SUPPLY CHAIN COORDINATION WITH RISK-AVERSE AGENTS UNDER REFUND-DEPENDENT OR PRICE-DEPENDENT DEMANDS

BY

THANG LOI NGUYEN

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF ENGINEERING (LOGISTICS AND SUPPLY CHAIN SYSTEMS ENGINEERING)

SIRINDHORN INTERNATIONAL INSTITUTE OF TECHNOLOGY

THAMMASAT UNIVERSITY

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A Thesis Presented

By

THANG LOI NGUYEN

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Abstract

SUPPLY CHAIN COORDINATION WITH RISK-AVERSE AGENTS UNDER REFUND-DEPENDENT OR PRICE-DEPENDENT DEMANDS

by

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Bachelor of Engineering (Industrial Management), Can Tho University, 2010.

This thesis examines a 2-stage supply chain that features a buyback contract between a manufacturer and a retailer under uncertain demand and consumer returns policy with partial refund amount. The supply chain is optimized using the utility of profit that includes the mean and variance of profit. The optimal values of buyback price, wholesale price, and retailer's order quantity are determined for the coordination situation of the decentralized supply chain when its members are risk averse. Two patterns of demands are considered including refund-dependent demand and price-dependent demand. Through a computational study, the impacts of the members' risk attitudes, refund amount, and/or retail price on the optimal decisions are investigated under the two patterns of demands for both coordinated and uncoordinated supply chains.

For a coordinated supply chain, the supply chain adjusts the optimal values of order quantity, buyback and wholesale prices to reach the optimal utilities of profits in the supply chain when the risk attitude(s), or refund amount, and/or retail price change. For an uncoordinated supply chain, where one of the agents makes off-optimal decision, the impacts of making such off-optimal decisions are investigated in terms of losses in the expected profit and utility of profit.

Further analysis of the price-dependent demand case shows that there are break-even points of the expected profit and utility values at different retail prices as the risk attitude parameters or refund amount change. The break-even points provide a

guideline on how to adjust the retail price to maximize the utilities of profits as function of other parameters.

Keywords: Customer returns; Supply chain coordination; Buