p_2} = (-1,1), F_{p_3} = (1,-1), F_{p_4} = (1,1)$

Table B.2 Computer generated design from Genetic Algorithm for weighted G-optimality criterion in cuboidal design, $k = 2$ (Continued)

Combination	**N**	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	**Other**
	9	1	1	1	1	(1, -0.4202), (-1, 0.4202), (0.4202, 1), (-0.4202, -1), (0, 0)
	10	1	1	1	1	(0, -1), (1.0, -0.4300), (-1, -0.4300), (0.5640, 1), (-0.5640, 1), (0, 0.1763)

Remark :$F_{p_1} = (-1, -1), F_{p_2} = (-1, 1), F_{p_3} = (1, -1), F_{p_4} = (1, 1)$

Table B.3: Computer generated design from Genetic Algorithm for weighted D-optimality criterion in cuboidal design, $k = 3$

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	Other
1	10	1	1	1	1	1	1	1	1	(-0.0001, 0.3863, -1), (1, -1, 0.9679), (0, -1, 1)
	11	1	1	1	1	1	1	1	1	(0.0414, -1, -0.8543), (0.0429, 1, 1), (-1, 0.1718, -1)
	12	1	1	1	1	1	2	1	1	(0.0214, 1, 0.8535), (-0.0232, -1, -1), (-1, -0.1580, 1)
	13	1	1	1	1	2	1	1	2	(-1, -0.1450, -1), (-0.0318, 1, -0.8581), (-0.0350, -1, 1)
	14	1	2	1	1	1	2	2	1	(0, 0.2833, 1), (0, -1, -1), (-1, 1, -0.9297)
2	10	0	0	1	0	1	0	1	1	(0.0001, 0.0122, -1), (0.0110, -1, 1), (-1, 0.0478, 1), (1, -0.1279, 0.3300), (-0.0311, 1, 0.0670) (-1, -1, -0.4109)
	11	1	1	0	1	0	1	0	1	(0.0282, -1, -0.0051), (-1, 0.1263, -0.3070), (1, 1, -0.1818), (1, -0.1269, -1), (-0.0273, 1, -1) (0.0062, 0.0025, 1)
	12	1	1	0	0	1	1	1	1	(-1, 1, -0.3104), (-0.0015, -0.0121, -1), (-0.0247, 1, 1), (1, 0, 0.1105), (-1, 0.0991, 1), (0, -1, 0.0384)
	13	1	1	0	1	1	1	1	1	(-1, 1, -0.0955), (-1, 0.0136, -1), (1, 0.0085, -0.0930), (-0.0034, 1, -1), (-0.0011, 0, 1), (-0.0004, -1, 0)
	14	1	1	1	1	1	1	1	1	(0, -1, -0.0002), (0, 0, -1), (1, 0, 0), (0, 1, 0), (-1, 0, 0), (0, 0, 1)
3	10	1	1	1	0	1	1	1	1	(0, -0.3276, -1), (-1, 1, 0.9785), (0, 1, 1)
	11	1	1	1	1	1	1	1	1	(-0.0238, -1, 1), (1, 1, 0.9550), (-0.0543, 0.2657, -1)
	12	1	1	1	1	1	2	1	1	(-1, -1, 0.9536), (0, 1, 1), (0, -0.1724, -1)
	13	2	1	1	1	2	1	1	1	(-0.0648, 1, -1), (-0.0243, -0.2263, 1), (1, 1, 0.9516)
	14	2	1	1	1	2	1	1	2	(0, -0.2511, 1), (-1, 1, 0.9516), (0, 1, -1)
4	10	1	0	1	1	0	0	1	0	(1, -1, -0.8282), (0.0515, 1, 0.1545), (1, 0.0401, 1), (-1, -0.2329, 0.4765), (-0.0008, 0.0055, -1) (-0.0010, -1, 1)
	11	1	1	1	1	1	1	1	0	(1, 0.0964, 0.2806), (0.0895, 1, 1), (-0.0362, -0.0183, -1), (0.0099, -1, -0.0016)
	12	1	1	1	1	1	1	1	1	(1, -0.0025, 0.0639), (-0.0243, 0, -1), (0.0111, -1, 1), (0.0086, 1, -0.0077)
	13	1	1	1	1	1	1	1	1	(1, 0.0050, -1), (0, 0, 1), (-0.0006, -1, -1), (0.0052, 1, -0.0285), (-1, -0.0093, -0.0645)
	14	1	1	1	1	1	1	1	1	(0, 0, 1), (0, -1, 0), (-1, 0, 0), (0, 0, -1), (1, 0, 0), (0, 1, 0)
5	10	1	1	1	0	1	1	1	1	(0, -0.3013, -1), (0, 1, 1), (-1, 1, 0.9949)
	11	1	1	1	1	1	1	1	1	(0, 1, -1), (-1, -1, -0.9915), (0.0266, -0.2715, 1)
	12	1	1	1	1	1	2	1	1	(1, 1, -0.9858), (-0.0238, -0.2914, -1), (-0.0414, 1, 1)
	13	1	1	1	1	2	1	1	2	(0.0163, -0.1913, 1), (-1, -1, 0.9896), (-0.0475, 1, -1)
	14	2	1	1	2	2	1	1	1	(1, 1, 0.9952), (0.0205, -0.1946, 1), (0.0610, 1, -1)
6	10	1	1	1	1	1	1	0	0	(1, 1, 0.8860), (1, 0.1857, -1), (-0.0316, -0.0966, -0.0079), (0.0570, 1, -1)
	11	1	1	1	1	1	1	0	1	(1, 0.1964, -1), (-0.0144, -0.0105, 1), (0.0059, -1, -0.0632), (0.1018, 1, -1)
	12	1	1	1	1	1	1	1	1	(-0.0118, -1, -0.0436), (-1, 0.0085, -0.2689), (0.0250, 0.0036, 1), (-0.0207, 1, -1)
	13	1	1	1	1	1	1	1	1	(1, -0.0141, -1), (0, -1, -1), (0, 1, -0.0490), (0, -0.0148, 1), (-1, -0.0112, -1)
	14	1	1	1	1	2	1	1	1	(1, 0.0315, 1), (-1, -0.0201, 1), (0, -0.0057, -1), (0, 1, 0.0213), (0, -1, 1)
7	10	0	1	1	1	1	0	1	1	(1, -1, 0.9983), (0, -1, 1), (1, 0.8318, 1), (-1, -1, -0.9991)
	11	1	1	1	1	0	1	1	1	(-1, 0.8980, -1), (-0.0261, -1, 1), (1, 1, 0.9930), (1, -1, -0.9961)
	12	1	1	1	2	1	2	1	1	(0, -1, -1), (1, 0.8512, -1)
	13	1	1	1	0	1	1	2	1	(1, -0.7593, 1), (-1, 1, 0.9107), (-1, 1, 0.9978), (-1, -1, -0.9928), (0, -1, 1)
	14	2	1	1	2	1	1	2	1	(-0.0165, -0.1458, 1), (-0.0709, 1, -1), (1, -1, 0.9903)

Remark :$F_{p_1} = (-1,-1,-1), F_{p_2} = (-1,-1,1), F_{p_3} = (-1,1,-1), F_{p_4} = (-1,1,1), F_{p_5} = (1,-1,-1), F_{p_6} = (1,-1,1), F_{p_7} = (1,1,-1), F_{p_8} = (1,1,1)$

Table B.3 Computer generated design from Genetic Algorithm for weighted D-optimality criterion in cuboidal design, $k = 3$ (Continued)

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	Other
8	10	0	0	1	1	1	1	1	1	(-1, -0.2594, -1), (-0.0678, -1, -1), (0.0295, 0.0915, 0), (-1, -1, 0.8701)
	11	1	1	1	1	1	1	1	1	(0.0143, 0.0523, -1), (-0.0111, -1, 0.0001), (-1, -0.0339, 0.0125)
	12	1	1	1	1	1	1	1	1	(0.0235, -1, -1), (-0.0245, -0.0127, 1), (0.0109, 1, -0.0786), (1, -0.0115, -0.3570)
	13	1	1	1	1	1	1	1	1	(1, 0, -1), (0, 0, 0), (0, -1, -1), (-0.0074, 1, -1), (-1, 0, -1)
	14	1	1	1	1	1	1	2	1	(-1, 0.0165, 1), (0, 0.0079, -0.0142), (-0.0058, 1, 1), (1, -0.0540, 1), (0, -1, 1)
9	10	1	1	1	1	1	0	1	1	(0, -1, 1), (1, -1, 0.9390), (0, 0.1403, -1)
	11	1	1	1	1	1	1	1	1	(0, -0.0081, 1), (0, 1, -0.8039), (0, -1, -1)
	12	1	1	1	1	1	1	1	2	(0.0087, -0.0501, -1), (-0.0419, 1, -0.8179), (-0.0024, -1, 1)
	13	1	1	1	1	1	1	2	1	(-1, 0.0704, -1), (-0.0186, 1, 1), (1, -1, 0.8707), (-0.0247, -1, -1)
	14	1	2	1	1	1	2	1	1	(-1, 1, -0.8628), (0.0441, 0.0228, -1), (-0.0035, -1, -1), (0.0377, 1, 1)
10	10	0	1	0	0	1	1	0	1	(-1, 1, 0.2419), (-0.0341, 0.0264, 1), (1, 0.1126, -0.1209), (0.0643, 1, -1), (-1, -0.1001, -1) (-0.0995, -1, -0.0767)
	11	1	0	1	0	1	0	1	1	(-1, 0.1217, 1), (0, 0, -1), (0.1010, -1, 1), (1, -0.1136, 0.0907), (-1, -1, 0.0824) (-0.0935, 1, 0.0071)
	12	1	0	1	1	1	0	1	1	(-1, -0.1115, 0.1075), (0.0280, 1, -0.0358), (1, -0.0377, 1), (1, -1, 0.0481), (0.0238, 0.0331, -1) (-0.0906, -1, 1)
	13	1	0	1	1	1	1	1	1	(-0.0203, 1, 0), (-1, -1, 0.0419), (-0.0128, 0, -1), (-1, -0.0300, 1), (1, -0.0281, 0.0262) (-0.0317, -1, 1)
	14	1	1	1	1	1	1	1	0	(1, 0.0295, 1), (1, 1, 0.0698), (-0.0198, -0.0067, 1), (-1, 0.0305, 0.0020), (0.0119, 0, -1) (0.0341, 1, 1), (0.0338, -1, -0.0148)
11	10	1	1	1	1	1	1	1	0	(0.0046, 1, 1), (0, -0.0822, -1), (1, 1, 0.9688)
	11	1	1	1	1	1	1	1	1	(0, 1, -1), (0, -1, -0.8908), (0, 0.0061, 1)
	12	1	1	2	1	1	1	1	1	(0.0265, 1, 0.8889), (0.0599, -0.0779, -1), (0, -1, 1)
	13	1	1	2	1	1	1	2	1	(0, -0.0813, -1), (-0.0204, -1, 1), (0, 1, 0.8956)
	14	2	1	1	2	2	1	1	1	(1, 0.1797, 1), (0.0189, 1, -1), (-0.0204, -1, 0.8769)
12	10	0	0	0	1	0	1	1	1	(-1, -1, 0.5437), (-0.0731, -0.0727, 1), (1, -0.2228, -0.2834), (0.1170, -1, -1), (-0.1765, 1, -0.2155) (-1, 0.1719, -1)
	11	1	1	1	1	1	1	1	1	(-1, -0.0159, -0.0073), (0.0138, 0.0210, 1), (-0.0099, -1, -0.0082)
	12	1	1	1	1	1	1	1	1	(-0.0393, 0.0083, 1), (1, -0.0229, -0.0587), (0.0392, -1, -1), (0.0049, 1, 0.0099)
	13	1	1	1	1	1	0	1	1	(1, -0.0945, 1), (1, -1, 0.2145), (0.0330, 1, 0.0684), (0.0787, -1, 1), (-1, -0.0377, 0.0889) (0.0419, -0.0386, -1.0000)
	14	1	1	1	1	1	1	1	1	(0, 0, 1), (-1, 0, 0), (0, 0, -1), (0, 1, 0), (1, 0, 0), (0, -1, 0)
13	10	1	1	1	1	1	1	1	0	(1, 1, 0.9586), (0, 1, 1), (0, -0.1199, -1)
	11	1	1	1	1	1	1	1	1	(0, -1, -0.8530), (0, 1, -1), (0, 0.0073, 1)
	12	1	1	1	1	2	1	1	1	(-0.0319, -1, 1), (0, 1, -0.8789), (1, 0.1200, 1)
	13	1	1	1	1	2	1	1	2	(0.0096, -1, 0.8785), (-0.0140, 1, -1), (-1, 0.0734, 1)
	14	2	1	1	1	2	1	1	1	(-0.0164, 1, -1), (1, 1, 0.9191), (-1, 0.1195, 1), (0.0237, -1, 1)
14	10	0	0	0	1	0	1	1	1	(-1, 0.1417, -1), (-0.1137, 1, -0.1158), (-1, -1, 0.3125), (1, -0.1327, -0.2028), (0.1029, -1, -1) (-0.0450, -0.0323, 1)
	11	1	1	0	0	1	1	0	1	(0.0768, 1, -1), (-1, 0.1062, -1), (1, 0.1284, -0.1934), (-0.0255, -1.0000, -0.0351), (-1, 1, 0.2844)

Remark :$F_{p_1} = (-1,-1,-1), F_{p_2} = (-1,-1,1), F_{p_3} = (-1,1,-1), F_{p_4} = (-1,1,1), F_{p_5} = (1,-1,-1), F_{p_6} = (1,-1,1), F_{p_7} = (1,1,-1), F_{p_8} = (1,1,1)$

Table B.3 Computer generated design from Genetic Algorithm for weighted D-optimality criterion in cuboidal design, $k = 3$ (Continued)

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	Other
	12	0	0	1	1	1	1	1	1	(-0.0636, -0.0112, 1) (-0.0299, -0.0114, 1), (-0.0882, -1, -1), (-0.0015, 1, -0.0627), (-1, -1, 0.2124), (1, -0.0238, -0.0620) (-1, -0.0968, -1)
	13	1	1	1	0	1	1	1	1	(1, 0.0265, 0.0523), (-1, 0.0474, 1), (-0.0244, -1, 0.0090), (-1, 1, 0.0770), (-0.0262, 0.0032, -1) (-0.0464, 1, 1)
	14	1	0	1	1	1	1	1	1	(-1, -1, 0.0908), (0, 1, 0.0443), (-1, -0.0627, 1), (-0.0290, -1, 1), (1, 0.0410, 1), (1, -0.0543, -0.0649) (-0.0289, -0.0127, -1)
15	10	1	1	1	1	0	1	1	1	(1, -1, -0.9753), (0.0025, -1, -1), (0, 0.0465, 1)
	11	1	1	1	1	1	1	1	1	(0, 0.0090, 1), (0, -1, -0.9068), (0, 1, -1)
	12	1	1	1	2	1	1	1	1	(0, -1, -0.8766), (1, 0.0112, 1), (0.0113, 1, -1)
	13	2	1	2	1	1	1	1	1	(1, 0, -1), (0.0098, 1, 1), (0.0142, -1, 0.8851)
	14	2	1	1	2	1	1	1	2	(1, -0.1585, -1), (0.0112, -1, 0.8682), (-0.0305, 1, -1)
16	10	1	1	1	1	0	1	0	0	(-1, -0.0130, -0.0432), (0.0999, 0.1756, 1), (1, -0.2398, -1), (0.1204, -1, -0.2753), (1, 1, 0.3348)
	11	1	1	1	1	1	1	1	1	(1, 0.0143, -0.0041), (-0.0160, -0.0226, 1), (0.0146, 1, -0.0096)
	12	1	1	1	1	1	1	1	1	(0.0038, -1, -0.0011), (1, 0.0212, 0.0691), (-0.0439, -0.0011, -1), (0.0459, 1, 1)
	13	1	1	1	1	1	0	1	1	(0.1000, -1, 1), (0.0225, 1, 0.0887), (0.0477, -0.0458, -1), (-1, -0.0427, 0.1172), (1, -0.1013, 1) (1, -1, 0.2964)
	14	1	1	1	1	1	1	1	1	(1, 1, -0.0520), (-1, 0.0303, 0.1129), (-0.0180, 1, 1), (0.0513, 0.0481, -1), (0.0304, -1, 0.0790) (1, -0.0287, 1)

Remark :$F_{p_1} = (-1,-1,-1), F_{p_2} = (-1,-1,1), F_{p_3} = (-1,1,-1), F_{p_4} = (-1,1,1), F_{p_5} = (1,-1,-1), F_{p_6} = (1,-1,1), F_{p_7} = (1,1,-1), F_{p_8} = (1,1,1)$

Table B.4: Computer generated design from Genetic Algorithm for weighted G-optimality criterion in cuboidal design, $k = 3$

Combination	**N**	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	**Other**
1	10	0	0	0	1	1	0	1	0	(-0.7071, 1, -1), (-1, -1, 0.9050), (-0.0429, -0.9530, 0.0395), (-1, -0.5250, -1) (-0.0126, 0.0636, 0), (0.7820, 1, 1), (1, -0.6061, 1)
	11	1	1	1	1	1	1	1	1	(0, 1, 0), (0, 0, 0), (0, -1, 0)
	12	1	1	1	1	1	1	1	1	(0, -1, -0.0039), (0, 0, -1), (0, 1, 0), (0, 0, 1)
	13	0	1	0	1	1	1	0	1	(-1, 1, -0.7050), (0, 0.0607, 1), (0, 0.9364, -1), (1, 1, -0.7751) (1, 0.0034, -0.2948), (0, -1, 0), (-1, 0.0120, -0.4436), (-1, -1, -0.9710)
	14	1	1	1	1	1	1	1	1	(-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0), (0, 0, -1), (0, 0, 1)
2	10	0	0	0	0	1	0	0	0	(1, 1, -0.8106), (1, 0.1383, 0.4076), (0, -0.0180, -0.9440), (1, -0.9970, 1) (-1, -0.0492, 1), (-0.0258, 1, 1), (-1, 1, -0.4503), (-0.0202, -1, 0.2965), (-1, -1, -0.8627)
	11	0	0	0	0	0	0	0	0	(1, -0.1681, -1), (0.1964, -1, -0.9330), (-1, -1, 0.7930), (-0.1073, 1, -1) (0.6048, 0.6978, 1), (-1, 1, -0.0172), (-0.1018, -0.2760, 0.1015), (-1, -0.4437, -1) (1, -1, 0.8709), (-1, 0.4118, 1), (1, 1, -0.1245)
	12	0	0	0	0	0	0	0	0	(1, -0.9397, -1), (0.0103, 0, -0.9514), (1, 0.2751, 1), (1, 1, -0.6960), (-1, 1, -0.9650) (0.0233, 0.0636, -0.1463), (0.0498, 1, 0.6850), (-1, 0.9212, 1), (-1, -1, -0.6838) (1, -1, 0.2851), (-1, -0.2265, 0.6669), (-0.0959, -1, 1)
	13	1	1	1	0	0	0	0	0	(1, 0.5237, 1), (1, -0.3407, -1), (1, 1, -0.7432), (0.1627, -1, 1), (-1, 0.0692, 0.1099) (0.0478, 1, 0.0316), (-0.0699, 0, 0), (-0.8574, 0.9540, 1), (1, -1, 0.0751), (0, -0.2605, -1)
	14	1	1	1	1	1	1	1	1	(-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0), (0, 0, -1), (0, 0, 1)
3	10	0	1	0	0	0	1	0	0	(1, -0.6299, -0.9959), (1, 1, -0.9588), (-0.7982, 1, -1), (-0.0595, 0.3082, 0.0846) (0.8587, 1, 1), (0.0179, -1, -0.9140), (-1, -0.6867, -0.9934), (-1, 1, 0.9157)
	11	1	1	1	1	1	1	1	1	(0, 1, 0), (0, 0, 0), (0, -1, 0)
	12	1	1	1	1	1	1	1	1	(0, 1, 0), (0, 0, -1), (0, -1, 0), (0, 0, 1)
	13	1	1	1	1	1	1	1	1	(1, 0, 0), (-1, -0.0004, 0), (0, -1, -0.0867), (0, 0, 1), (0, 1, -0.0855)
	14	1	1	1	1	1	1	1	1	(-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0), (0, 0, -1), (0, 0, 1)
4	10	0	1	0	0	0	0	0	0	(0.0369, -0.1875, 1), (-1, 0.0444, -1), (-1, 1, 0.7646), (1, 1, 0.9598), (0, 1, -0.8920) (1, -1, 0.9120), (0.4546, -1, -1), (1, 0.0947, -0.8586), (-0.4577, -0.9300, -0.1785)
	11	0	0	1	0	0	0	0	1	(-0.6397, 1, 0.6767), (0.0421, 0.0883, -0.9552), (1, -0.4547, 0.5408), (0.9358, 1, -0.9114) (-1, -1, -0.6955), (-0.1045, -1, -0.2701), (0.1816, -1, 1), (1, -1, -0.9382), (-1, -0.4255, 1)
	12	0	0	0	0	0	0	0	0	(-0.0369, -0.9494, -1), (1, -1, -0.8310), (-0.1487, 1, 0.9056), (0.0716, -0.0732, -0.0482) (-1, -0.4271, -1), (-1, 1, -0.7778), (-1, 0.3893, 1), (1, 0.7822, 0.9244), (-0.0411, -1, 1) (0.9907, 1, -1), (-1, -1, 0.2749), (1, -0.5859, 0.8328)
	13	0	0	0	1	0	0	0	0	(-0.9907, -1, -0.9737), (1, -0.9054, -0.5624), (0.0268, 0.0180, -1), (-1, -0.9792, 1) (0.1840, -1, -0.8861), (1, -0.8481, 1), (1, 0.6957, -1), (0.0017, -0.0280, 0.9282) (-0.7703, 1, -1), (1, 1, 0.8115), (0.1772, 1, -0.0887), (-1, 0.0627, -0.3027)
	14	1	1	1	1	1	1	1	1	(-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0), (0, 0, -1), (0, 0, 1)
5	10	1	1	1	1	1	1	1	1	(0, 1, 0), (0, -1, 0)
	11	1	1	1	1	1	1	1	1	(0, 1, 0), (0, -1, 0), (0, 0, 0)
	12	1	1	1	1	1	1	1	0	(0.0223, -1, -0.0356), (1, 1, 0.9691), (0, 1, 0), (0.5798, 0.0463, 0), (-0.5664, 0, 0)
	13	1	1	1	1	1	1	1	1	(0, 0, 0), (-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0)

Remark : $F_{p_1} = (-1,-1,-1), F_{p_2} = (-1,-1,1), F_{p_3} = (-1,1,-1), F_{p_4} = (-1,1,1), F_{p_5} = (1,-1,-1), F_{p_6} = (1,-1,1), F_{p_7} = (1,1,-1), F_{p_8} = (1,1,1)$

Table B.4 Computer generated design from Genetic Algorithm for weighted D-optimality criterion in cuboidal design, $k = 3$ (Continued)

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	Other
	14	1	1	1	1	1	1	1	1	(-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0), (0, 0, -1), (0, 0, 1)
6	10	0	0	0	0	0	0	1	1	(1, -0.4613, -0.9705), (-1, -0.1152, -1), (-1, -1, 0.3375), (-0.5873, 1, -0.6717), (-1, 0.9346, 1) (1, -1, 0.8409), (-0.0476, -1, -1), (0.1150, 0.1612, 0.4493)
	11	0	0	0	0	1	0	0	0	(-0.0152, -1, 1), (-1, 0.0862, 1), (-1, -1, 0.1975), (-0.7824, -0.7762, -1), (-0.5750, 1, 0.8842) (1, -0.6718, 0.8437), (0.2014, 0.1224, -0.0922), (0.9688, 1, -1), (-1, 1, -0.9296), (1, 0.9194, 1)
	12	1	0	1	0	0	0	0	0	(0, 1, -0.9257), (-0.0934, -1, 0), (0.1336, 0.0423, 1), (1, -0.8431, 1), (-1, -0.0283, 0), (1, 0.8548, -1) (1, -0.8877, -1), (-0.9294, 1, 0.9934), (1, 0.9708, 0.8788), (-0.9746, -1, 1)
	13	1	0	1	0	1	0	1	0	(1, 1, 0.6876), (1, -0.0270, 1), (-1, 1, 0.4723), (1, -1, 0.4644), (-1, -1, 0.6841), (0.0073, -1, 1) (0, 0, -0.3338), (0, 1, 1), (-1, 0, 1)
	14	1	1	1	1	1	1	1	1	(-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0), (0, 0, -1), (0, 0, 1)
7	10	1	1	1	1	1	1	1	1	(0, 1, 0), (0, -1, 0)
	11	1	1	1	1	1	1	1	1	(0, 1, 0), (0, -1, 0), (0, 0, 0)
	12	1	1	1	1	1	1	1	1	(-0.7763, 0, 0), (0.8916, -0.0013, 0), (-0.0683, 1, 0), (-0.0563, -1, 0)
	13	1	1	1	1	1	1	1	1	(0, 1, 0), (-1, 0, 0), (0, 0, -1), (1, 0 ,0), (0, -1, 0)
	14	1	1	1	1	1	1	1	1	(0.1857, -1, 0.0004), (1, 0, -0.0014), (-1, 0, 1), (-1, 0, -1), (0.1805, 1, 0), (0, -0.0054, 1)
8	10	0	0	0	0	0	0	0	0	(-0.9980, 0.4518, 0.9519), (1, 1, 0.4834), (1, -1, -0.8832), (-0.8810, -1, 0.9900), (-1, -0.7485, -1) (0.0167, 1, 1), (1, -0.6217, 1), (-0.9511, 1, -0.7944), (0.8660, 0.6621, -1), (0, -0.5508, -0.0283)
	11	0	0	0	0	0	0	0	0	(-0.7179, -0.7996, -1), (-0.7412, 1, 0.8246), (1, -0.6562, 0.8742), (0.1415, 0.1902, 0.0177) (1, -0.9905, -0.9122), (0.9917, 1, -1), (-1, 0.9649, -0.9972), (1, 0.9092, 1), (-1, -0.0851, 1) (-1, -1, 0.0911), (0, -1, 1)
	12	0	0	0	0	0	0	0	0	(1, -0.9297, 1), (-0.8565, 1, -1), (1, 0.2726, -1), (0.7529, 1, -0.7632), (-0.9055, 1, 1) (1, -1, -0.9475), (-1, 0.0724, -0.1065), (1, 1, 0.9470), (-1, -0.8131, -1), (0.0513, -0.0299, 1) (-0.0055, -1, -0.2500), (-0.9988, -1, 1)
	13	1	0	0	0	1	0	1	0	(0, 1, 0.9526), (-0.0514, -0.0654, -0.3234), (-1, 0.0548, 1), (-1, 1, 0.5759), (1, 1, 0.6839) (1, 0.0192, 0.8913), (-1, 0.9924, -1), (1, -1, 0.6162), (-1, -1, 0.6533), (-0.0132, -1, 1)
	14	1	1	1	1	1	1	1	1	(-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0), (0, 0, -1), (0, 0, 1)
9	10	1	0	0	0	0	1	0	0	(0.0803, -0.0010, 0.1379), (0.9828, 1, 1), (-1, 1, -0.1328), (-0.2971, 1, -1), (-1, 0.1639, 1) (1, 0.5789, -1), (-0.5041, -1, 1), (0.9345, -1, -1)
	11	0	1	1	0	0	1	1	0	(1, 0.8176, 1), (0.3220, -1, -1), (-1, -0.8212, -1), (-0.2029, 1, 1), (-0.0560, -0.0044, -0.0071) (-1, 0.5565, 1), (1, -0.4669, -0.9575)
	12	1	0	1	0	1	0	0	0	(1, -0.7278, 1), (1, 0.8637, 1), (-1, -0.9628, 1), (0.2263, -1, 0.8430), (-0.9711, 1, 1), (1, 0.9569, -1) (-1, 0, 0.2119), (0.1188, 1, 0.1950), (0.2459, -0.1294, -1)
	13	1	1	0	1	1	1	0	0	(0, -1, 0.0764), (1, 0.9908, 1), (1, 1, -0.4390), (-0.9842, 0, -0.6446), (0, 0, 1), (-1, 1, -0.4657) (0.0077, 1, -1), (0.9583, 0, -0.6671)
	14	1	1	1	1	1	1	1	1	(-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0), (0, 0, -1), (0, 0, 1)
10	10	0	0	0	0	0	0	0	1	(1, -1, -0.9006), (-1, 0.2169, 1), (1, -0.2178, 0.1984), (0.9521, 0.9573, -1), (-0.1875, 1, 0.0904) (-1, 1, -0.9276), (0.1265, -1, 1), (-0.0446, 0, -1), (-1, -1, -0.5143)
	11	0	1	0	0	0	0	0	1	(1, -0.7625, -1), (-0.2562, 0, 1), (0.3204, 1, -1), (-1, 0.3479, -1), (-1, 1, 0.4011), (1, -0.9726, 1) (-1, -1, -0.7188), (1, 0.3174, -0.1772), (0, -0.9467, -0.1061)
	12	0	0	1	0	0	0	0	0	(1, 0, -1), (-1, -1, 0.6693), (1, 1, 0.9603), (0.0261, -0.0091, 0.9887), (0.5314, -1, 1)

Remark : $F_{p_1} = (-1,-1,-1), F_{p_2} = (-1,-1,1), F_{p_3} = (-1,1,-1), F_{p_4} = (-1,1,1), F_{p_5} = (1,-1,-1), F_{p_6} = (1,-1,1), F_{p_7} = (1,1,-1), F_{p_8} = (1,1,1)$

Table B.4 Computer generated design from Genetic Algorithm for weighted D-optimality criterion in cuboidal design, $k = 3$ (Continued)

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	Other
	13	0	0	0	1	0	0	0	1	(-0.9590, -0.0435, -0.0713), (0.5720, 1, -0.7750), (0.1429, 1, -0.0379), (1, -1, -0.11730) (-0.6256, -1, -1), (-1, 1, 0.9734) (0.1728, -1, -1), (0.9841, -0.1226, -1), (1, -1, -0.1252), (-1, 0.0109, -0.10450), (0.1298, 1, 0.0051) (0.0043, 0, 1), (1, -0.9076, 1), (1, 1, -0.8689), (-1, -1, -0.9341), (-1, -1, 0.8900), (-1, 0.9475, -1)
	14	1	1	1	1	1	1	1	1	(-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0), (0, 0, -1), (0, 0, 1)
11	10	0	0	1	0	0	0	1	0	(1, -0.6403, 1), (-0.7619, 1, 1), (-0.0965, -0.0070, -1), (1, -1, -0.9909), (-0.1509, -1, 1) (-1, -0.4856, 1), (0.8119, 0.9839, 1), (-1, -1, -0.8206)
	11	0	0	1	0	1	0	1	0	(-0.1096, -1, 1), (-1, -0.1677, 1), (-1, -1, 0.2172), (-0.8777, -0.9277, -1) (-0.7550, 1, 1), (1, -0.7550, 1), (0.1194, 0.1638, -0.8372), (0.9795, 1, 1)
	12	0	0	0	0	0	1	0	0	(-1, 0.7705, -1), (-1, -0.9361, -1), (-0.2443, 1, -0.8768), (1, 0.9269, -0.9842), (-0.0692, -1, -0.0675) (1, 0, 0.0349), (0.9900, 1, 1), (-1, -1, 0.9713), (-0.3788, 0.0893, 1), (-1, 1, 0.9840), (0.9457, -1, -1)
	13	1	1	1	1	1	1	1	1	(1, 0, 0), (0, 1, -0.1969), (-1, 0, -0.00430, (0, 0.0029, 1), (0, -1, -0.1649)
	14	1	1	1	1	1	1	1	1	(-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0), (0, 0, -1), (0, 0, 1)
12	10	0	0	0	0	0	0	0	0	(-0.9980, 0.4518, 0.8786), (1, 1, 0.3059), (1, -1, -0.9110), (-0.8810, -1, 0.9900), (-1, -0.7809, -1) (0.0615, 1, 1), (1, -0.6490, 1), (-0.9558, 1, -0.7944), (0.7592, 0.6918, -1), (-0.0413, -0.5508, -0.0283)
	11	0	0	1	0	1	0	1	1	(-0.1140, -1, 1), (-1, -0.1140, 1), (-1, -1, 0.1140), (-0.7550, -0.7550, -1), (-0.7550, 1, 0.7550) (1, -0.7550, 0.7550), (0.1858, 0.1858, 0.1858)
	12	0	0	0	0	0	0	0	0	(1, -0.7188, 1), (1, -1, -0.9960), (-1, -1, -0.9960), (0.9930, 0.9841, 0.8816), (-1, 1, -0.8898) (-1, -0.5815, 1), (-0.4144, -1, 0.9600), (-0.7357, 1, 0.9983), (0.9998, 1, -0.8238), (-1, 0.0100, 0.0940) (0.3524, -1, 0.0469), (0.0928, 0.0913, -1)
	13	1	1	1	1	0	1	1	1	(0.3938, -1, -1), (1, -0.3938, -1), (1, -1, -0.3938), (-1, 0, 0), (0, 1, 0), (0, 0, 1)
	14	1	1	1	1	1	1	1	1	(-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0), (0, 0, -1), (0, 0, 1)
13	10	0	0	0	0	0	0	0	0	(1, -0.5519, -1), (-0.8728, 1, 1), (0.9817, 0.9832, 1), (0.8112, 1, -1), (-0.0339, 0.1619, 1) (-1, -0.6006, -1), (-1, -0.9425, 1), (-0.9357, 1, -1), (0, -1, -0.9459), (1, -1, 0.9198)
	11	1	0	0	1	0	1	1	0	(-1, -0.4660, 1), (-0.9650, 0.9783, -1), (1, 0.5598, 0.9589), (0.9505, -0.9866, -1), (-0.4407, -1, 0.9396) (0.3186, 1, 1), (0.0109, -0.0217, -1)
	12	1	1	1	1	0	1	0	1	(0.2264, 0.0076, 1), (1, 0.7713, -1), (1, -0.8165, -1), (-1, 0.0261, -0.3808), (0.2359, 1, -0.5717) (0.1193, -1, -0.5302)
	13	1	1	1	0	1	1	1	1	(0, -0.9379, 0.6237), (-1, -0.0160, 0), (1, 0.0223, 0.0068), (0.0217, -0.0840, -1), (-1, 0.9965, 1) (-0.0087, 1, 0.4064)
	14	1	1	1	1	1	1	1	1	(-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0), (0, 0, -1), (0, 0, 1)
14	10	0	0	0	0	1	0	0	0	(-1, 0.1864, 0.3550), (0.1126, 0, -1), (1, 1, -0.3880), (0.1977, -1, 0.1665), (-1, -0.9335, 1) (-1, 1, -0.9442), (-0.9391, -1, -1), (1, -0.3333, 1), (-0.1059, 1, 1)
	11	1	0	0	0	1	1	0	0	(1, 0.2774, 0.2073), (-1, -0.9890, 0.9455), (-1, 1, -0.4120), (1, 1, -0.9319), (-0.3353, 0.1080, -1) (0.4132, 1, 1), (0.0031, -1, 0.0326), (-0.9964, 0.4432, 1)
	12	0	1	1	0	0	0	0	0	(-0.1280, -0.1587, -0.1480), (1, -1, -0.2542), (1, 1, 0.1415), (1, 0.1966, 1), (-0.8264, -1, -1) (1, -0.1692, -1), (-0.7347, 1, 1), (0.2663, 1, -1), (0.3524, -1, 1), (-0.9756, 0.0532, 0.0881)
	13	0	0	0	1	0	0	0	0	(0.2088, -1, -1), (1, -0.1061, -1), (1, -1, -0.0707), (-0.9886, -0.0952, -0.0700), (0.0917, 0.9757, -0.0559) (0.0904, -0.0578, 1), (1, -0.8716, 1), (1, 1, -0.9373), (1, 1, 0.9205), (-1, -1, -0.9387), (-1, -1, 0.9302) (-1, 0.8883, -1)

Remark :$F_{p_1} = (-1,-1,-1), F_{p_2} = (-1,-1,1), F_{p_3} = (-1,1,-1), F_{p_4} = (-1,1,1), F_{p_5} = (1,-1,-1), F_{p_6} = (1,-1,1), F_{p_7} = (1,1,-1), F_{p_8} = (1,1,1)$

Table B.4 Computer generated design from Genetic Algorithm for weighted D-optimality criterion in cuboidal design, $k = 3$ (Continued)

Combination	**N**	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	**Other**
	14	1	1	1	1	1	1	1	1	(-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0), (0, 0, -1), (0, 0, 1)
15	10	0	1	0	0	0	0	0	0	(1, 0.5698, -0.9985), (0.1625, 1, -1), (0, 0, 1), (1, 1, 0.8679), (0.8500, -1, -1), (-0.9014, -0.9480, -1) (-1, 0.6921, -1), (-0.9981, 1, 1), (1, -1, 0.9478)
	11	0	0	0	1	0	0	0	1	(-0.9046, 1, -1), (0.0642, 1, -0.2487), (1, 0.8376, -1), (0.9922, -1, 1), (0, -0.1361, 1) (-1, -1, 0.9899), (1, -0.9391, -1), (-0.7418, -1, -1), (-1, -0.0401, -0.8676)
	12	0	1	1	1	0	0	0	1	(-1, -0.9574, -1), (1, -0.7459, -1), (0.3870, -1, -0.7105), (0.1604, 1, -0.2720), (1, -1, 0.9900), (-1, 0, 0) (1, 0.9032, -1), (0.3424, -0.0613, 1)
	13	1	1	1	1	1	1	1	1	(1, -0.0307, 0), (-1, -0.0513, 0.0080), (-0.0092, 0.4838, 1), (0, -1, 0), (0, 0.5952, -1)
	14	1	1	1	1	1	1	1	1	(-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0), (0, 0, -1), (0, 0, 1)
16	10	0	0	0	0	0	0	0	0	(-1, 0.4518, 0.8786), (1, 1, 0.3059), (1, -1, -0.9110), (-0.8733, -1, 0.9900), (-1, -0.7756, -1) (1, -0.6490, 1), (-0.9558, 1, -0.7944), (0.7592, 0.6918, -1), (-0.0227, -0.5508, -0.0254), (0.0615, 1, 1)
	11	0	0	1	0	1	0	0	1	(-0.1485, -1, 1), (-1, -0.1140, 1), (-1, -1, 0.1140), (-0.7382, -0.7550, -1), (-0.7402, 1, 0.7546) (1, -0.7417, 0.7719), (0.1954, 0.1688, -0.1071), (0.9391, 0.9647, -1)
	12	0	0	0	0	0	0	0	0	(1, 1, 0.5700), (0.8838, -0.9532, -1), (-1, 0.9942, 1), (0.0139, 1, 0.0042), (1, 0.5035, 1) (-0.9192, 0.9921, -1), (-0.8414, -1, -1), (1, -1, 0.7246), (0.9893, 0.7720, -1), (-0.8807, -1, 1) (0.0426, -0.1295, 1), (-1, -0.0952, 0)
	13	0	1	0	1	0	0	0	1	(0.3031, -1, -1), (1, -0.2230, -1), (1, -1, -0.2331), (-1, 0.0071, -0.1754), (0.0114, 1, -0.2239) (0.2555, -0.1961, 1), (0.9762, -0.8556, 1), (0.9741, 1, -0.8412), (-1, -0.9303, -0.8779), (-1, 0.9941, -1)
	14	1	1	1	1	1	1	1	1	(-1, 0, 0), (1, 0, 0), (0, -1, 0), (0, 1, 0), (0, 0, -1), (0, 0, 1)

Remark :$F_{p_1} = (-1, -1, -1), F_{p_2} = (-1, -1, 1), F_{p_3} = (-1, 1, -1), F_{p_4} = (-1, 1, 1), F_{p_5} = (1, -1, -1), F_{p_6} = (1, -1, 1), F_{p_7} = (1, 1, -1), F_{p_8} = (1, 1, 1)$

Table B.5: Computer generated design from Genetic Algorithm for weighted D-optimality criterion cuboidal design, $k = 4$

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	F_{p_9}	$F_{p_{10}}$	$F_{p_{11}}$	$F_{p_{12}}$	$F_{p_{13}}$	$F_{p_{14}}$	$F_{p_{15}}$	$F_{p_{16}}$
		Other															
1	16	1	1	0	1	1	1	0	1	1	0	0	1	1	1	1	1
		(0.0078, -1, -1, 1), (-1, -0.0557, 1, -1), (-0.0214, 1, 0.7254, -0.9673), (1, -1, 0.9842, -1)															
	17	1	1	1	0	1	1	0	1	1	1	1	0	1	1	1	1
		(0.0100, -1, 0.6439, 1), (-0.0094, -0.0192, 1, -1), (-1, 1, 1, -0.9573), (0, 1, -1, -1)															
	18	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1
		(0.0136, 1, -0.2867, 1), (0.0084, 0, -1, -1), (1, -1, -1, 0.9838), (0.0771, -1, 1, -1), (1, -1, 1, 0.9964)															
	19	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1
		(0, -1, -1, -1), (0, 0, 1, -1), (0, 1, 0.1445, 1), (-1, 1, -1, 0.9734), (-1, -1, 1, 0.9995)															
	20	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
		(1, 0, -1, -1), (1, -1, -1, -0.9487), (-0.0181, 1, -1, 1), (-0.0056, -1, -0.2659, -1), (-0.0131, 0.0026, 1, 1)															
2	16	0	1	0	0	1	0	0	1	0	0	0	1	0	1	1	0
		(0.0240, 0.0319, -1, -1), (-1, -1, 0.1690, -1), (0.0135, 0.0655, 1, 0.0036), (-0.0043, -1, -0.0597, 0), (-0.0122, -1, 1, -1), (1, -0.0238, 0.0277, -1) (-1, -0.0624, 1, -1), (-0.0001, 1, -0.0029, 1), (1, -1, -1, -0.9682), (-1, 0, -1, 1)															
	17	1	0	0	1	0	0	0	1	1	0	0	1	0	0	0	1
		(0.0242, 1, -1, 1), (0.0212, 1, -1, -1), (1, 1, -0.1600, -1), (1, 0, 1, -1), (-0.0315, -0.0190, 0.0758, 1), (-1, -0.0326, -1, 1), (1, 0.0501, -1, 1) (-1, 1, 0.0717, -1), (-0.0485, 0, 1, -0.0991), (0.0073, -1, 0.0097, 0.0605), (-1, 1, -1, 0)															
	18	1	0	0	1	0	0	1	0	1	0	0	1	0	1	0	0
		(0.0440, 1, -0.0650, -1), (0, -1, 1, -1), (1, -1, 0, 0.0165), (-1, 0.0387, 0, -1), (-1, -1, 0, 1), (1, 0.0553, 1, -1), (-1, 1, -1, 0.9898), (1, 1, 1, -0.0090) (0.0148, 1, -0.0560, 1), (0, -0.1171, -1, 0.0147), (-0.0078, -0.0782, 1, 0.9245), (1, -0.0015, -1, 1)															
	19	1	0	1	1	1	0	0	0	1	0	1	0	0	0	0	1
		(1, 1, 0.0270, -1), (-1, 0.0725, -0.0566, -1), (1, 1, -1, -0.0711), (0, 0, 1, 0), (1, 0, -1, 1), (0.0100, 0.0017, 1, -1), (-1, 1, 1, 0.1413), (0, 1, -1, -1) (-0.0199, -1, -0.1235, 0.0516), (-1, 0, -1, 0.9954), (-0.0294, 1, -0.0449, 1), (1, -1, 0.0854, 0.9984)															
	20	0	1	1	0	0	1	0	0	1	0	1	1	0	1	0	1
		(-1, 1, 0, -1), (0.0370, 0.0052, 1, -1), (0.0357, 0, 0, 1), (-0.0185, -1, -1, 1), (1, 1, 0.0480, -1), (1, 0.0138, -1, 0.0350), (-0.0769, -1, 0.0019, 0) (-1, 0.0253, -1, -1), (-0.0033, 1, -0.9991, -1), (-1, -0.0374, 1, 1), (-1, 1, 1, 0.1279), (0, 1, 1, -0.0354)															
3	16	1	1	1	0	1	1	1	1	0	1	0	1	1	0	1	1
		(1, -1, -0.8989, -1), (1, 1, -1, 0.9948), (0, 0.0813, -1, -1), (-0.0224, -1, 1, 1)															
	17	1	1	1	1	1	0	1	0	1	1	1	1	1	0	0	1
		(-1, 1, -0.9172, 1), (1, 0.9997, -1, 1), (1, 1, 1, -0.9904), (0, -0.0785, -1, 1), (-0.0504, 1, 1, 1)															
	18	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1	1
		(0, -1, -1, 1), (-1, 1, -1, 0.9892), (0, 0.0361, 1, -0.9653), (1, -1, -0.8961, -1)															
	19	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
		(0, 1, -0.7384, 1), (0.0160, -1, -1, -1), (-1, 1, -1, -0.9956), (-0.0056, 0.0009, 1, -1)															
	20	1	1	0	1	1	1	1	1	2	1	1	1	1	1	1	1
		(-0.0110, 0, -1, 1), (-1, -1, 1, -0.9570), (-0.0103, -1, 0.8594, -0.9766), (0, 1, 1, -1)															
4	16	1	0	0	1	0	1	1	1	1	0	0	1	0	1	1	0

Remark :$F_{p_1} = (-1,-1,-1,-1), F_{p_2} = (-1,-1,-1,1), F_{p_3} = (-1,-1,1,-1), F_{p_4} = (-1,-1,1,1), F_{p_5} = (-1,1,-1,-1), F_{p_6} = (-1,1,-1,1), F_{p_7} = (-1,1,1,-1)$
$F_{p_8} = (-1,1,1,1), F_{p_9} = (1,-1,-1,-1), F_{p_{10}} = (1,-1,-1,1), F_{p_{11}} = (1,-1,1,-1), F_{p_{12}} = (1,-1,1,1), F_{p_{13}} = (1,1,-1,-1), F_{p_{14}} = (1,1,-1,1)$
$F_{p_{15}} = (1,1,1,-1), F_{p_{16}} = (1,1,1,1)$

Table B.5 Computer generated design from Genetic Algorithm for weighted D-optimality criterion cuboidal design, $k = 4$ (Continued)

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	F_{p_9}	$F_{p_{10}}$	$F_{p_{11}}$	$F_{p_{12}}$	$F_{p_{13}}$	$F_{p_{14}}$	$F_{p_{15}}$	$F_{p_{16}}$
		Other															
		(-1, -0.0267, -0.1709, -1), (1, 1, 0.0364, 0), (-0.0040, -1, -1, 1), (1, 0.0047, -1, -1), (0.0072, 1, -1, -1), (0.0250, 0.0119, 1, -0.1009) (0, -1, 0.1544, -1)															
	17	1	0	1	1	0	1	1	1	1	0	0	1	0	0	0	0
		(1, 1, -1, -0.1967), (0, -1, -1, 1), (1, -0.0631, -1, 1), (-0.0167, 1, -1, -1), (0.0226, 0.0382, 1, 0), (-1, -0.0010, -0.1800, 1), (1, 1, 0.0956, 1) (1, 1, 1, -0.9933), (0.0498, -1, -0.0060, -0.0334)															
	18	1	1	1	0	0	0	1	0	1	0	1	0	1	1	1	0
		(0, 1, 1, 1), (0, -0.0047, -1, 1), (1, 0.0129, 1, 1), (-1, 1, -0.1128, 1), (-1, 0, 1, 1), (-1, 1, -1, -0.0940), (0, 0, -0.0288, -0.2372), (1, -1, -0.0905, 1) (0, -1, 1, 0.0521)															
	19	0	1	0	1	1	1	0	1	1	1	0	1	0	1	0	1
		(0.0214, -1, 0.0678, -0.0330), (1, 1, -0.1468, -1), (-1, -1, -0.0598, -1), (-0.0029, -1, 1, -1), (0, 1, 1, -1), (-1, 0.0011, 1, -0.8983), (0, 0, -1, -1) (-0.0010, 0.0288, 0, 1), (1, -0.0224, 1, -1)															
	20	0	1	1	1	0	1	0	1	0	1	1	1	0	1	0	1
		(0.0454, -1, -0.0815, -1), (0, 1, -1, -1), (-1, -0.0915, -0.9768, -1), (1, 0, -1, -0.9892), (1, 1, 0.0643, -1), (0, 0, 0.9945, -1), (-0.0412, 1, 1, -0.0914) (-1, 1, 0, -1), (0.0117, 0.0132, 0.0008, 0.1782), (0.0316, -1, -1, 0)															
5	16	1	0	1	1	1	0	1	0	1	1	1	0	1	1	1	1
		(-0.0199, 0.1787, 1, 1), (-1, 1, -0.9446, 1), (1, -1, 1, 0.9992), (-0.0839, -1, -1, 1)															
	17	1	1	0	0	1	1	1	0	1	1	1	1	1	1	1	1
		(-1, -0.1043, 1, 1), (-0.0557, -1, 1, 1), (-1, -1, 1, -0.9976), (-0.0626, 1, 0.9101, 1)															
	18	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1
		(1, -1, -0.9393, 1), (1, 1, 1, -0.9996), (0, -1, -1, -1), (-0.0052, 0.0807, 1, -1)															
	19	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
		(1, -1, -1, 0.9946), (-1, 0.0385, -1, 1), (0, -1, -0.4253, 1), (-0.0056, 1, 1, 1)															
	20	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	0
		(-1, 0.0092, -1, -1), (1, 1, 1, 0.9995), (0.0195, -1, -1, -1), (0, 1, 0.6951, -1)															
6	16	1	0	1	1	0	1	1	0	1	1	1	0	1	0	1	0
		(1, 1, -0.4478, 1), (0.0412, -1, 0.0846, 1), (-0.0033, 1, 1, 1), (1, -0.0749, 1, 1), (-1, 0.0825, 0.1732, 1), (0, 0.0058, -1, 0.1198)															
	17	1	0	1	1	1	1	1	0	1	0	1	0	1	0	1	1
		(-0.0050, -1, -1, 1), (1, -1, 0.3771, 1), (0.0131, -0.0108, 1, 0.2029), (0.0256, 1, -0.0550, 1), (-1, -0.0416, -0.1895, 1), (1, 0.0699, -1, 1)															
	18	0	1	1	1	0	1	1	1	0	1	1	1	0	1	0	1
		(-0.0481, 1, -1, -0.9510), (0.0039, -1, -0.1777, -1), (1, 0, -1, -1), (-0.0157, -0.0248, 0.0348, 0.0149), (0.0173, 0.0183, 1, -1), (-1, 0.0149, -1, -1) (1, 1, -0.0136, -1)															
	19	1	1	1	0	1	1	1	1	1	0	1	0	1	1	1	0
		(1, 0.0610, 1, 1), (0.0125, 1, 0.1897, 1), (-0.0178, 0.0283, 0, -1), (0.0230, 0, -1, 0.0513), (0, -1, 1, 1), (1, -1, -0.0578, 1), (-1, -0.1010, 1, 0.9955)															
	20	1	1	1	1	1	1	0	0	1	1	1	1	0	1	0	1
		(0, -1, 0, -1), (-1, 1, 1, 0.9979), (1, -0.0264, 1, -1), (-0.0057, 1, 1, -1), (-1, 0.0625, 1, -1), (-0.0152, -0.0337, -0.0074, -1), (0, 0.0103, -1, -0.0810) (1, 1, -0.0968, -1)															
7	16	1	1	1	0	0	1	1	1	1	1	0	1	1	1	0	1
		(1, -0.2888, 1, -1), (-1, 1, -0.9455, -1), (0.0332, -1, 1, -1), (-1, -1, 1, 0.9802)															

Remark : $F_{p_1} = (-1,-1,-1,-1), F_{p_2} = (-1,-1,-1,1), F_{p_3} = (-1,-1,1,-1), F_{p_4} = (-1,-1,1,1), F_{p_5} = (-1,1,-1,-1), F_{p_6} = (-1,1,-1,1), F_{p_7} = (-1,1,1,-1)$
$F_{p_8} = (-1,1,1,1), F_{p_9} = (1,-1,-1,-1), F_{p_{10}} = (1,-1,-1,1), F_{p_{11}} = (1,-1,1,-1), F_{p_{12}} = (1,-1,1,1), F_{p_{13}} = (1,1,-1,-1), F_{p_{14}} = (1,1,-1,1)$
$F_{p_{15}} = (1,1,1,-1), F_{p_{16}} = (1,1,1,1)$

Table B.5 Computer generated design from Genetic Algorithm for weighted D-optimality criterion cuboidal design, $k = 4$ (Continued)

Combination	**N**	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	F_{p_9}	$F_{p_{10}}$	$F_{p_{11}}$	$F_{p_{12}}$	$F_{p_{13}}$	$F_{p_{14}}$	$F_{p_{15}}$	$F_{p_{16}}$
		Other															
	17	1	0	1	0	1	1	1	1	1	1	0	1	1	1	1	1
		(-1, -1, 0.9373, 1), (-0.1646, -1, -1, 0.9774), (1, -1, 1, -0.9962), (0, 0.0791, 1, 1)															
	18	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
		(0, -0.0972, 1, 0.9340), (0, 1, -1, 1), (-1, 1, 0.9478, 1)															
	19	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
		(-0.0559, -1, -0.8808, -1), (-1, 0, 1, -1), (0.0159, 1, -1, -1), (1, -1, -1, -0.9968)															
	20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		(0, -1, -0.8375, 1), (0.0190, 1, -1, 1), (-1, 0.0095, -1, 1), (1, -1, 1, 0.9921)															
8	16	1	0	1	1	1	1	0	1	0	1	1	0	0	1	1	1
		(-0.0661, -1, -1, 1), (-0.0019, -0.0033, 1, 0), (-1, 0.0482, -0.0172, -1), (1, -0.0664, -1, -1), (0.0376, 1, -0.1243, 0.0150)															
	17	1	1	0	1	1	1	0	1	0	1	0	1	0	1	1	1
		(0.0003, -1, 1, -1), (1, -1, 0.1423, -1), (-0.0168, 1, 0, -1), (-0.0595, -0.0305, -1, 0), (-1, 0, 1, -1), (1, 0.0724, -1, -1)															
	18	1	1	0	1	1	1	0	1	0	1	0	1	1	1	0	1
		(0, 1, 1, -1), (-1, 0.0321, 1, -1), (1, -1, 0, -1), (0.0215, -0.0585, -1, 0), (0, 0.0051, 0.0164, 1), (0, -1, 1, -1), (1, 0.0483, 1, -1)															
	19	1	1	0	1	1	0	1	1	1	1	1	1	1	0	1	0
		(0.0352, 1, -1, 1), (-0.0042, 0.0195, 1, 0.0246), (1, 1, 0, 1), (0, -1, -0.1367, 1), (-1, -1, 1, -0.9873), (1, 0.0234, -1, 1), (-1, 0.0957, -0.0650, 1)															
	20	1	1	1	1	0	1	1	1	0	1	1	0	0	1	1	1
		(0.0075, 1, -1, -1), (-1, 0.1376, -1, -0.9867), (1, 1, -0.1110, -1), (1, 0, -1, -1), (0.0176, 0, 1, -0.0751), (0, -1, -1, -1), (0, 0, 0.0534, 0.9990) (1, -1, 0.9787, 1)															
9	16	1	1	0	1	0	1	0	1	0	0	1	1	1	1	0	1
		(-0.0218, -0.0158, -1, 1), (-0.0468, -1, 0, -1), (0, 1, 1, -0.9912), (-1, -0.0510, 1, -1), (-1, 1, -1, -0.9895), (1, -1, -1, 0.9469)															
	17	0	1	1	0	0	0	1	0	1	1	1	0	0	1	0	0
		(-1, 1, -1, 0.9514), (-1, -1, -1, -0.9893), (-1, 0.0319, 1, 1), (-0.0184, 1, 0.1102, 1), (-1, 0.9966, -1, -1), (0.0339, -1, 1, 1), (1, 1, -1, -0.9949) (0.9941, 1, 1, -1), (0, 0.0461, -1, -1), (1, 0, 1, 1)															
	18	0	1	1	1	0	1	1	1	1	0	1	1	1	1	1	1
		(-0.0909, -1, -1, -1), (0, -0.0432, 1, 1), (-0.0275, 1, -0.0203, -1), (-1, 0.0588, -1, -1), (1, -1, -1, 0.9426)															
	19	1	1	0	1	1	1	1	1	1	0	1	1	1	1	0	1
		(0, 0.0197, -1, 1), (-1, -0.1140, 1, -1), (-0.0682, -1, 1, -0.9745), (1, -1, -1, 0.9574), (0, 1, 0, -1), (1, 1, 1, -0.9560															
	20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		(-0.0165, 0, 1, 0.9544), (0, 1, -1, 1), (1, 0, -1, -1), (0, -1, 0, -1)															
10	16	0	1	0	0	0	0	1	0	1	0	0	1	0	1	1	0
		(0.0558, 0, 0, -1), (1, 0.0118, 0, 0.0622), (-1, -1, 0.0416, -1), (-0.1236, 1, -1, -0.0369), (-1, -1, 1, -0.0207), (0.0419, -1, 0, 1), (-1, 0.0820, -1, -1) (0.0143, 0.0022, 1, 1), (-1, 0, 1, 1), (-1, 1, -0.0723, 1)															
	17	0	1	1	0	1	0	0	1	0	1	0	0	1	0	0	1
		(1, -1, 1, 0.0421), (-0.0074, -1, 0.9932, 1), (1, 0.0698, -0.0593, 1), (1, -1, -0.0462, -1), (-1, -1, -0.0712, -0.0741), (-1, 0.0662, 1, -0.0522) (-0.1298, -0.0649, -1, 0.0232), (1, 0.0902, 1, -1), (-0.0723, -0.0078, 0, -1), (0, 1, 0.0730, -0.0059)															
	18	0	1	0	0	1	0	0	1	0	0	1	0	0	1	1	0
		(1, 1, -0.0256, -0.0774), (0.0667, -1, -1, -1), (1, -1, -0.2243, 0.1954), (-1, -1, 0.0600, -1), (-1, 1, -0.0902, 1), (-1, -1, 1, 0.1118), (-1, 0.0067, 1, -1)															

Remark : $F_{p_1} = (-1,-1,-1,-1), F_{p_2} = (-1,-1,-1,1), F_{p_3} = (-1,-1,1,-1), F_{p_4} = (-1,-1,1,1), F_{p_5} = (-1,1,-1,-1), F_{p_6} = (-1,1,-1,1), F_{p_7} = (-1,1,1,-1)$ $F_{p_8} = (-1,1,1,1), F_{p_9} = (1,-1,-1,-1), F_{p_{10}} = (1,-1,-1,1), F_{p_{11}} = (1,-1,1,-1), F_{p_{12}} = (1,-1,1,1), F_{p_{13}} = (1,1,-1,-1), F_{p_{14}} = (1,1,-1,1)$ $F_{p_{15}} = (1,1,1,-1), F_{p_{16}} = (1,1,1,1)$

Table B.5 Computer generated design from Genetic Algorithm for weighted D-optimality criterion cuboidal design, $k = 4$ (Continued)

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	F_{p_9}	$F_{p_{10}}$	$F_{p_{11}}$	$F_{p_{12}}$	$F_{p_{13}}$	$F_{p_{14}}$	$F_{p_{15}}$	$F_{p_{16}}$
		Other															
		(0, -0.0269, 0.0075, 1), (-0.0182, 1, 1, -0.0997), (-1, -0.0068, -1, 0.1116), (1, -0.0236, -1, -1), (1, -0.0663, 0.9974, 1)															
	19	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0
		(-1, -1, 0.0096, 1), (1, 1, 0.1471, 1), (-1, 1, 1, 0), (-1, 0.0008, -1, -1), (-1, 0, 1, 1), (-1, -1, -1, -0.0326), (1, 1, -1, -0.0333), (0, 0.0262, 1, 0.0201) (0.0243, 1, -1, 1), (1, 0, -1, 1), (0, 0.0141, -0.0322, -1),(1, -0.0830, -0.0231, -0.0526), (0.0357, -1, 0, 0.0319), (-1, 1, 0, -1)															
	20	0	1	0	0	0	0	1	0	1	0	0	1	0	1	1	0
		(0, 0.0472, 1, 1), (-1, 1, -1, 0.1079), (1, -0.0406, 0.0334, -0.9769), (1, -1, 1, 0), (-1, -1, 1, 0), (-1, -1, -0.0189, -1), (0.0414, -1, -0.0661, 1) (1, -0.0368, -1, 1), (-0.0023, -0.0142, 0, -0.0503), (-1, -0.0568, -1, -1), (0.0134, 1, -1, -1), (-1, 0, 1, 1), (1, 1, 0.1323, 0), (-1, 1, -0.0771, 1)															
11	16	0	1	1	0	1	0	1	1	0	0	1	1	1	0	1	0
		(-0.1994, -1, 1, 1), (1, 1, 1, 0.9912), (0.1465, -1, -1, -1), (1, 0.0745, -1, 1), (1, -1, -1, 0.9619), (-0.1046, 1, -1, 1),(-1, -1, -0.7625, -0.9975)															
	17	0	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0
		(-1, 0.1560, 1, 1), (0, 0, -0.1138, -1), (0.9978, 1, 1, 1), (-0.1186, -1, 1, 1), (-1, -1, -1, -0.9818), (1, -1, 0.9935, 1)															
	18	1	1	0	1	1	1	1	1	1	1	0	1	0	1	1	1
		(0.0831, -1, 1, -1), (1, -0.3205, 1, -1), (-1, -1, 1, -0.9751), (1, 1, -1, -0.9605), (-0.0005, 0.0377, -0.0818, 1)															
	19	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1
		(0.0064, 0.0474, 1, 1), (1, -1, -1, -0.9847), (1, 0.3926, -1, -1), (0, -1, 0.1293, -1), (0.1899, 1, -1, -1)															
	20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		(0.0112, 1, -0.0816, 1), (-0.0264, -0.0487, 1, -1), (0, -1, -1, 0.9515), (1, 0.0292, -1, 1)															
12	16	0	0	1	0	0	1	1	0	1	0	0	1	0	0	1	1
		(-1, 0, 1, 1), (0, -1, -1, 1), (-1, -1, -0.1738, 0.0921), (-1, -0.0652, -1, -1), (1, -0.0143, 0.1664, -1), (0.0781, 1, -0.0021, 1), (1, 0.0679, -1, 1) (0.0265, -0.1090, 1, -0.0941), (1, 1, -1, -0.9843)															
	17	0	1	1	1	0	0	0	0	0	0	0	1	1	0	0	0
		(1, -0.0357, 1, -1), (-0.0148, -0.1270, 0.0011, 0.0743), (-0.0903, 1, 1, -1), (-1, 1, 0.0905, -1), (1, -1, 0.0830, -1), (1, 1, 0.9972, 1), (-1, 0, -1, -1) (1, -0.1106, -1, 1), (-1, 0.9702, 1, 1), (-1, 1, -0.9736, 0.1589), (1, -1, -1, -0.1363), (-0.0719, 1, -1, 1)															
	18	1	1	0	1	0	1	1	0	0	0	1	0	0	1	0	0
		(1, 1, 1, -0.1131), (-0.9957, 1, -0.9993, -1), (1, -1, -1, -0.1102), (1, 1, -0.1842, -1), (-0.0810, -1, 1, -0.1449), (-1, 1, 1, 0.9628), (1, 0, 1, 1) (0.0701, 1, 0.1881, 1), (-1, -0.0590, 0, 0.0202), (0.0165, 0.0302, -1, -1), (1, -1, 0.1133, 1)															
	19	0	0	0	1	1	0	0	0	0	1	1	1	0	0	0	0
		(-1, -0.1010,, -1, 1), (-1, 1, -0.0071, 1), (-1, -0.0614, 1, -1), (1, -0.0586, -1, -1), (-1, 1, 1, 0.6180), (-0.0261, -1, -1, -1), (-0.0305, 1, 1, -0.9744) (0.1056, 0.0077, 0, 0.0475), (0.1038, 1, -1, 1), (1, 0.9293, 1, 1), (-1, -1, 0.1687, -1), (-1, -1, -1, 0.0453), (1, 1, -0.0181, -1), (1, 1, -1, 0.1450)															
	20	1	1	0	1	1	0	0	0	1	1	0	0	1	1	1	0
		(-1, 1, -0.9979, 1), (0, -1, 1, -1), (0.0395, 1, 1, 1), (-1, 0.0254, 1, -1), (1, -0.0711, -0.0389, -1), (-0.0601, 1, -0.0016, -1), (1, -0.0248, 1, 1) (-0.0309, -0.0335, -1, 0.0318), (1, -1, 1, -0.0249), (-1, 1, 1, 0.0959), (-1, -1, 0.0564, -0.1496)															
13	16	0	1	1	0	0	1	1	1	0	1	1	1	1	1	0	1
		(0, -1, -1, -1), (-1, -1, 1, 0.9867), (-1, 0.0550, -1, -1), (0.1094, 1, 0, -1), (-0.0138, -0.0707, 1, 1)															
	17	0	1	1	0	1	1	1	0	0	1	1	0	1	0	1	1
		(1, -1, -1, -0.9766), (-1, -1, -0.9549, -1), (-1, 0.0051, 1, 1), (1, 1, -0.9991, 1), (-0.0923, 1, 0.1575, 1), (0.0375, -1, 1, 1), (0, 0.0086, -1, -1)															
	18	1	1	1	1	1	0	1	1	1	1	1	0	1	1	1	0
		(1, 0, 1, 1), (0, 0, -1, -1), (0.0485, -1, 1, 1), (-1, 1, -1, 0.9700), (0.1087, 1, 0, 1)															

Remark : $F_{p_1} = (-1,-1,-1,-1), F_{p_2} = (-1,-1,-1,1), F_{p_3} = (-1,-1,1,-1), F_{p_4} = (-1,-1,1,1), F_{p_5} = (-1,1,-1,-1), F_{p_6} = (-1,1,-1,1), F_{p_7} = (-1,1,1,-1)$
$F_{p_8} = (-1,1,1,1), F_{p_9} = (1,-1,-1,-1), F_{p_{10}} = (1,-1,-1,1), F_{p_{11}} = (1,-1,1,-1), F_{p_{12}} = (1,-1,1,1), F_{p_{13}} = (1,1,-1,-1), F_{p_{14}} = (1,1,-1,1)$
$F_{p_{15}} = (1,1,1,-1), F_{p_{16}} = (1,1,1,1)$

Table B.5 Computer generated design from Genetic Algorithm for weighted D-optimality criterion cuboidal design, $k = 4$ (Continued)

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	F_{p_9}	$F_{p_{10}}$	$F_{p_{11}}$	$F_{p_{12}}$	$F_{p_{13}}$	$F_{p_{14}}$	$F_{p_{15}}$	$F_{p_{16}}$
		Other															
	19	1	0	1	0	1	0	1	1	1	1	1	1	1	1	0	0
		(-1, -0.9905, -1, 1), (0.1754, 1, 1, 1), (0, -1, -1, 1), (-0.0221, -0.0238, -0.1311, -1), (1, 0.1610, 1, 1), (1, 1, 1, -0.9712), (-0.9956, 1, -0.9807, 1) (-1, -0.9878, 1, 1)															
	20	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1
		(1, -0.0937, -1, 1), (1, -1, -1, -0.9708), (0, -0.0004, 1, -1), (0.1395, -1, -1, 1), (0, 1, 1, 1), (0, 1, 0.0090, -1)															
14	16	1	0	0	1	0	0	0	1	0	1	0	1	1	0	0	0
		(-1, 1, -0.0205, -1), (-1, 1, -1, 0.0158), (1, 0.1221, 1, 0), (0.0944, 1, 1, -1), (-0.9930, -0.0370, 1, -1), (-0.0528, -1, 0.0108, -0.0236), (0, 0, -1, 1) (1, 1, 0.1463, 1), (-1, -0.0538, -0.0813, 1), (1, -1, 0.0163, -1)															
	17	1	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0
		(1, -1, 0.0947, 1), (-1, 1, -0.0557, 1), (0.0625, 1, 1, 1), (0, -1, 0.0006, -0.0982), (-1, -0.0246, 1, 1), (-1, 1, -1, -0.0690), (-1, -0.0797, -0.0742, -1) (1, 1, -1, 0.9902), (1, -1, -1, -0.9992), (1, 0.1257, 1, 0.1485), (1, 1, -0.0680, -1), (0.0516, -0.0779, -1, -1), (-1, -1, 1, 0.0506)															
	18	1	1	1	0	1	0	0	1	0	0	1	0	0	1	0	0
		(1, -1, -1, -0.0990), (0.2279, 0, -1, -1), (1, -1, 0.0011, 1), (0, -1, 0, -0.0746), (1, 1, -0.0650, -1), (1, 0.0133, 1, 1), (-0.0222, 1, -0.0936, 1) (1, 1, 1, 0.0118), (-1, 0.0049, 0.0979, 0.1082), (-1, 0.9910, 1, -1), (-0.0664, -1, 1, 0.9546)															
	19	1	0	0	1	0	1	0	1	0	1	0	1	1	0	0	0
		(-1, 1, -0.0126, -1), (-1, -1, -0.0340, 0.1183), (1, 1, 1, -0.0266), (-0.0081, -1, 1, -1), (1, -1, -0.0157, -1), (-1, 0.0497, 1, -1), (1, -0.0353, -1, 0) (1, 0.0693, 1, -1), (0.0225, 1, -1, -1), (1, 1, 0.0437, 1), (-0.0751, 1, 1, 0), (-0.0261, -0.0280, -0.0228, 1)															
	20	0	0	0	0	0	0	1	0	1	0	0	1	0	1	0	1
		(-1, 1, 0.0352, 1), (-1, 1, -0.9983, 0), (-1, -1, -0.0792, -1), (0.0090, 1, 1, 0.0031), (-1, -0.0047, -1, -1), (0.9990, 1, -0.0077, -1), (1, 0.0407, -1, 0) (0.0753, -0.0825, 0, 1), (0.0096, 1, -1, -1), (1, -1, -0.0169, 0.0255), (0.0534, -1, 1, -1), (-1, -0.9896, -1, 1), (1, 0.0225, 1, -1), (-1, -0.0094, 1, 1) (-1, -1, 1, 0.0325)															
15	16	0	0	1	1	1	1	1	0	1	0	1	1	0	1	1	0
		(-0.1727, -1, -1, 1), (-1, -1, -1, -0.9733), (0.2953, 1, -1, -1), (1, 0, -1, 1), (0.2787, 1, 1, 1), (-1, 1, 0.8719, 1)															
	17	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1
		(0, 0, -0.1871, 1), (1, 0.1900, 1, -1), (1, -1, 1, 0.9881), (0.1607, -1, 1, -1)															
	18	1	1	0	1	1	1	0	0	1	0	1	1	1	1	1	1
		(-1, 1, 1, 0.9871), (-1, -1, 0.9840, -1), (-0.0128, 1, 1, -0.9688), (1, -1, -1, 0.9556), (0, -0.0422, -0.0059, 1), (-1, 0.2318, 1, -1)															
	19	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
		(-1, -1, 1, 0.9632), (0, 1, 0, -1), (0.0035, -1, -1, -1), (-0.0153, 0.0128, 1, 1)															
	20	1	1	1	1	1	1	1	0	1	1	1	1	0	1	1	1
		(0.9942, 1, -1, -1), (-1, 1, 1, 0.9987), (0.0094, 1, -1, -1), (0.0153, 0.0342, 1, 0.9449), (1, 0, -1, -1), (0.0051, -1, -0.0329, -1)															
16	16	1	0	1	0	0	1	0	0	0	0	0	1	1	0	1	0
		(1, -1, -0.9994, -1), (-1, 1, 0.7073, -1), (0.0332, -1, -1, 1), (0.0405, -0.0966, 0.0187, -1), (-0.0356, 1, 1, 0.0517), (-1, 0.0179, -1, -0.0548) (1, 1, 0.0997, 1), (-1, 0.1595, 1, 1), (-1, -1, -0.0114, 1), (1, -0.0524, -1, 1)															
	17	1	1	0	0	0	0	1	1	0	0	0	1	1	1	0	1
		(1, 0.0815, 1, -1), (-1, -1, 1, 0.2245), (-1, 1, -1, 0.3496), (1, -1, -0.0988, -1), (0, 1, 0.1131, -0.0998), (1, -1, -1, 0.2757), (-0.0245, -1, 1, -1) (0.0350, -0.0259, -1, -1), (-1, -0.0068, 0.0576, -1)															
	18	0	0	0	1	1	1	0	0	1	1	0	0	0	1	0	1

Remark : $F_{p_1} = (-1,-1,-1,-1), F_{p_2} = (-1,-1,-1,1), F_{p_3} = (-1,-1,1,-1), F_{p_4} = (-1,-1,1,1), F_{p_5} = (-1,1,-1,-1), F_{p_6} = (-1,1,-1,1), F_{p_7} = (-1,1,1,-1)$
$F_{p_8} = (-1,1,1,1), F_{p_9} = (1,-1,-1,-1), F_{p_{10}} = (1,-1,-1,1), F_{p_{11}} = (1,-1,1,-1), F_{p_{12}} = (1,-1,1,1), F_{p_{13}} = (1,1,-1,-1), F_{p_{14}} = (1,1,-1,1)$
$F_{p_{15}} = (1,1,1,-1), F_{p_{16}} = (1,1,1,1)$

Table B.5 Computer generated design from Genetic Algorithm for weighted D-optimality criterion cuboidal design, $k = 4$ (Continued)

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	F_{p_9}	$F_{p_{10}}$	$F_{p_{11}}$	$F_{p_{12}}$	$F_{p_{13}}$	$F_{p_{14}}$	$F_{p_{15}}$	$F_{p_{16}}$
		Other															
		(-1, -1, -0.1027, -1), (0.0731, 0, -1, -0.1400), (1, 1, -0.1645, -1), (-0.0014, -1, 1, -1), (-0.0881, 1, 1, -1), (-1, 1, 1, 0.1077), (1, -1, 1, 0.2092) (-0.9838, -1, -1, 1), (-1, -0.0485, 1, -1), (1, 0, 1, -1), (0, 0, 0.0636, 1)															
	19	0	0	1	0	0	0	1	0	0	1	0	0	1	0	1	0
		(1, -1, 1, -0.1730), (-0.0129, -1, 1, 1), (-1, 0.0876, -1, 1), (-0.9985, 1, -1, -1), (-1, -1, 0.0536, 1), (1, 0.0662, -1, 0), (0, -0.0430, 0.0327, -0.1390) (1, 1, 0.0273, 1), (1, 0.0435, 1, 1), (-1, 0.9799, 0.9779, 1), (-0.0662, -1, -1, -1), (1, -1, 0.0376, -1), (-0.0792, 1, -1, 1), (-1, -1, -1, -0.0961)															
	20	0	1	0	1	1	1	0	1	0	1	1	1	1	1	0	1
		(-0.1399, -1, 1, -0.0384), (0.1204, 1, 1, -1), (-1, 1, 0.1555, -0.0551), (1, 0.0810, 0.1565, -1), (-1, -0.0042, 1, -1), (0.0429, -1, -1, -1), (-1, -1, 0, -1)															
		(-0.0004, 0.0316, -0.1212, 1), (-1, -0.0952, -1, -0.0347)															

Remark :$F_{p_1} = (-1,-1,-1,-1), F_{p_2} = (-1,-1,-1,1), F_{p_3} = (-1,-1,1,-1), F_{p_4} = (-1,-1,1,1), F_{p_5} = (-1,1,-1,-1), F_{p_6} = (-1,1,-1,1), F_{p_7} = (-1,1,1,-1)$
$F_{p_8} = (-1,1,1,1), F_{p_9} = (1,-1,-1,-1), F_{p_{10}} = (1,-1,-1,1), F_{p_{11}} = (1,-1,1,-1), F_{p_{12}} = (1,-1,1,1), F_{p_{13}} = (1,1,-1,-1), F_{p_{14}} = (1,1,-1,1)$
$F_{p_{15}} = (1,1,1,-1), F_{p_{16}} = (1,1,1,1)$

Table B.6: Computer generated design from Genetic Algorithm for weighted G-optimality criterion cuboidal design, $k = 4$

Combination	**N**	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	F_{p_9}	$F_{p_{10}}$	$F_{p_{11}}$	$F_{p_{12}}$	$F_{p_{13}}$	$F_{p_{14}}$	$F_{p_{15}}$	$F_{p_{16}}$
		Other															
1	16	0	0	0	1	0	0	1	0	1	0	0	0	0	0	1	0
		(1, 1, -1, 0.8104), (-0.5017, 0.1366, 1, 0.0940), (-0.0750, -1, 1, -1), (-1, 1, -0.6073, 1), (0.4554, -0.1339, 0.0934, -0.9251), (-0.0020, -1, -1, 1) (0.1322, 1, 0.7567, 1), (1, -0.0226, -0.0269, 0.4511), (-1, 0, -1, -0.0666), (-1, -1, -0.6479, -1), (-0.0416, 1, -1, -0.9043), (1, -1, 1, 0.8895)															
	17	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0
		(0.0608, 1, 1, 1), (1, 0.0633, -0.9931, 1), (-1, 0, -1, -0.9227), (1, -1, -0.9307, -1), (1, 1, -1, -0.0478), (-1, 1, -1, -0.9046), (0.0007, 0, 0.0446, -0.9193) (0, 1, -1, -0.9403), (-0.0833, -1, 0.9714, -1), (-1, 0.0204, 1, 1), (0.0316, -1, -0.9990, 0.9295), (1, -1, 0.9545, 0), (-1, 1, 1, 0.9200), (1, -0.1087, 1, 1)															
	18	1	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0
		(-1, -0.9552, 1, -1), (-1, 1, -1, -0.9288), (0, 0.0225, 0.5263, -0.9286), (1, 0, 1, -1), (1, 1, -1, -0.9521), (0.1956, 1, -0.9509, 1), (-0.9976, 1, 1, 1) (1, 1, 1, 0.7744), (-1, 0.9768, 0.9522, -0.9687), (0.1509, -1, 1, -0.9052), (-1, -0.0196, -1, 1), (1, 0, -1, 1), (0.1771, -1, -1, 1), (1, -1, -1, -0.9612) (0.1357, 0.9648, 1, 0)															
	19	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
		(-0.0008, 0.9330, 1, 0.9570), (0.0098, -1, -0.1583, -0.9949), (-1, -1, 0.9549, 1), (-0.0702, -1, -1, 0.9089), (1, 1, 0.0203, 0), (1, -1, 1, -0.9389) (1, 1, 0.9350, 1), (-1, 1, 0, 0.9625), (-1, 0.0188, -0.9630, 1), (-1, 0.9788, 1, -1), (-1, 1, -1, -0.9822), (1, 0, -1, -1), (-0.0618, 1, -0.9952, -0.0249) (0, 0.0834, 1, -1), (1, 1, -0.9696, -0.9633), (1, -1, -0.9964, 1), (-1, -1, 1, -0.0764), (-1, -0.9822, -0.9898, -0.9467)															
	20	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
		(1, -0.9424, 1, 0.9341), (-1, -1, -0.9499, 0.9735), (1, 1, 1, -0.7411), (1, 0.9164, -1, 1), (-0.0477, -1, 1, -1), (-1, 1, -1, 0.9777), (1, -1, -0.1858, 1) (-1, 1, 1, 0.8725), (1, -0.0115, 1, -0.8442),(-0.1144, 1, 0.0187, -1), (0.1809, -1, -1, -1), (-0.0704, 0.0708, -1, 0), (1, 1, -0.9533, -0.7244) (-0.2173, -0.0430, -0.1113, 1), (-1, -1, -0.6753, -1), (-1, 1, -0.3936, -1), (0.2674, 1, 1, 1), (1, -1, -1, -0.9816), (-1, -0.0282, 1, -1)															
2	16	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
		(-1, 1, 0.8340, 1), (1, 1, -1, -0.5054), (1, -0.0841, 1, -0.9442), (-1, -1, -0.9835, 1), (-0.0702, 0.0398, -0.2715, 0.7141), (-1, 1, -1, -0.6105) (1, -1, 0.1841, -0.0776), (1, 0.1261, -0.0491, -1), (0, 1, 0.0114, -1), (0.9427, 1, 0.9688, 1), (0, -1, 1, -1), (-1, -0.0782, 0.9974, -0.4479) (-1, -0.9528, 0.1271, -0.9783), (-0.1243, 0.0876, -1, -1), (0, -1, 1, 0.1273)															
	17	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0
		(0.0092, 0.0782, -1, -1), (-1, -1, -0.9335, -0.9394), (0.0114, -1, -0.9105, -0.0021), (0.8077, -0.1360, -0.3789, 1), (0.1023, 1, 1, 0.9810) (1, 0.1248, 1, -0.3721), (-1, -0.0299, -0.0271, 0.3095), (1, -1, 0.9455, 0.8594), (1, -1, -0.9154, -0.7949), (1, 1, -1, 0.7020), (1, 1, 0.3232, -1) (-0.0351, 1, -0.0801, 0.0201), (-1, 1, 0.6702, -1), (0, -1, 0.0479, -1), (-0.7608, 0.0688, 1, -0.2019)															
	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		(0, 0.0093, -0.9586, 0.0910), (-1, -1, -1, 0.9180), (1, 0, 0, -0.9879), (-1, 0, -0.0319, -0.8138), (0.0097, 1, 1, -0.9911), (-0.0033, 1, 1, -0.1057) (1, 1, -0.9561, -1), (-1, 1, -1, -0.8666), (-0.0121, -1, -0.0214, 1), (1, -1, -1, 0.8057), (-1, -1, 1, -0.9591), (0, -1, 0.0946, 0), (-1, 0.0265, 1, 1) (1, -1, 1, -0.9305), (0, 0.0425, -1, -1), (1, 1, 0.1026, 0.4656), (-1, 1, 0.2016, 0.9269), (1, 0.0426, 1, 1)															
	19	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
		(1, 0.0757, -0.3116, 1), (0.0109, -1, 0.0945, -0.1309), (0, 1, 0, -1), (-0.0417, -1, 1, 0), (-0.1252, 0, 1, 1), (-0.0305, 1, -1, 1), (-1, 1, 1, 0.7488) (-1, 0.2594, -1, 0.8182), (1, -0.2271, -0.0709, -0.0344), (-1, -1, 0.3014, 1), (-1, -1, -0.9881, -0.9017), (1, 0.1803, 0.8440, -1), (-1, -0.2560, 1, -1) (-0.0766, -0.0103, -0.9865, -0.0287), (1, 1, 1, -0.3241), (1, 1, -1, 0.4098), (-1, 1, -0.3571, -0.6600)															
	20	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		(-1, 0.8328, -0.2273, 0.8954), (-1, -0.2709, -0.1434, -0.9616), (1, -1, -0.2290, 1), (0.0702, 0.9669, 0, -0.9879), (1, 1, -1, 0.9850), (-1, -0.3492, 1, -1)															

Remark : $F_{p_1} = (-1,-1,-1,-1), F_{p_2} = (-1,-1,-1,1), F_{p_3} = (-1,-1,1,-1), F_{p_4} = (-1,-1,1,1), F_{p_5} = (-1,1,-1,-1), F_{p_6} = (-1,1,-1,1), F_{p_7} = (-1,1,1,-1)$
$F_{p_8} = (-1,1,1,1), F_{p_9} = (1,-1,-1,-1), F_{p_{10}} = (1,-1,-1,1), F_{p_{11}} = (1,-1,1,-1), F_{p_{12}} = (1,-1,1,1), F_{p_{13}} = (1,1,-1,-1), F_{p_{14}} = (1,1,-1,1)$
$F_{p_{15}} = (1,1,1,-1), F_{p_{16}} = (1,1,1,1)$

Table B.6 Computer generated design from Genetic Algorithm for weighted G-optimality criterion cuboidal design, $k = 4$ (Continued)

Combination	**N**	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	F_{p_9}	$F_{p_{10}}$	$F_{p_{11}}$	$F_{p_{12}}$	$F_{p_{13}}$	$F_{p_{14}}$	$F_{p_{15}}$	$F_{p_{16}}$
		Other															
		(0.0588, 1, 1, 0.0365), (1, 0.3038, 1, 1), (-1, -1, 0.7161, 0.0252), (0.1116, -1, 1, -1), (-1, 1, -1, -0.8744), (1, 1, 0.4432, -0.8529), (-1, 1, 1, 0.9694) (-0.0658, 0.0699, 0, 1), (1, 0.0956, -1, -1), (1, -1, -0.2077, -0.0238), (0.0708, -1, -1, -1), (0, 0.0310, -1, 0.0251), (1, -0.3210, 1, -0.9151)															
3	16	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
		(0.0308, -1, 0.4319, -0.9072), (1, 1, 1, 0.8441), (1, 0.0034, -1, 1), (-0.9976, -1, 1, -1), (-1, 1, -0.9472, 0.9796), (0.0114, -0.6578, -1, 0.9954) (0.0288, 0.7198, 1, 0.1030), (1, -1, 0.9662, 1), (1, 1, -1, -0.9499), (-1, -0.0637, 1, 1), (-1, 0.0181, -1, -1), (1, -0.0717, 1, -1), (-1, -1, -1, 0.9251) (-0.0782, 1, -1, 0.1270), (-1, 1, 1, -0.9809)															
	17	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0
		(-0.0209, 0.9985, -1, -1), (-1, -1, -1, -0.9428), (-1, -1, 0.9189, 0.9426), (1, 1, 1, -0.9692), (0.9779, 0.0781, -1, -1), (-1, 1, -0.9904, -0.9428) (1, 0.0360, 1, 1), (-1, 0.0129, -1, 0.9641), (0.0693, -1, 1, 0.8587), (0.0793, -1, -1, -0.0475), (0.0125, 0, -0.1028, 1), (1, -1, 0.9736, -1) (-1, -0.1443, 1, -1), (1, -1, -1, 0.9841), (-0.1048, 0.9986, 1, -1)															
	18	0	1	0	0	0	0	0	1	0	0	0	0	1	0	0	0
		(1, -1, 0.1966, -0.9915), (1, -1, -1, 0.8615), (1, -1, 1, 0.0677), (-1, -0.9599, 1, -1), (-0.0149, 0, 1, 1), (-1, 1, 0.1929, -0.9564), (-1, 1, -1, 0.8487) (-0.0189, -1, -0.2925, -1), (0.0065, -0.0462, 0.1263, -1), (1, 0.9403, 0.9584, 1), (0, 1, -1, -1), (0.9955, 0.9937, 1, -0.9852), (-1, -0.0231, -1, -0.9332) (1, 0.0873, -1, 0.9136), (-1, -1, 1, 0.0676)															
	19	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0
		(-1, -1, 1, 0.7426), (1, 1, 1, -0.1147), (-1, 1, -0.8279, -1), (-1, 1, -1, 0.1091), (1, -0.1682, 0.9923, 1), (-1, -1, -0.8856, -1), (1, -1, -1, -0.9525) (-0.0241, -0.1763, -0.2399, 0.9570), (1, -1, -0.8396, 0.9675), (-1, 0, 1, -1), (-0.1291, -0.0399, -1, 0.0855), (0.2446, 1, 1 -1), (1, 1, -0.9861, -0.9939) (-0.0327, 1, 0.9093, -0.9700), (-0.0710, -1, 1, 0)															
	20	0	1	0	0	0	1	1	0	1	0	0	0	1	0	0	0
		(-1, -1, 1, -0.0381), (0.1043, -1, 0.9895, -1), (1, 1, 1, 0.9117), (0.0827, 0.0456, 0.0898, 1), (-1, 0.1319, 1, 1), (1, -1, 1, 0.3029), (-1, 0.5500, -1, -1) (-0.5497, 1, 0.1291, 0.0871), (1, 0.8551, -0.9884, 1), (1, 0.2457, 1, -1), (0.0260, -1, -1, 1), (1, -1, -0.2145, 1), (0.1444, 1, 1, 1), (-1, -1, -0.0989, -1) (0.1856, -0.0060, -0.9796, 1)															
4	16	0	0	1	0	0	0	0	0	0	1	1	0	1	0	0	1
		(1, 0.2369, -0.3219, 0.7172), (-1, -1, -1, 0.3052), (-1, 0, 0.0165, 1), (0, -1, -0.8721, -0.9179), (-1, 1, -1, -0.9998), (-0.1565, 1, 0.2707, -0.8930) (1, 0.0543, 0.4847, -1), (-0.0389, -0.1244, 1, -0.6055), (-0.0352, -1, 0.3000, 0.9842), (-1, 1, 1, 0.6739), (-0.0764, 0.3763, -1, 1)															
	17	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
		(1, -0.2571, 1, 1), (-1, 1, 0.7662, -1), (-1, 0.2712, -0.1083, 1), (-0.0724, 1, 0.9010, 1), (0.0261, -1, -1, 1), (-1, -0.1384, 1, 1), (0, 0.3174, -1, -1) (1, 0.3677, -0.3373, 1), (1, 1, 0.8888, -0.9554), (1, -1, 0.9473, -0.9784), (-0.0732, -1, 0.0685, 0.7732), (-1, -1, -1, -0.2217), (1, 1, -1, -0.5090) (0.0003, 0.0346, 1, -0.0697), (-1, 1, -1, -0.0508), (1, -1, -1, -0.4260)															
	18	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0
		(-1, -1, -0.3551, -0.8969), (-1, 1, 0.5869, 0.9919), (0.0352, 1, 1, -0.0481), (-0.0443, -1, -1, -1), (1, 0.1364, 0.9309, 1), (0.0709, 1, -1, -0.3798) (-0.0419, -0.0348, 1, 0.0282), (1, -1, -1, 0.4254), (-1, -0.5329, -1, 0.9448), (0.0839, 0.9151, -0.0182, 0.0506), (1, -1, 1, -0.8819), (-1, 0.4415, 1, -1) (1, 1, 1, -0.9426), (1, -0.0616, -0.4317, -1), (0.2338, -1, 0.5317, 1)															
	19	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0
		(-0.1997, -1, 1, -0.1611), (1, -1, 0.0631, 1), (-1, -1, 0.8108, 1), (-0.2021, -1, -1, 0.9506), (1, -1, -0.9917, -0.1229), (0.0367, 0.0597, -0.2283, -0.0536) (-1, -0.1360, -0.8310, 0.7262), (1, 0.8915, -0.2220, -1), (-0.1870, 1, 0.2240, -1), (1, 1, 1, -0.0043), (0.1531, 0.1902, -1, -1), (1, 1, -1, 0.9677) (-1, -1, -0.8630, -1), (-1, 0.1316, 0.9825, -1), (-1, 1, 1, 0.9751), (1, 0, 1, 0.3186), (-0.2668, 1, -1, 1)															
	20	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0

Remark :$F_{p_1} = (-1,-1,-1,-1), F_{p_2} = (-1,-1,-1,1), F_{p_3} = (-1,-1,1,-1), F_{p_4} = (-1,-1,1,1), F_{p_5} = (-1,1,-1,-1), F_{p_6} = (-1,1,-1,1), F_{p_7} = (-1,1,1,-1)$
$F_{p_8} = (-1,1,1,1), F_{p_9} = (1,-1,-1,-1), F_{p_{10}} = (1,-1,-1,1), F_{p_{11}} = (1,-1,1,-1), F_{p_{12}} = (1,-1,1,1), F_{p_{13}} = (1,1,-1,-1), F_{p_{14}} = (1,1,-1,1)$
$F_{p_{15}} = (1,1,1,-1), F_{p_{16}} = (1,1,1,1)$

Table B.6 Computer generated design from Genetic Algorithm for weighted G-optimality criterion cuboidal design, $k = 4$ (Continued)

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	F_{p_9}	$F_{p_{10}}$	$F_{p_{11}}$	$F_{p_{12}}$	$F_{p_{13}}$	$F_{p_{14}}$	$F_{p_{15}}$	$F_{p_{16}}$
		Other															
		(-1, -1, 0.2871, 1), (1, 1, 1, -0.8354), (0.0442, 1, 0.1813, -1), (-1, 0.2542, 1, -1), (1, -1, 0.9968, -1), (-1, 1, -1, -0.9328), (-0.0887, -1, -1, 0.0479) (0.0651, -0.0691, -1, -1), (1, -1, -0.0429, -0.0467), (1, 0.4931, -1, -1), (1, 1, -0.9493, 1), (-1, 0.3470, -1, 0.9153), (-1, -1, -0.9802, -0.9744) (-0.8100, -1, 1, 0.0246), (-0.0025, -0.1350, -0.0479, -1), (1, 0.2705, 1, 0.9929), (-0.0207, -1, 1, 1), (-0.1767, 1, 0, -0.0857)															
5	16	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
		(-1, -0.3591, 1, -1), (1, -0.9355, -1, -1), (-1, 1, -0.5785, -1), (1, 1, -1, -0.7452), (-1, -1, 0.4860, 1), (0.9977, 1, -0.6171, 1), (1, -0.1310, 1, 1) (-0.1008, -1, 0.8986, -0.0569), (-0.2463, 1, 1, -0.1419), (1, -1, 1, -0.5312), (0.9723, -1, -1, 1), (-1, 0.2080, -1, 1), (-1, 1, 0.6366, 0.9440) (0.0042, 0.0993, -1, 0.1952)															
	17	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	1
		(1, -1, 1, 0.9414), (-1, 1, 0.7010, 1), (1, 0.2740, -1, 1), (-0.2163, 1, -1, 1), (-1, 1, -1, -0.9690), (1, -1, 0.3455, -1), (1, 1, -0.9798, -1) (-0.6315, 1, 1, -1), (-1, -0.1813, -1, -1), (1, 0.2501, 1, -1), (-1, -0.2344, 1, 0.9794), (0.0947, -0.8966, -0.9897, -1), (0, -0.6421, 1, 1)															
	18	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0
		(-1, 1, 0.4082, 0.9728), (-0.9948, -1, -1, 1), (1, 0.7985, 1, -1), (-0.6493, 1, 1, -1), (0.0121, -0.4167, -1, -1), (1, -1, 1, -0.9961), (1, 0.8612, -1, 1) (-1, -0.7006, -0.9267, -1), (0.9854, -1, -1, -1), (-0.3269, 0.8447, -1, 1), (-1, 0, 1, 1), (-0.1066, -1, 1, 1), (1, -0.4059, 1, 1), (-1, -1, 1, -0.8732) (0.8830, 0.9995, 1, 1)															
	19	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
		(-1, -0.9977, 0.9703, 0.9466), (-1, -0.5967, 1, -1), (0.2453, -0.0355, -0.0923, -0.9382), (0, 0.0612, -1, 0.0356), (1, 1, 1, -0.6254), (1, 0.0435, 1, 1) (0.2392, 1, 1, 0.1979), (-1, 0.8315, 1, 1), (1, -1, 1, -0.3175), (-0.0027, -1, 1, 0.2448), (1, -1, -0.3987, -1), (1, 1, -0.7455, 1), (1, 0.7904, -0.9630, -1) (-1, -0.7955, -1, 1), (1, -1, -1, 0.9727), (-0.9543, 1, 1, -1)															
	20	0	1	0	0	0	1	0	0	0	0	0	0	1	0	0	1
		(1, -0.9779, 1, 0.6899),(-1, -1, 0.9164, -1), (-1, 0.9485, 0.9999, -1), (1, 0.2446, 1, -0.9896), (-0.9580, 1, -1, -1), (-0.0683, 0.0548, -0.0614, 0.8970) (1, -0.1586, -1, 0.7757), (1, -1, -1, 0.5005), (0.7739, 1, -0.9667, 1), (-1, -0.5729, -1, -1), (1, -1, 0.1743, -1), (-1, 0.8509, 1, 1), (0.1135, -1, -1, -1) (0.0591, -0.9834, 1, 0.7467), (-1, -0.8502, 1, 0.9631), (0.3326, 1, 0.9614, -1)															
6	16	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
		(1, 1, -0.0380, 0.9061), (-1, -1, 1, 0.0876), (-1, 0.8207, 0.1177, -1), (-0.0197, 0.1740, 1, -0.7770), (0.1678, -1, -0.0733, -0.8591), (1, 0, 1, 1) (1, -1, 1, -0.9930), (1, -1, -0.8597, 1), (1, 0.9411, -1, 1), (-1, -0.5258, -0.0466, 1), (-1, 0.8350, -1, 0), (1, 0, -0.9825, -0.9470), (0, 1, -1, -1) (-0.9960, 1, 1, 1)															
	17	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
		(0.0361, -1, 0.0273, -0.9418), (-1, -0.9506, -1, 1), (-1, 0.9326, 0.2468, -1), (-1, -1, 1, 0.1089), (1, -1, -0.7297, 0.8893), (-1, 0.7682, -1, 0.1523) (1, 0.9923, -1, 1), (-1, 1, 0.9932, 1), (0.0274, 1, -1, -1), (-0.0539, 0.0707, 1, -1), (1, -0.2104, -0.8670, -0.9347), (-1, -0.1096, 0.1848, -0.1445) (1, 1, 1, 0.1984), (1, -0.4141, 1, 1), (1, 1, -0.2768, -1)															
	18	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0
		(1, -0.6213, -0.4681, 0.3927), (-1, -1, -0.5931, 1), (0.0428, -1, 0.4232, -1), (-0.9457, 0, 1, 1), (-1, 0.1647, -0.3414, -1), (0.8874, 1, 1, 0.7173) (0.0661, -0.4423, -1, 0.8325), (0.9948, 0.0106, 1, -1), (-1, -1, 1, -0.0957), (0.0502, 0.9143, -1, -0.8563), (1, -1, -1, -0.8665), (1, 0.9123, -1, 1) (1, 1, -0.0677, -1), (-1, -0.8398, -1, -0.9740), (-0.6456, 1, 0.0822, 0.2506)															
	19	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0
		(-0.1013, 1, -0.0210, 0), (0, -0.1700, 1, -0.1474), (0.9099, 1, 1, -0.1140), (1, -0.0462, -1, -1), (0.0190, -0.7413, 0.2090, 1), (1, -0.0504, 1, 1) (0.1219, -0.9283, -1, -1), (1, -0.9014, -0.0109, -0.0196), (-1, 1, 0.8931, 1), (-1, 1, -1, -0.9797), (-1, -0.8971, 0.1035, -1), (0.9867, 1, -1, 1) (-1, -1, -1, 0), (-1, 0.3789, 1, -0.9187), (-1, 0.2862, -0.9188, 1), (1, -1, 1, -0.9100), (1, 1, 0.0506, -1)															

Remark :$F_{p_1} = (-1,-1,-1,-1), F_{p_2} = (-1,-1,-1,1), F_{p_3} = (-1,-1,1,-1), F_{p_4} = (-1,-1,1,1), F_{p_5} = (-1,1,-1,-1), F_{p_6} = (-1,1,-1,1), F_{p_7} = (-1,1,1,-1)$
$F_{p_8} = (-1,1,1,1), F_{p_9} = (1,-1,-1,-1), F_{p_{10}} = (1,-1,-1,1), F_{p_{11}} = (1,-1,1,-1), F_{p_{12}} = (1,-1,1,1), F_{p_{13}} = (1,1,-1,-1), F_{p_{14}} = (1,1,-1,1)$
$F_{p_{15}} = (1,1,1,-1), F_{p_{16}} = (1,1,1,1)$

Table B.6 Computer generated design from Genetic Algorithm for weighted G-optimality criterion cuboidal design, $k = 4$ (Continued)

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	F_{p_9}	$F_{p_{10}}$	$F_{p_{11}}$	$F_{p_{12}}$	$F_{p_{13}}$	$F_{p_{14}}$	$F_{p_{15}}$	$F_{p_{16}}$
		Other															
	20	0	0	0	0	0	1	0	1	0	0	0	1	0	1	0	0
		(1, 1, 1, -0.9287), (0, 0.0451, 1, 0.2719), (-1, 0.1148, -0.8032, -0.9101), (1, -0.1263, 0.8403, -1), (0.0409, 0.9396, -0.8887, -1), (1, -1, -1, 0.0474) (0.0616, -1, -1, 0.9768), (0.0737, -1, 0.4179, -1), (-1, -0.0350, 1, -1), (-0.9956, -1, -1, -1), (-1, -1, 1, 0.9194), (1, -1, -0.0257, 0.0239), (-1, -1, 0, 1) (0.9265, 1, 0.1311, 1), (1, 0.0455, -0.8334, -1), (-0.9811, 1, 0.1892, -1)															
7	16	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		(-1, 0.8951, 1, -1), (1, 1, 1, -0.8356), (1, -1, 0.5648, -1), (-1, 1, -1, -0.4194), (-1, -0.9035, 1, -1), (0.2578, 0.2437, 1, 0.7105), (1, -0.7893, 1, 1) (1, 1, -0.2340, 1), (0.2353, -1, -1, 1), (-1, -1, 0.9777, 1), (1, 0.5758, -1, -0.4705), (-1, -0.2729, -1, 1), (1, -0.8843, -1, 0.0210), (-1, 1, 0.6658, 1) (0.0615, 1, -1, 0.0523)															
	17	0	1	1	0	1	0	0	1	0	0	0	0	1	0	0	0
		(0.5056, -0.3308, -1, -1), (-1, 0.2776, -1, 1), (1, -1, -1, 0.2201), (1, -0.9343, 0.1979, 1), (-0.6584, -1, -1, -1), (-1, 0.4375, 1, -1), (-0.0045, 1, -1, 1) (1, 1, -0.1962, 0.3843), (-0.6025, -1, 0.9802, 0.9770), (1, 0.7999, 1, 1), (1, -0.9823, 1, -1), (0.4095, 1, 1, -0.9869)															
	18	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
		(1, 1, 1, 0.4264), (0.1087, -1, -0.3227, -1), (1, -0.7799, 1, 1), (-1, 1, -1, -0.9073), (1, -0.3002, -1, -1), (1, 1, 0.2827, -1), (-1, -0.2853, 1, -1) (0.0919, 0.0935, 0.9063, 0.7753), (-1, -1, -0.3385, -1), (-1, 0.3136, -1, 1), (-1, -1, -1, 0.3779), (1, -1, 0.9885, -1), (1, -1, -1, 0.9855) (-0.7432, 1, 1, -1), (-1, 1, 0.8975, 1), (1, 1, -1, 0.9554), (0.2151, 1, -1, -0.1990)															
	19	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
		(1, -1, -1, 0.9104), (0.9131, 1, -1, -0.9338), (1, 1, 0.9397, 1), (0.9072, 1, 1, -1), (1, -0.7787, 1, 0.9572), (1, -0.3704, -0.9987, -1), (-1, 1, 0.9956, 1) (1, -1, 0.7903, -1), (0.7795, 1, -1, 1), (0.0642, -1, 0.7537, 1), (-0.8419, 1, -0.9764, -1), (-1, -0.8767, 1, 0.9889), (-1, -1, 0.9943, -1), (0, -1, -1, -1) (-1, -0.4661, -1, -0.9957), (-1, -1, -1, 0.9054), (-1, 1, 0.9333, -1), (-0.0248, 0, 0.9245, -1)															
	20	0	1	0	0	0	0	1	0	1	0	0	0	1	0	0	1
		(-0.0415, 1, -0.9814, 0), (1, 0.9342, -0.9413, 1), (-0.9936, -1, 1, 1), (-1, 0.8521, -1, 1), (1, -1, 1, -0.9402), (0.8364, 1, 1, -1), (1, -0.8231, -1, 1) (-1, -0.6339, -1, -1), (1, 0.0122, -0.1136, -0.9251), (0.9593, -0.9554, 0.9336, 1), (0.0109, -1, -0.7851, -0.6804), (-1, -0.9666, 0.9617, -1) (-1, 0.8882, 1, 1), (-1, 0.9861, -1, -1), (-0.0673, 0, 1, 0)															
8	16	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
		(-1, 0.0019, -0.9422, -0.9785), (1, 1, 1, 0.9254), (0, 0.9581, 0, -0.1062), (-1, 1, -1, 0.9777), (0.9504, 0.0943, -1, 1), (-1, -1, 0.8526, -0.9456) (1, -0.2051, 1, -0.9428), (1, -1, 0.8656, 1), (0, -0.9901, 0.1481, -0.9384), (-1, 1, 0.9838, -1), (0.0530, -0.0593, 1, 0.9155), (1, 1, -1, -0.9682) (1, 0.1287, 0.0422, -1), (-1, -0.1162, 1, 1), (-0.9731, -1, -0.9995, 0.9865)															
	17	1	0	0	0	0	0	0	1	0	1	0	0	0	0	1	0
		(0.1833, 0, 1, 1), (1, 0.9989, -1, 0.9819), (-0.0263, 0.9520, -1, 0.0520), (1, -0.2722, 1, 0.2933), (1, -1, 0.9567, -1), (-1, -0.1967, -0.8136, 1) (1, 0.9120, -0.0045, 1), (-1, 1, -0.1417, -0.8131), (-1, -0.1461, 1, -0.9929), (1, -0.1271, -1, -0.9931), (-0.9865, 1, -1, -1), (-1, -1, 1, 0.9809) (-0.0865, -1, -0.0177, 0.0546)															
	18	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0
		(-1, 1, 1, 0.9457), (1, 0.7410, 1, 1), (1, -1, 0.0114, 0.8565), (1, 0.2874, -0.5489, -1), (-0.0158, -0.1511, 0.1780, -0.0742), (-1, -1, -0.0277, -1) (-1, 0.0626, -1, 1), (-1, 0.1658, 1, -0.9480), (-0.3271, 1, 0, 1), (1, 1, -1, 0.5184), (1, -1, -1, 0.8043), (-1, -0.8876, -1, -0.1105), (0.8721, 1, 1, -1) (0.1221, -0.9186, 1, 0.9623), (0.0146, -1, -1, -1)															
	19	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0
		(-1, -1, -1, -0.9981), (1, 0.9052, 0.1805, 1), (0.9788, 0.0946, 1, 0.1393), (1, 0, -1, -1), (0.1143, -1, 1, -1), (1, -1, 0.1688, -1), (1, 1, -1, 0.9068) (0.0184, 1, -0.7848, -1), (-1, -0.3794, -0.8818, 1), (-1, -0.1646, 1, -0.9891), (1, -0.9488, 1, 1), (-1, 0.9564, 0.0879, -1), (0.9710, -1, -1, 0.9797)															

Remark :$F_{p_1} = (-1,-1,-1,-1), F_{p_2} = (-1,-1,-1,1), F_{p_3} = (-1,-1,1,-1), F_{p_4} = (-1,-1,1,1), F_{p_5} = (-1,1,-1,-1), F_{p_6} = (-1,1,-1,1), F_{p_7} = (-1,1,1,-1)$
$F_{p_8} = (-1,1,1,1), F_{p_9} = (1,-1,-1,-1), F_{p_{10}} = (1,-1,-1,1), F_{p_{11}} = (1,-1,1,-1), F_{p_{12}} = (1,-1,1,1), F_{p_{13}} = (1,1,-1,-1), F_{p_{14}} = (1,1,-1,1)$
$F_{p_{15}} = (1,1,1,-1), F_{p_{16}} = (1,1,1,1)$

Table B.6 Computer generated design from Genetic Algorithm for weighted G-optimality criterion cuboidal design, $k = 4$ (Continued)

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	F_{p_9}	$F_{p_{10}}$	$F_{p_{11}}$	$F_{p_{12}}$	$F_{p_{13}}$	$F_{p_{14}}$	$F_{p_{15}}$	$F_{p_{16}}$
		Other															
		(-1, 1, -1, 0.2558), (-0.3843, -1, -0.1597, 0.1108), (0.2101, 0.2089, 1, 1)															
	20	0	0	0	1	0	0	0	1	0	1	0	0	0	0	0	0
		(-0.9798, 0.0707, -1, -1), (1, 0, 0.9890, -1), (-1, 1, -1, 0.6514), (0, -1, -0.8966, -1), (1, 0.2729, -1, -1), (0.0846, -1, 1, 0.9809), (-1, -0.2653, 1, -1) (1, 1, 1, 0.9871), (-0.1109, 0.9990, 1, -0.9017), (-0.0125, 0.1610, -0.0084, 0.1109), (1, -1, -0.2464, -0.5465), (1, -1, 1, 0.1295), (-1, -1, -1, 0.4538) (0.9700, 1, -1, 1), (-1, 1, -0.1504, -1), (-1, -1, 0.1114, -0.2768), (1, 1, 0.0232, -1)															
9	16	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0
		(0.3304, -1, -0.3893, 0.9990), (-1, -0.4972, 1, 1), (0.0748, 0.0905, -0.9123, 1), (0.1375, 1, 1, 0.4885), (-0.9576, 1, 0.0156, 1), (-1, 0.5904, 1, -1) (1, -0.9117, 1, 1), (-1, -0.9169, -1, 0.5396), (1, -0.0287, 0.3824, -0.1090), (0.2570, 0.2754, -1, -1), (0.0129, -1, 1, -0.8143), (1, 1, 0.8530, -1) (-1, -1, -0.0465, -1), (-1, 1, -1, -0.6635)															
	17	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	1
		(1, -0.9975, 1, -1), (1, 0.0245, 0.0332, 0.1163), (-1, 0.0382, -1, -1), (0.1940, 0.9007, -1, 1), (0.2293, -1, -1, -0.9956), (-1, 1, 0.5550, -0.9940) (-1, 0.2112, 1, 1), (-0.9985, -1, -1, -0.0891), (0.2020, -0.0171, 0.0071, -1), (0.1527, 1, 1, -0.9464), (-1, -1, 0.0134, 1), (-1, 1, -0.7333, 1) (0.1989, -0.9128, 1, 1)															
	18	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		(-1, -0.8095, 1, 1), (0.0743, 1, -1, 1), (0.0583, -1, -0.2448, -1), (-1, 1, 0.2474, 1), (-0.9625, -1, 0.6073, -0.9838), (-0.0294, 0.0629, 1, 0.3664) (1, 1, -0.9785, -1), (1, -1, 0.2726, 1), (1, 0.0775, -0.9029, 1), (1, -1, -0.9942, -0.7359), (-1, 0.9642, -0.9720, -0.5821), (1, -0.9761, 1, -0.6807) (-1, -0.1258, -0.8354, -1), (0.0894, 0.0785, 0.1738, 0), (1, 1, 0.7585,), (-1, 1, 1, -0.9178), (1, 0.8441, 0.8677, -1)															
	19	0	0	0	0	0	1	1	0	0	0	0	1	0	0	1	0
		(0.0440, -1, 1, -1), (-0.0549, 0.4762, 0.1121, 1), (1, -0.4085, -0.8244, 0.3341), (-1, -1, -0.1299, 0.7982), (1, -1, -0.9553, -0.5140), (1, 1, 0.0685, 1) (-1, -1, -0.0611, -1), (-0.1320, -1, -1, 1), (0.1725, 0.1681, 1, 0.8624), (1, 1, -1, 0.0270), (-1, -0.2069, -1, -1), (0, 0.8233, -1, -1), (-1, -0.5754, 1, 1) (1, -0.4078, 0.3528, -1), (-1, 1, 0.1657, 0.1605)															
	20	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
		(-0.0386, 0.2697, -1, -0.9966), (-1, -0.2026, 0.1096, 1), (0, 1, 0.1830, -0.2464), (1, -1, 1, 0.2743), (-1, -1, -0.9813, 1), (0.0289, -1, 0.8593, -1) (-1, 1, -1, 0.0848), (0.0369, -1, 1, 1), (0.3091, 0, -1, 1), (1, -1, -0.7027, -1), (-0.8973, 1, 1, 1), (-1, -0.0268, 1, -0.3619), (1, 0.9546, 1, 0.4179) (1, 0, 1, -1), (1, -0.8952, -1, 1), (-1, -1, 1, -0.7962), (-1, 1, -0.2051, -1), (1, 1, -0.1371, 1)															
10	16	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
		(-1, 1, -0.1796, -1), (-1, -1, 0.8734, -0.9876), (1, 0.0960, -0.4092, -1), (1, 1, 0.3180, -0.2713), (1, 0.0236, 1, 1), (-0.1206, -0.9850, -0.0418, 0) (1, -1, -0.9966, 0.4016), (0.1617, 0.6992, -1, -1), (0.8773, 1, -1, 1), (0.0999, 1, 1, -0.0278), (0.1888, -0.4402, 0.0669, 1), (-1, 1, 0.1598, 1) (-1, 0.2401, -1, -0.2097), (-1, 0.2458, 1, 0.6813), (-1, -1, -0.9445, 1)															
	17	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
		(0.0572, 1, 1, 1), (1, 0.9913, 0.1111, -0.4113), (0.1152, 0.1959, -1, -0.0528), (-0.0165, -0.0916, -0.0182, 1), (1, 1, -1, 0.9655), (-1, 1, 0.4770, -1) (1, -1, 1, 0.2039), (-1, 1, -1, 0.7645), (0, -1, 0.0502, -1), (1, -0.7344, 0.0770, 1), (-0.8954, -1, 1, -0.2491), (-1, 0.0680, 0.1415, 0.1383), (-1, 0, 1, 1) (-1, -0.5469, -1, -1), (-1, -1, -0.7973, 1), (1, 0.3961, 1, -1)															
	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		(-0.9990, 1, -1, -0.7415), (1, 1, -0.9913, -1), (1, -1, -0.9551, 0.9811), (0, 0.2109, -0.1001, 0.0982), (0.8004, 1, -1, 0.1430), (1, -1, 1, -0.3039) (-0.6436, -1, 1, 1), (-1, 0.0097, 1, -1), (-1, 0.3148, 1, 0.0624), (-1, -1, -0.3103, 0), (1, 0.9725, 1, 1), (0.2827, 0.0956, -0.0827, 1), (0, 1, 1, -1) (-1, -0.2384, -1, 1), (0.0371, -0.9500, -1, -1), (-1, -1, 0, -1), (1, -0.3181, 0.2794, -1), (-1, 1, 0.1988, 1)															
	19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Remark :$F_{p_1} = (-1,-1,-1,-1), F_{p_2} = (-1,-1,-1,1), F_{p_3} = (-1,-1,1,-1), F_{p_4} = (-1,-1,1,1), F_{p_5} = (-1,1,-1,-1), F_{p_6} = (-1,1,-1,1), F_{p_7} = (-1,1,1,-1)$
$F_{p_8} = (-1,1,1,1), F_{p_9} = (1,-1,-1,-1), F_{p_{10}} = (1,-1,-1,1), F_{p_{11}} = (1,-1,1,-1), F_{p_{12}} = (1,-1,1,1), F_{p_{13}} = (1,1,-1,-1), F_{p_{14}} = (1,1,-1,1)$
$F_{p_{15}} = (1,1,1,-1), F_{p_{16}} = (1,1,1,1)$

Table B.6 Computer generated design from Genetic Algorithm for weighted G-optimality criterion cuboidal design, $k = 4$ (Continued)

Combination	**N**	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	F_{p_9}	$F_{p_{10}}$	$F_{p_{11}}$	$F_{p_{12}}$	$F_{p_{13}}$	$F_{p_{14}}$	$F_{p_{15}}$	$F_{p_{16}}$
		Other															
		(-0.8982, 0.1328, -0.9008, 1), (-0.9614, -0.9854, 0.8482, 1), (0.2439, -1, 1, 0.3580), (0.1394, 0.2336, -1, 0.0858), (1, 1, 1, 0.9869), (1, -0.9168, -1, 1) (1, -0.5800, 1, -1), (1, -1, 0.1405, -0.1679), (0, -0.1979, 0.2222, 0.1681), (1, 0.5170, -0.0489, 0.1479), (-1, 1, -1, -0.7985), (-1, -0.9144, -0.0787, -1) (-0.9601, 1, 1, 0.0214), (-1, 0, 1, -1), (0.2228, 1, 0.3882, -1), (-1, -1, -1, -0.1715), (-0.6980, 1, 0.0834, 1), (1, 1, -1, -0.7548), (0.4241, -1, -1, -1)															
	20	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
		(-0.2779, -1, -1, 1), (1, -0.0369, 1, 1), (1, 1, 0.8666, -1), (0.0673, 1, 1, -0.1044), (1, -0.1294, -1, -1), (-1, -1, -1, 0.2476), (-1, -0.5423, -0.2381, 1) (-1, -0.3786, 0.4319, -0.8804), (1, 0.7350, -1, -0.1969), (-1, -1, 1, -0.9040), (1, -1, 0, -0.0218), (-1, 1, -0.3877, 0.0867), (-0.0399, 1, -0.0875, -1) (0.0821, -0.0478, 1, 0.1091), (1, 1, 0, 1), (-0.0582, -0.0682, -0.1717, -1), (1, -1, 0.8537, -0.9718), (0.7091, -1, -0.8932, 0.9983), (-1, 0.8432, -1, -1)															
11	16	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
		(0.1650, 0.5877, 1, 1), (0.4079, 1, -0.2670, -1), (1, -1, 0.2429, 1), (0.0710, -1, -1, 1), (-0.7108, -0.5706, 0.8899, -1), (-1, 1, 1, -0.8223) (1, 1, -1, 0.4588), (-1, -0.1968, -1, 1), (1, 0.2530, -0.0204, 1), (1, -0.7566, -1, -1), (0.9296, -1, 1, -1), (-0.8713, 1, -1, -1), (1, 1, 1, -0.4518) (-0.9017, 1, 0.0501, 1)															
	17	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
		(-1, -0.2808, -1, -1), (-1, -1, -0.7877, 1), (0, 0.1921, 1, 0.2585), (-0.3670, 1, -0.2722, -1), (0.9707, -1, -1, -1), (-1, 1, -1, 0.8846), (1, 1, 1, -0.2102) (-0.5104, -1, 1, -1), (1, 1, -0.0252, 1), (-0.9420, 0.6016, 1, 1), (1, -0.4673, 0.6335, -1), (-1, -1, 0.9481, 0.0076), (-0.3122, -1, -1, -0.3972) (1, -0.3223, -1, 1), (0.8780, 1, -1, -0.9441), (-1, 1, 0.9749, -1)															
	18	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
		(1, -0.9837, -0.7564, 1), (0.9930, -0.4448, 1, -1), (-1, 0.1194, 1, -0.9294), (0.1139, -0.0789, -0.0642, 0.9284), (-0.0902, 1, 1, -0.9843) (-1, -1, 0.3155, -1), (-1, 0.9544, 1, 1), (1, 0.9772, -0.9943, 0.3036), (1, 1, -0.0903, -1), (-1, 0.8661, -1, -1), (1, -1, 1, 0.2094), (1, 0.8442, 1, 1) (1, -0.7185, -1, -1), (-0.7932, -1, 1, 1), (0, 1, -1, 1), (-1, -0.8093, -1, 0.9580), (-0.3052, -1, -1, -0.9468)															
	19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		(0.5521, 0.1156, 1, 0.0642), (-0.9544, 0.9448, 0.9371, -1), (1, 1, -0.9766, 0.0392), (-0.0087, -0.0997, -1, 0.9924), (0.9975, -0.9384, 1, 1) (1, 1, 0.2260, 1), (-0.9800, -1, -1, -1), (-0.9872, -1, -0.4398, 1), (-1, -0.9266, 1, -1), (-0.6635, -1, 1, 1), (1, 0, -1, -1), (0.9573, -1, -1, 1) (-0.3214, 1, -1, -1), (1, -1, 0.2562, -1), (-1, 0.3173, -0.3466, -0.9023), (0.2814, -1, -0.0108, -1), (-0.8282, 0.9968, 1, 1), (1, 1, 1, -0.8346)															
	20	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	1
		(0.7246, -1, -1, -1), (-1, 0.3031, 1, 1), (1, -1, 0.9971, -1), (-1, -1, -0.2324, 1), (-1, -1, -1, 0.0128), (1, 0.6543, -0.8330, 1), (0.9633, -1, -1, 1) (-1, 1, 0.3290, 1), (-0.2431, -0.9644, 1, 1), (-1, 1, 0.1287, -1), (-0.3692, 1, 1, -0.9506), (1, 0.5627, 1, -1), (-0.0403, -0.3898, -0.0185, -0.8803) (-0.1056, 1, -1, 0), (-1, 0.1103, -1, -1), (-0.8301, 0.9981, -1, 1)															
12	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		(-0.0506, -0.1310, 1, -1), (-0.9733, 0.7635, -1, -1), (-1, -1, 0.1522, -1), (-1, 1, 1, 0.1025), (-0.8059, 1, -0.6950, 1), (0.9726, -1, 1, -0.1520) (1, 0.1558, -1, 0.9658), (1, 1, 0.9566, 0.7792), (-0.0800, -0.0699, -0.1890, 0.1906), (-0.9370, -0.8949, 1, 1), (1, 0.1210, 0.0999, -1) (0.9887, 1, -1, -0.0287), (0.1848, 1, 0.4611, -1), (-1, -1, -1, 0.9744), (0.4512, -1, -1, -1), (1, -1, -0.6197, 0.9859)															
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		(-1, 0.6456, -1, 0.7584), (-0.1813, 1, -1, -0.7488), (1, -0.4246, -1, -1), (1, -0.6847, 1, 1), (0.0096, 0, 0.0540, -0.2988), (1, 1, 1, -0.9570) (0.3941, -1, 1, -0.9756), (-1, -1, -0.9019, -1), (0.7267, -0.3405, -1, 1), (0.0836, -1, -1, 0.4325), (-1, -1, 0.7013, 0.9714), (-0.9266, 1, 1, 0.9615) (1, 1, -0.9313, 0.7286), (-1, -0.0333, 1, -0.4363), (-1, 1, 0.1360, -1), (1, -1, -0.2105, -0.1663), (0.3179, 0.8740, 0.3760, 0.5547)															
	18	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
		(1, -0.1503, 1, 1), (-1, -0.8968, -1, -0.3242), (0.8153, 1, -1, -1), (-1, -0.9681, 0, 0.9498), (1, 1, 0.9068, -0.9453), (1, -0.0560, -0.8711, -0.0542) (0.9080, 1, -0.0375, 1), (-0.7921, 1, 0.9794, 0.3520), (-1, 0.1396, 0.9956, 1), (0.1265, -1, 0.9565, 0.3871), (-1, 1, -0.2922, -1), (1, -1, 0.7980, -1)															

Remark :$F_{p_1} = (-1,-1,-1,-1), F_{p_2} = (-1,-1,-1,1), F_{p_3} = (-1,-1,1,-1), F_{p_4} = (-1,-1,1,1), F_{p_5} = (-1,1,-1,-1), F_{p_6} = (-1,1,-1,1), F_{p_7} = (-1,1,1,-1)$
$F_{p_8} = (-1,1,1,1), F_{p_9} = (1,-1,-1,-1), F_{p_{10}} = (1,-1,-1,1), F_{p_{11}} = (1,-1,1,-1), F_{p_{12}} = (1,-1,1,1), F_{p_{13}} = (1,1,-1,-1), F_{p_{14}} = (1,1,-1,1)$
$F_{p_{15}} = (1,1,1,-1), F_{p_{16}} = (1,1,1,1)$

Table B.6 Computer generated design from Genetic Algorithm for weighted G-optimality criterion cuboidal design, $k = 4$ (Continued)

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	F_{p_9}	$F_{p_{10}}$	$F_{p_{11}}$	$F_{p_{12}}$	$F_{p_{13}}$	$F_{p_{14}}$	$F_{p_{15}}$	$F_{p_{16}}$
		Other															
		(-0.1344, 0.0889, 1, -1), (-0.9242, 0.9949, -1, 0.9123), (-0.0532, 0.0477, -0.1353, -0.0132), (0.4200, -1, -1, -0.9300)															
	19	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0
		(0.9591, 1, 1, -0.2601), (-1, 1, -1, -0.1371), (-0.1626, -0.7794, 1, -1), (-1, -1, 1, 0.8628), (1, -0.1098, 1, 1), (1, -1, 0.2270, 1), (0, -0.0242, -1, 1) (1, -1, -1, -0.2096), (-1, -0.9512, -0.9054, -0.9965), (1, 1, -0.8999, 1), (0.5714, 1, -1, -1), (1, 0.0872, 0, -0.9945), (-0.3007, 1, -0.0951, 0.8891) (-0.1846, -1, 0.1490, -0.0978), (-1, -0.8483, -1, 1), (-1, 1, 0.9212, -1), (-1, -0.2338, 0.7298, 0.0028)															
	20	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
		(-1, 1, 0.1985, 0.0091), (1, -0.9746, -1, -0.9880), (1, 0.9925, -0.1475, -1), (-0.9726, 0.1408, -0.1092, 0.9536), (1, 1, 1, 0.3623), (-0.2767, -1, 1, 1) (0.4969, 1, 0.1746, 1), (1, -0.8608, -0.7977, 1), (-1, 1, -0.9526, -0.9449), (1, -0.1819, 1, 0.7596), (-1, -0.7901, -1, -0.7765), (0.1501, 0.3828, 1, -1) (1, 0.8587, -1, 0.0830), (-1, 1, 1, -0.1794), (-0.9868, -1, -1, 1), (1, -1, 1, -0.5278), (-0.3852, 1, -1, 0.9776), (0.0434, -0.5854, -0.1855, 0.0510) (-0.9904, 0, 1, 1)															
13	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		(1, 1, -0.7526, -0.9637), (1, -1, -1, -0.5866), (-1, 0.1549, 1, 1), (1, 0.9967, 0.9059, 0.9618), (0.0290, 1, -1, 0.5298), (-0.9866, -1, -0.7850, 1) (-0.9678, -0.4344, -1, -0.9486), (-1, 1, -0.4556, 1), (0.2017, -1, 1, 0.5479), (0, 0.0562, -0.1043, -1), (1, -0.0585, -1, 1), (-1, -1, 0.8120, -0.9232) (1, -1, 0.6357, 0.2210), (-0.9648, 1, 1, -1), (-1, 0.9978, -1, -1), (1, 0.1278, 1, -1)															
	17	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	1
		(-1, -0.1731, -1, 0.1398), (-0.2150, -0.9253, 1, 1), (1, 1, -1, -0.7733), (-1, 1, -1, -0.9099), (1, -1, -0.8743, 1), (0, 1, -1, 1), (1, -0.0675, -1, -0.9319) (-1, -1, -0.6664, 0.9081), (-0.1265, -1, -1, -1), (0.7606, 1, 0.2658, -0.9619), (0.0835, -0.0111, 1, -1), (-1, 1, 0.4591, -0.8486), (-1, 0.9006, 1, 1) (0.9391, -0.2516, 0.2247, 1)															
	18	0	1	1	0	0	0	0	0	0	0	1	0	1	0	0	0
		(-0.9695, -0.1698, -1, -1), (-0.0508, 0.0335, 1, -0.1992), (1, -0.0514, -0.1696, 0.1455), (-1, -0.2492, 0.6981, 1), (1, 1, 0.9149, 1), (0.5303, -1, 1, 1) (0.5726, 1, 1, -1), (-0.3402, -1, -0.0406, -0.0046), (1, -0.9025, -1, 1), (-1, 1, 0.1080, -1), (0.3048, 1, -1, 1), (0.5041, -1, -1, -1), (-1, 1, 1, 0.9663) (-1, 1, -1, 0.3229)															
	19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		(1, -0.9921, -1, -1), (0.9032, -1, 1, 0.9049), (-1, 0.2394, 0.1420, 1), (1, -1, -0.3546, 1), (-1, -1, -0.5310, -1), (0.3041, 0.9215, -0.8650, -0.9870) (-1, 0.9801, -1, -1), (0.1187, -1, 1, -1), (-0.2781, 1, 0.8325, 1), (1, 1, -0.0389, -1), (-1, 1, -0.7759, 1), (-1, -1, 1, 0.9786), (-1, 1, 1, -0.9631) (-0.9779, -0.1914, 0.3886, -1), (1, 1, 1, 0.9260), (1, 0.0224, 1, -1), (1, 0.8093, -1, 1), (0.1876, -0.1240, -1, 0.8858), (-0.8393, -1, -1, 0.6579)															
	20	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0
		(-1, 0.2321, 1, -1), (1, -0.3995, 1, 1), (-0.7688, -1, 0.8747, -1), (-0.3665, -1, -0.9346, 1), (-1, -1, 0.9116, 1), (0.0363, -0.2172, -0.0811, -1) (-1, 1, -0.0577, 0.9955), (1, -1, -0.7948, -1), (1, 0.5708, -1, -1), (-1, 0.0976, -1, 1), (0.0247, -0.1497, 0.9137, 0.0423), (-1, -0.9940, -1, -0.8950) (0.9929, 1, 0.0807, 0.9679), (0.8762, 0.9125, -0.9979, 1), (1, -1, 1, -0.4279), (-0.5091, 0.9976, -1, -1), (-0.3831, 1, 1, 1), (-1, 1, 0.1751, -1)															
14	16	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
		(1, -1, -1, 0.1906), (-0.4379, 1, 0.9387, 0.6505), (-1, -1, 0.2145, -0.2776), (-0.2828, -1, 0.0390, -1), (-1, 0.1086, 1, -1), (-1, -0.9777, -1, 1) (1, 1, 0.9525, 0.3079), (-0.9933, 1, -0.1074, 0.8784), (-1, 0.8990, -1, -1), (-1, -0.2791, 1, 0.4203), (1, -0.9493, 1, -1), (-0.0013, -1, 1, 1) (1, -0.1308, 0.2163, 1), (0.9995, 0.7243, -0.7706, -1), (-0.1631, 0.0577, -0.9342, 0.0435)															
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		(0.0728, 1, -0.1551, -1), (1, -0.7401, -0.9531, -1), (-0.7263, -0.2623, 1, -1), (-1, 1, -0.8555, -0.8666), (1, 1, -1, 0.4782), (-1, -1, -0.1180, -1) (1, 1, 1, -0.7382), (1, -1, 0.5869, -0.3727), (-1, 1, 1, 0.1954), (-1, 0.1863, -0.9627, 1), (-0.4396, -0.9879, -1, -0.2420), (-1, -1, 0.3446, 0.8720) (-0.0253, 1, 0.1629, 1), (0.9263, -1, -1, 1), (0.0380, -0.0165, -0.1394, -0.0469), (0.2317, -1, 1, 1), (1, 0.0617, 0.5295, 1)															

Remark : $F_{p_1} = (-1,-1,-1,-1), F_{p_2} = (-1,-1,-1,1), F_{p_3} = (-1,-1,1,-1), F_{p_4} = (-1,-1,1,1), F_{p_5} = (-1,1,-1,-1), F_{p_6} = (-1,1,-1,1), F_{p_7} = (-1,1,1,-1)$
$F_{p_8} = (-1,1,1,1), F_{p_9} = (1,-1,-1,-1), F_{p_{10}} = (1,-1,-1,1), F_{p_{11}} = (1,-1,1,-1), F_{p_{12}} = (1,-1,1,1), F_{p_{13}} = (1,1,-1,-1), F_{p_{14}} = (1,1,-1,1)$
$F_{p_{15}} = (1,1,1,-1), F_{p_{16}} = (1,1,1,1)$

Table B.6 Computer generated design from Genetic Algorithm for weighted G-optimality criterion cuboidal design, $k = 4$ (Continued)

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	F_{p_9}	$F_{p_{10}}$	$F_{p_{11}}$	$F_{p_{12}}$	$F_{p_{13}}$	$F_{p_{14}}$	$F_{p_{15}}$	$F_{p_{16}}$
		Other															
	18	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
		(-1, -1, -0.9208, 0.0034), (1, -0.0306, 0.9897, 1), (0.1470, 1, 1, 0.8488), (-1, -0.1771, 0.9880, 1), (-0.2260, -1, 1, 0.3511), (1, 0.9544, -1, 0.8414) (-0.3661, 0.0542, -0.1140, -0.1881), (0.8209, -1, 0.0236, 1), (1, 1, -0.1560, -1), (-1, 1, 1, -0.9360), (-1, 0.9805, -0.2607, 0.7749), (0.9420, -1, 1, -1) (1, -0.9492, -1, -0.2422), (-0.6649, -0.9546, -1, 1), (-1, -0.9786, 0.5154, -1), (0.2563, -0.0639, -0.9490, -1), (1, 0.5900, 0.9695, -0.0502)															
	19	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
		(1, 0.0370, 0.0795, -0.0575), (0, -0.1256, -1, -0.0393), (0.1081, -0.9906, 1, 1), (0.5195, -0.0750, -1, 1), (-1, -0.9289, 1, 1), (0.9996, 0.1080, 1, -1) (-0.0547, -0.9637, 0.9207, -0.3264), (-0.9848, 1, 0.9523, 0.0752), (-1, -0.9979, 0, -0.9304), (1, -0.9944, -0.1631, 1), (0.3703, 1, 0.0183, -1) (-1, -0.8744, -1, 0.6251), (1, -0.9929, 1, -0.0423), (-1, 0.8602, -0.0887, 1), (-1, -0.1188, 1, -1), (1, 1, -1, -0.1298), (1, -0.9354, -1, -1)															
	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		(0.9909, 1, 1, -0.0872), (1, -1, -0.0319, -0.1225), (-1, -0.1554, 0.4545, 0.0749), (-1, -0.1489, -1, -0.8926), (-0.8883, 1, -1, 1), (-1, -1, 0.0854, 1) (1, -0.0760, 1, -1), (1, -0.2120, -0.4362, -1), (0.0776, -0.2290, 1, 1), (1, 1, 0.1913, 1), (0.2520, -0.9962, -0.9533, -1), (1, -0.6292, -1, 0.9901) (-1, 1, 1, 0.6335), (0.0629, 0.1573, 0.0896, 0.0041), (1, -1, 1, 0.4628), (1, 1, -1, -0.4371), (-0.0558, 1, 0.3532, -1), (-0.6407, -1, -1, 0.3217) (-1, -1, 1, -0.9561), (-1, 1, 0.0614, -0.9082)															
15	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		(-0.6906, 1, -1, -1), (1, -0.8651, -1, -1), (-0.1467, 0.9998, 1, 1), (-1, 0.9896, 1, -1), (-0.2041, -1, 0.9846, -1), (1, 1, -0.0348, -1), (-1, -1, -1, -0.1714) (-0.1093, -1, -1, 1), (-1, 1, -0.6719, 1), (1, 0.9788, -1, 0.9466), (-1, -0.4360, 0, -1), (1, 0.8311, 1, 0.3367), (1, -1, 0.4401, 1), (-1, -0.8403, 1, 1) (0.1000, 0, -0.9113, 0.9681), (1, -0.4214, 1, -0.8592)															
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		(1, -0.7225, 1, -1), (-0.2366, -1, 0, -1), (-0.0804, -1, 1, 1), (-1, -0.9715, -0.9881, 1), (1, -1, 0.6022, 1), (1, 1, 1, 0.7345), (-1, 1, -0.9872, 0.0823) (1, -0.3350, -1, 1), (-1, 1, 0.2929, -0.2966), (-0.2749, 1, 1, -1), (1, -0.0160, 0.1235, -1), (1, 1, -0.8410, -1), (-1, -1, 1, -0.9116), (-1, 0.4233, 1, 1) (1, -1, -1, -0.8489), (0.1634, 1, -0.9662, 1), (-0.9384, -0.1881, -1, -1)															
	18	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
		(-1, -0.9671, 1, 1), (1, -0.1600, -0.3506, 1), (-0.9767, 1, -1, -0.1849), (0.0538, -1, -1, 1), (0.0408, 0.8315, -1, -1), (1, -1, 0.6978, 0.9878) (-0.9826, 1, 1, -0.8759), (-1, -1, -0.9562, -1), (-1, -0.2309, -1, 1), (-1, 1, 0.1219, 1), (1, 1, -0.0054, -1), (1, 0.8823, -1, 1), (1, 1, 1, 0.3823) (-1, -0.3310, 0.8500, -0.9963), (-0.1123, 0.3478, 1, 1), (1, -0.2672, 1, -0.9774), (0.0617, -1, 0.9292, -1)															
	19	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0
		(1, 1, 0.9337, 1), (-1, 1, 0.0568, -1), (-1, -1, 0, 0.9920), (0.0272, 1, -0.8913, 1), (1, -0.0084, 1, -1), (0.9284, -1, 1, 0.9450), (1, 0.2120, -0.8157, 1) (-1, 1, 1, 0.8419), (-0.7953, -0.5054, 1, 1), (1, -1, -1, 0.6951), (-1, 0.0066, -1, -1), (1, -1, 0.0058, -1), (-0.0964, -1, -1, -1), (0.0626, 1, 1, -1) (-0.0340, -0.0432, 0, 0.3710), (-1, -1, 0.9747, -1), (-1, 1, -1, 0.8815)															
	20	1	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0
		(0.7707, 1, -0.9919, -1.0000), (-1, 1, -0.8613, -0.3016), (0.3200, 1, 0.9553, 1), (-0.0086, -1, 1, -1), (-1, 0.7571, -1, 1), (1, 0.1444, 1, -0.9510) (1, 1, 1, -0.4501), (1, -1, -1, -0.8895), (1, -0.1167, -1, 1), (-0.1685, -1, -1, 1), (-0.9765, 0.5833, 1, 1), (0.0639, 0.2464, -0.9856, -0.9546) (0.9710, -1, -0.0088, -0.6067), (-0.4992, -0.0309, 0.1781, -1), (-1, -1, 1, -0.0354), (1, 1, -0.7917, 1), (-1, -1, 0, 1)															
16	16	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
		(0.0238, 1, 0.6508, 0.6592), (1, -0.5275, 1, 1), (-1, -0.0851, -0.6110, 0), (-1, 1, 0.3590, -1), (-0.4946, -0.7530, 1, -1), (1, 1, 0.9708, -1) (-1, 0.4040, 1, 1), (-1, -1, 0.2292, 1), (0.4421, 0.8722, -1, -1), (-0.1102, -1, 1, 0.1168), (0.9267, -1, -1, 0.7291), (-0.9592, -1, -1, -1) (-0.0653, -0.2624, -0.5199, 1), (1, 1, -0.9242, 0.9062), (1, -0.8424, -0.1705, -1)															
	17	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0

Remark : $F_{p_1} = (-1,-1,-1,-1), F_{p_2} = (-1,-1,-1,1), F_{p_3} = (-1,-1,1,-1), F_{p_4} = (-1,-1,1,1), F_{p_5} = (-1,1,-1,-1), F_{p_6} = (-1,1,-1,1), F_{p_7} = (-1,1,1,-1)$
$F_{p_8} = (-1,1,1,1), F_{p_9} = (1,-1,-1,-1), F_{p_{10}} = (1,-1,-1,1), F_{p_{11}} = (1,-1,1,-1), F_{p_{12}} = (1,-1,1,1), F_{p_{13}} = (1,1,-1,-1), F_{p_{14}} = (1,1,-1,1)$
$F_{p_{15}} = (1,1,1,-1), F_{p_{16}} = (1,1,1,1)$

Table B.6 Computer generated design from Genetic Algorithm for weighted G-optimality criterion cuboidal design, $k = 4$ (Continued)

Combination	N	F_{p_1}	F_{p_2}	F_{p_3}	F_{p_4}	F_{p_5}	F_{p_6}	F_{p_7}	F_{p_8}	F_{p_9}	$F_{p_{10}}$	$F_{p_{11}}$	$F_{p_{12}}$	$F_{p_{13}}$	$F_{p_{14}}$	$F_{p_{15}}$	$F_{p_{16}}$
		Other															
		(1, -1, 0.9664, 0.5948), (-0.2449, -1, 0.7933, -1), (1, 0.1152, 1, -0.7707), (-0.6206, 0.9415, -1, -0.9857), (-1, 1, 0.3139, 0.0140), (1, 1, -1, 0.2443) (0.1719, -0.0296, -0.0768, 0.0607), (-0.9320, 1, -0.7027, 1), (-0.9137, -0.8133, -1, 0.9703), (-0.3122, 1, 1, -0.0723), (-1, -1, -0.7743, -0.9097) (1, 1, 0.2076, -0.9916), (-1, 0.1844, 1, -1), (-1, -0.8325, 1, 1), (0.8150, 0.9184, 0.8155, 1), (1, -0.8672, -0.8849, 1)															
	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		(-1, 1, -1, -0.6180), (0.9694, 0.3744, -0.0808, -0.2252), (0.3416, -0.9927, 1, 0.3155), (0.7872, 1, -1, -1), (1, -1, -0.2873, 1), (-1, 0.2952, 1, -1) (-0.0318, -0.3342, -1, -1), (-1, -1, -1, 0.7018), (-1, -0.9407, 0.9820, 0.7984), (-1, 1, 0.7731, 0.9071), (1, -1, -1, -0.2593), (1, -1, 1, -0.9633) (-0.8184, 0.1841, -0.3680, 1), (0.6624, 1, -1, 1), (-0.9574, -1, -0.1122, -1), (1, 1, 1, -0.9861), (1, 0.5581, 1, 1), (0.0855, 1, 0.8218, -0.6159)															
	19	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0
		(1, 0.3146, -1, -1), (-1, -1, -0.2348, -1), (0.2614, 1, 1, -0.4570), (-1, -0.7637, -1, 1), (1, -0.3228, 0.2017, 0.6318), (0.1064, -1, -1, -0.7028) (-1, -1, 1, 0.2545), (-1, 0.8352, -0.0663, 0.1332), (1, -1, -1, 0.9960), (-0.3064, 0.5754, -0.9562, 0.8101), (1, 1, -1, 0.9867), (0, -1, 1, 1) (0.1056, 0.0613, 0.8446, 1), (0.6896, 1, 0.1917, -1), (-1, 0.1046, 1, -1), (1, 1, 1, 0.6943)															
	20	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0
		(-1, -0.9788, -1, 0.2128), (1, -0.2542, -1, 1), (0.9706, 0.5973, 1, 1), (-1, 1, -0.9666, 1), (1, 1, -1, -0.6138), (-1, 0.8374, -0.8946, -1) (-1, -0.3646, 0.6727, 0.9852), (-1, -1, 0.8756, -1), (0.8310, 1, 0.0808, 1), (-1, 1, 1, 0.0766), (1, -1, 1, -0.1120), (1, 0.2993, 0.4058, -1) (-0.0912, 1, 0.9749, -1), (0.9003, -1, -1, -1), (-0.1487, -1, 0.9464, 1), (1, -0.9977, -0.1352, 1), (-0.9876, -1, -1, -0.0212) (0.1743, 0.0723, -1, -0.1378), (0.1451, -0.0542, 0.0196, -0.2522)															

Remark :$F_{p_1} = (-1,-1,-1,-1), F_{p_2} = (-1,-1,-1,1), F_{p_3} = (-1,-1,1,-1), F_{p_4} = (-1,-1,1,1), F_{p_5} = (-1,1,-1,-1), F_{p_6} = (-1,1,-1,1), F_{p_7} = (-1,1,1,-1)$
$F_{p_8} = (-1,1,1,1), F_{p_9} = (1,-1,-1,-1), F_{p_{10}} = (1,-1,-1,1), F_{p_{11}} = (1,-1,1,-1), F_{p_{12}} = (1,-1,1,1), F_{p_{13}} = (1,1,-1,-1), F_{p_{14}} = (1,1,-1,1)$
$F_{p_{15}} = (1,1,1,-1), F_{p_{16}} = (1,1,1,1)$

APPENDIX C

CATALOG OF COMPUTER GENERATED DESIGNS FROM GENETIC ALGORITHM FOR SPHERICAL DESIGN

Table C.1: Computer generated design from Genetic Algorithm for weighted D-optimality criterion in spherical design, $k = 2$

Combination	**N**	**Near Boundary**	**Near Center**	**Other**
1	6	6	0	
	7	7	0	
	8	8	0	
	9	8	0	$(-0.0386, -1.3842)$
	10	9	0	$(0.9908, -0.9542)$
2	6	5	1	
	7	6	1	
	8	7	1	
	9	8	1	
	10	9	1	
3	6	6	0	
	7	7	0	
	8	8	0	
	9	5	0	$(0.0683, 1.3805), (1.3805, 0.0683)$ $(-1.3393, -0.5526), (-0.5526, -1.3393)$
	10	10	0	
4	6	5	1	
	7	6	1	
	8	5	1	$(1.3088, -0.4165), (-1.3088, -0.4165)$
	9	8	1	
	10	9	1	
5	6	6	0	
	7	7	0	
	8	8	0	
	9	9	0	
	10	9	0	$(0.9482, 1.0050)$
6	6	5	1	
	7	6	1	
	8	7	1	
	9	8	1	
	10	9	1	
7	6	6	0	
	7	7	0	
	8	8	0	
	9	9	0	
	10	10	0	
8	6	5	1	
	7	6	1	
	8	7	1	
	9	8	1	
	10	9	1	
9	6	5	0	$(0.8789, -0.9929)$
	7	6	0	$(-0.8638, -0.9830)$
	8	7	0	$(0.3890, 1.2291)$
	9	8	0	$(0.7059, -1.0363)$
	10	9	0	$(0.2147, -1.1807)$
10	6	5	1	
	7	6	1	
	8	7	1	
	9	8	1	
	10	8	2	
11	6	5	0	$(0.2268, -1.3344)$
	7	6	0	$(-0.3925, -1.2801)$
	8	7	0	$(-0.5615, -1.2007)$
	9	8	0	$(0.9898, -0.8275)$
	10	9	1	
12	6	5	1	
	7	6	1	
	8	7	1	
	9	8	1	
	10	9	1	
13	6	5	0	$(0.9675, -0.9378)$
	7	6	0	$(0.9822, 0.8835)$
	8	7	0	$(-0.7996, 1.0155)$

Remark : Near Boundary refer to 98% of distance ($\sqrt{k}$) and Near Center refer to $(0 \pm 0.1, 0 \pm 0.1)$

Table C.1 Computer generated design from Genetic Algorithm for weighted D-optimality criterion in spherical design, $k = 2$ (Continued)

Combination	N	Near Boundary	Near Center	Other
	9	8	0	$(-1.1969, -0.4490)$
	10	9	1	
14	6	5	1	
	7	6	1	
	8	7	1	
	9	8	1	
	10	8	2	
15	6	5	0	$(0.4798, -1.2720)$
	7	6	0	$(-1.1830, -0.6150)$
	8	7	0	$(-1.1102, 0.7248)$
	9	8	0	$(-0.8111, -1.0366)$
	10	9	1	
16	6	5	1	
	7	6	1	
	8	7	1	
	9	8	1	
	10	8	2	

Remark : Near Boundary refer to 98% of distance ($\sqrt{k}$) and Near Center refer to $(0 \pm 0.1, 0 \pm 0.1)$

Table C.2: Computer generated design from Genetic Algorithm for weighted G-optimality criterion in spherical design, $k = 2$

Combination	N	Near Boundary	Near Center	Other
1	6	6	0	
	7	7	0	
	8	8	0	
	9	9	0	
	10	10	0	
2	6	5	1	
	7	6	1	
	8	7	1	
	9	7	0	$(-0.1015, 0), (0.7828, -0.0796)$
	10	8	0	$(0.1238, 0.2984), (-0.1327, -0.2214)$
3	6	6	0	
	7	7	0	
	8	8	0	
	9	9	0	
	10	10	0	
4	6	5	1	
	7	6	1	
	8	7	1	
	9	8	1	
	10	8	0	$(0.7137, -0.1880), (0.0315, 0.1093)$
5	6	6	0	
	7	7	0	
	8	8	0	
	9	9	0	
	10	10	0	
6	6	5	1	
	7	6	1	
	8	7	1	
	9	7	0	$(-0.0241, 0.1401), (0.0355, -0.7974)$
	10	8	1	$(-0.0547, 0.6384)$
7	6	6	0	
	7	7	0	
	8	8	0	
	9	9	0	
	10	10	0	
8	6	5	1	
	7	6	1	
	8	7	1	
	9	7	0	$(0.1659, -0.0174), (-0.8027, -0.0129)$
	10	8	0	$(-0.4749, -0.1668), (0.1629, 0)$
9	6	6	0	
	7	7	0	
	8	7	1	
	9	8	1	
	10	9	1	
10	6	5	1	
	7	5	0	$(0.0212, 0.2752), (0.8878, -0.5009)$
	8	6	0	$(0.2320, -0.1179), (-0.6942, 0.5782)$
	9	7	0	$(0.1286, 0.0840), (-0.7596, -0.0179)$
	10	8	0	$(-0.0979, -0.3118), (-0.0031, 0.4828)$
11	6	6	0	
	7	6	1	
	8	8	0	
	9	8	1	
	10	9	1	
12	6	5	1	
	7	5	1	$(-0.8180, 0.9034)$
	8	6	0	$(-0.1209, 0), (0.9608, -0.0626)$
	9	7	0	$(0.1233, -0.5298), (-0.1677, 0.5093)$
	10	8	0	$(0.5758, 0.1444), (-0.2117, -0.0580)$
13	6	5	1	
	7	6	1	
	8	8	0	
	9	8	1	

Remark : Near Boundary refer to 98% of distance ($\sqrt{k}$) and Near Center refer to $(0 \pm 0.1, 0 \pm 0.1)$

Table C.2 Computer generated design from Genetic Algorithm for weighted G-optimality criterion in spherical design, $k = 2$ (Continued)

Combination	N	Near Boundary	Near Center	Other
	10	9	1	
14	6	5	1	
	7	5	1	$(0.7721, -0.7653)$
	8	6	1	$(0.6603, -0.6838)$
	9	7	0	$(0.1812, 0.7468), (-0.0708, -0.1074)$
	10	8	0	$(-0.2434, -0.3035), (0.1213, 0.3759)$
15	6	6	0	
	7	7	0	
	8	8	0	
	9	9	0	
	10	9	1	
16	6	5	1	
	7	5	1	$(1.2034, -0.0021)$
	8	6	1	$(-0.0053, 0.9677)$
	9	7	1	$(0.5167, 0.5992)$
	10	8	0	$(-0.0445, -0.3209), (0.0249, 0.5280)$

Remark : Near Boundary refer to 98% of distance ($\sqrt{k}$) and Near Center refer to $(0 \pm 0.1, 0 \pm 0.1)$

Table C.3: Computer generated design from Genetic Algorithm for weighted D-optimality criterion in spherical design, $k = 3$

Combination	N	Near Boundary	Near Center	Other
1	10	10	0	
	11	11	0	
	12	12	0	
	13	13	0	
	14	14	0	
2	10	9	0	$(0, 0.0002, 0.1464)$
	11	10	1	
	12	11	1	
	13	12	1	
	14	13	1	
3	10	10	0	
	11	11	0	
	12	12	0	
	13	13	0	
	14	14	0	
4	10	9	0	$(0.0008, -0.0057, 0.1314)$
	11	10	0	$(0, -0.0137, -0.1394)$
	12	11	1	
	13	12	1	
	14	13	1	
5	10	10	0	
	11	11	0	
	12	12	0	
	13	13	0	
	14	14	0	
6	10	9	1	
	11	10	1	
	12	11	1	
	13	12	1	
	14	13	1	
7	10	10	0	
	11	11	0	
	12	12	0	
	13	13	0	
	14	14	0	
8	10	9	1	
	11	10	1	
	12	11	1	
	13	12	1	
	14	13	1	
9	10	10	0	
	11	10	0	$(-0.1234, -1.6806, -0.1856)$
	12	11	0	$(-0.1129, -1.6478, -0.3688)$
	13	12	0	$(0.3491, 0.0527, 1.6349)$
	14	13	0	$(0.0448, -0.5775, -1.5365)$
10	10	9	1	
	11	10	1	
	12	11	1	
	13	12	1	
	14	13	1	
11	10	9	0	$(-0.1810, 0.6580, -1.5319)$
	11	11	0	
	12	11	0	$(0.5439, 0.8315, 1.3680)$
	13	12	0	$(-0.5226, -0.9705, -1.2901)$
	14	13	0	$(-0.4765, -0.8235, -1.3747)$
12	10	9	1	
	11	10	1	
	12	11	1	
	13	12	1	
	14	13	1	
13	10	9	0	$(-0.2611, -0.4331, 1.6020)$
	11	10	0	$(0.0393, -0.8815, 1.4168)$
	12	11	0	$(0.3850, -1.1513, 1.1775)$

Remark : Near Boundary refer to 98% of distance ($\sqrt{k}$) and
Near Center refer to $(0 \pm 0.1, 0 \pm 0.1, 0 \pm 0.1)$

Table C.3 Computer generated design from Genetic Algorithm for weighted D-optimality criterion in spherical design, $k = 3$ (Continued)

Combination	**N**	**Near Boundary**	**Near Center**	**Other**
	13	12	0	$(0.4426, -0.9542, -1.2916)$
	14	13	0	$(0.0813, 0.3590, -1.6054)$
14	10	9	1	
	11	10	1	
	12	11	1	
	13	12	1	
	14	13	1	
15	10	10	0	
	11	11	0	
	12	12	0	
	13	12	0	$(-0.8331, 0.1774, 1.4547)$
	14	13	0	$(-0.7672, -0.5466, -1.3794)$
16	10	9	1	
	11	10	1	
	12	11	1	
	13	12	1	
	14	13	1	

Remark : Near Boundary refer to 98% of distance ($\sqrt{k}$) and
Near Center refer to $(0 \pm 0.1, 0 \pm 0.1, 0 \pm 0.1)$

Table C.4: Computer generated design from Genetic Algorithm for weighted G-optimality criterion in spherical design, $k = 3$

Combination	N	Near Boundary	Near Center	Other
1	10	10	0	
	11	11	0	
	12	12	0	
	13	13	0	
	14	14	0	
2	10	10	0	
	11	11	0	
	12	11	0	$(-0.3460, 0.0193, 0)$
	13	12	0	$(-0.1667, 0, -0.0184)$
	14	13	1	
3	10	9	0	$(-0.2259, -0.1277, -0.9979)$
	11	11	0	
	12	11	0	$(-0.2589, -0.0277, -1.5687)$
	13	12	0	$(-0.1939, -0.0286, 1.3380)$
	14	14	0	
4	10	10	0	
	11	11	0	
	12	11	0	$(-0.1412, -0.0356, 1.1933)$
	13	12	0	$(0, 0.0932, -0.2044)$
	14	12	0	$(-0.2091, 0.1432, 0.0903)$, $(0.0334, -0.5704, -0.7017)$
5	10	10	0	
	11	11	0	
	12	12	0	
	13	13	0	
	14	14	0	
6	10	9	0	$(0.6006, 0.4294, 0.0467)$
	11	10	0	$(0.2663, 0.5550, 0.0514)$
	12	11	1	
	13	12	1	
	14	13	0	$(-0.0105, -0.1022, 0.0399)$
7	10	10	0	
	11	11	0	
	12	12	0	
	13	13	0	
	14	14	0	
8	10	10	0	
	11	10	0	$(-0.2640, 0.6188, -0.0236)$
	12	11	0	$(0.0932, -0.3895, 0)$
	13	12	0	$(0.2434, 0.5428, 0.2832)$
	14	13	0	$(0.0892, -0.1678, -0.0556)$
9	10	10	0	
	11	11	0	
	12	12	0	
	13	13	0	
	14	14	0	
10	10	9	1	
	11	9	1	$(-0.9580, 0.8131, -0.5217)$
	12	10	0	$(-0.0126, -0.1549, -0.1181)$, $(0.8781, 0.5688, 0.4890)$
	13	11	0	$(-0.0736, -0.0513, -1.0222)$, $(0.0445, 0, 0.1364)$
	14	12	0	$(0.0512, 0.0154, 0.3182)$, $(-0.2621, -0.5700, -0.3530)$
11	10	10	0	
	11	10	0	$(0, 0, -0.3230)$
	12	12	0	
	13	12	0	$(0.1159, 0.1862, -0.0094)$
	14	14	0	
12	10	9	0	$(-0.2653, 0.1347, -0.0304)$
	11	10	1	
	12	11	1	
	13	11	0	$(0.1916, 0.0434, 1.2804)$, $(0.0550, -0.0090, -0.1407)$
	14	12	0	$(-0.5646, 0.2657, 0.0549)$, $(0.4426, -0.1070, -0.2013)$
13	10	10	0	
	11	11	0	
	12	12	0	

Remark : Near Boundary refer to 98% of distance ($\sqrt{k}$) and
Near Center refer to $(0 \pm 0.1, 0 \pm 0.1, 0 \pm 0.1)$

Table C.4 Computer generated design from Genetic Algorithm for weighted G-optimality criterion in spherical design, $k = 3$ (Continued)

Combination	N	Near Boundary	Near Center	Other
	13	13	0	
	14	13	0	$(-0.0401, -0.1003, 0.1293)$
14	10	9	1	
	11	10	1	
	12	10	0	$(-0.7279, -0.7582, -0.0100), (0.4180, 0, -0.0854)$
	13	11	0	$(-0.0412, 0, -0.2018), (0.4230, 0.0337, 0.9777)$
	14	12	0	$(0.1559, 0.7758, -0.3459), (-0.1562, -0.1781, 0.3098)$
15	10	10	0	
	11	10	0	$(-0.2150, -0.0931, 0.1964)$
	12	11	0	$(0.0731, 0.9473, -1.3943)$
	13	12	0	$(-0.1586, -0.0113, -0.1136)$
	14	13	0	$(-0.0746, 0.1529, -0.3355)$
16	10	9	0	$(0.0897, 0.4734, -0.2379)$
	11	10	1	
	12	10	1	$(-1.4817, 0.0944, -0.1953)$
	13	11	1	$(0.8646, 0.8966, 0.1848)$
	14	11	1	$(1.5511, 0.0387, -0.4017), (-0.0611, -1.2100, 0)$

Remark : Near Boundary refer to 98% of distance ($\sqrt{k}$) and
Near Center refer to $(0 \pm 0.1, 0 \pm 0.1, 0 \pm 0.1)$

Table C.5: Computer generated design from Genetic Algorithm for weighted D-optimality criterion in spherical design, $k = 4$

Combination	N	Near Boundary	Near Center	Other
1	16	16	0	
	17	17	0	
	18	18	0	
	19	19	0	
	20	20	0	
2	16	15	0	$(-0.0450, -0.0103, 0.0716, -0.4156)$
	17	16	1	
	18	17	0	$(0.0212, 0, -0.0426, -0.2457)$
	19	18	1	
	20	19	0	$(0.0233, 0.1171, -0.0181, 0.2033)$
3	16	16	0	
	17	17	0	
	18	18	0	
	19	19	0	
	20	20	0	
4	16	15	0	$(-0.0242, 0.1170, -0.0046, -0.0482)$
	17	16	1	
	18	17	1	
	19	18	0	$(0, -0.0115, 0, 0.1332)$
	20	19	0	$(0.0301, -0.0357, 0.0475, 0.1656)$
5	16	16	0	
	17	17	0	
	18	18	0	
	19	19	0	
	20	20	0	
6	16	15	1	
	17	16	1	
	18	17	1	
	19	18	0	$(0.0549, 0, -0.0072, -0.1259)$
	20	19	0	$(0, 0, -0.1085, 0.0098)$
7	16	16	0	
	17	17	0	
	18	18	0	
	19	19	0	
	20	20	0	
8	16	15	1	
	17	16	1	
	18	17	0	$(0.0358, 0.0983, 0.1080, -0.0594)$
	19	18	1	
	20	19	0	$(0, 0.0809, 0.1161, 0)$
9	16	16	0	
	17	17	0	
	18	18	0	
	19	19	0	
	20	20	0	
10	16	15	1	
	17	16	1	
	18	17	1	
	19	18	1	
	20	18	2	
11	16	16	0	
	17	17	0	
	18	18	0	
	19	19	0	
	20	20	0	
12	16	15	1	
	17	16	0	$(0.0734, 0.0321, -0.1138, 0)$
	18	17	1	
	19	18	1	
	20	19	1	
13	16	16	0	
	17	17	0	
	18	18	0	

Remark : Near Boundary refer to 98% of distance ($\sqrt{k}$) and
Near Center refer to $(0 \pm 0.1, 0 \pm 0.1, 0 \pm 0.1, 0 \pm 0.1)$

Table C.5 Computer generated design from Genetic Algorithm for weighted D-optimality criterion in spherical design, $k = 4$ (Continued)

Combination	N	Near Boundary	Near Center	Other
	19	19	0	
	20	19	0	$(0.0699, -0.2032, -0.1289, -1.9394)$
14	16	15	1	
	17	16	1	
	18	17	0	$(0.0099, 0.0037, 0.1243, -0.0197)$
	19	18	1	
	20	19	1	
15	16	16	0	
	17	17	0	
	18	18	0	
	19	19	0	
	20	20	0	
16	16	15	0	$(-0.2253, 0, 0, -0.0372)$
	17	16	0	$(0.1241, -0.0659, -0.0085, 0.0805)$
	18	17	1	
	19	18	1	
	20	19	0	$(0.0585, -0.1039, 0, 0.0600)$

Remark : Near Boundary refer to 98% of distance ($\sqrt{k}$) and
Near Center refer to $(0 \pm 0.1, 0 \pm 0.1, 0 \pm 0.1, 0 \pm 0.1)$

Table C.6: Computer generated design from Genetic Algorithm for weighted G-optimality criterion in spherical design, $k = 4$

Combination	N	Near Boundary	Near Center	Other
1	16	16	0	
	17	17	0	
	18	18	0	
	19	19	0	
2	16	15	1	
	17	16	1	
	18	16	1	(-0.1005, 0.1367, 1.9472, -0.0850)
	19	18	1	
3	16	15	0	(-0.0236, 0.0843, -0.2168, 0.1168)
	17	17	0	
	18	16	0	(0.1485, -0.1642, 1.9050, 0.1875), (0.7777, -0.1331, -1.5429, 0.3790)
	19	19	0	
4	16	14	1	(-0.1017, -0.2345, -0.0792, 0.1375)
	17	16	0	(-0.5618, 1.4905, -0.9894, -0.0539)
	18	16	1	(-0.0461, -0.0475, -0.1504, -0.0579)
	19	17	1	(-0.0493, 0.0385, 1.8886, 0.5059)
5	16	16	0	
	17	17	0	
	18	18	0	
	19	19	0	
6	16	15	0	(-0.1489, -0.1751, 0.2057, 0.1706)
	17	16	0	(0, 0.0681, 0.1520, 0.0800)
	18	17	0	(0, 0.1408, 0.0869, 0)
	19	18	0	(-0.1416, -0.0027, -0.0240, 0.0443)
7	16	15	0	(0.2418, -1.8329, -0.6168, -0.0073)
	17	17	0	
	18	18	0	
	19	19	0	
8	16	15	0	(0, -0.1297, -0.1335, 0.0439)
	17	16	0	(-0.1258, 1.9131, -0.1079, 0.0206)
	18	16	0	(0, 0.0568, -0.0735, 0.4586), (0, -1.2320, -0.6373, 1.3533)
	19	17	1	(1.4872, -0.9959, -0.6403, 0.2024)
9	16	15	0	(0.0779, 0.0105, -0.1883, 0.1854)
	17	17	0	
	18	18	0	
	19	19	0	
10	16	14	0	(0.0920, 0.4963, -0.0983, 0.1090), (-0.7446, 0, 0.3627, -0.0136)
	17	15	0	(-0.0869, -0.1667, 0.0775, 0.3242), (0.0386, 0.1623, 0.0685, 0)
	18	16	0	(0.0217, -0.0062, -0.1109, -0.0662), (0.0092, 0, 0.5155, 0.0776)
	19	17	0	(0.0811, 0.0617, -0.0241, -0.1138), (0.0063, -0.1274, -0.0767, -0.0115)
11	16	15	0	(0, -0.6039, 1.8472, -0.0111)
	17	17	0	
	18	18	0	
	19	19	0	
12	16	13	0	(-0.2896, 0.7877, 1.6605, 0.3659), (-1.2469, -0.3021, 0.3402, -1.4333) (0.0316, -0.0547, 0.1140, 0.0619)
	17	13	1	(0.3304, 0, 1.7514, 0.1044), (1.6473, 0.1144, -0.4203, -0.0314) (-1.4345, 0.6523, 0.1892, -1.0110)
	18	14	1	(-0.0735, 1.3396, 1.2792, 0.0647), (-1.2368, -0.9137, -0.9437, 0) (0, 1.0585, -1.2662, -0.0609)
	19	11	0	(0.1846, 0.0554, -0.0443, -1.3246), (-0.4009, -0.5674, -1.7391, 0.0675) (-0.2385, 1.4391, 0.1265, 1.1541), (1.6285, 0.2790, 0.8313, 0.0108) (-0.2967, 0, 1.9013, 0.0956), (1.6981, -0.1811, -0.8122, 0.0641) (0.0459, -0.0722, 0.0161, 0.1897), (-0.0025, -1.8827, 0.2774, 0.2891)
13	16	16	0	
	17	17	0	
	18	17	0	(0.0675, -0.0228, -0.0749, -0.1185)
	19	19	0	
14	16	13	0	(-0.0176, 0.3528, 1.0561, 0.0484), (0, 0.0211, -0.0512, -0.1171) (-1.2898, -0.0231, -1.3395, 0.5229)
	17	14	0	(0.4383, 0, 1.8742, 0.0568), (-0.0240, 0.0447, -0.1272, 0.2242) (-0.0073, -0.3101, -0.0260, -0.7972)

Remark : Near Boundary refer to 98% of distance ($\sqrt{k}$) and
Near Center refer to $(0 \pm 0.1, 0 \pm 0.1, 0 \pm 0.1, 0 \pm 0.1)$

Table C.6 Computer generated design from Genetic Algorithm for weighted G-optimality criterion in spherical design, $k = 4$ (Continued)

Combination	**N**	**Near Boundary**	**Near Center**	**Other**
	18	16	0	(0.0260, -0.0938, 0.3873, -0.2300), (-0.1756, 0.1116, -0.0427, 0.2845)
	19	16	0	(-0.3077, 0.0327, -0.0974, 0.0874), (0, 0.1045, -0.1024, -0.0079)
				(0, -0.4367, 1.5644, -1.0638)
15	16	15	0	(0, -0.0335, 0.2335, -1.9447)
	17	16	0	(-0.3306, -0.0753, -1.4185, 1.1804)
	18	16	0	(0.0811, 1.2769, 1.4191, -0.3047), (-0.1006, -0.5753, -0.9275, 1.5523)
	19	17	0	(1.9130, 0, -0.2100, 0.3462), (-0.2508, 0.0393, 0.0739, -0.1101)
16	16	14	1	(-0.0940, 1.6616, 1.0316, 0.0129)
	17	15	1	(0, -1.5371, -0.2295, -1.1752)
	18	15	0	(0.0958, -0.0718, -1.5385, -0.0396), (0.0461, 0, 0, 0.1179)
				(-0.0604, -1.4203, 0.3253, -0.8880)
	19	14	0	(0, 0.1030, -0.2004, -0.0099), (0.0567, 0.1499, 1.7448, 0.3284)
				(-1.2414, -0.4691, -1.3510, 0.3388), (-0.0791, 0.1764, 0, 0)
				(-0.4872, 0, 0.4730, -1.7614)

Remark : Near Boundary refer to 98% of distance ($\sqrt{k}$) and
Near Center refer to $(0 \pm 0.1, 0 \pm 0.1, 0 \pm 0.1, 0 \pm 0.1)$

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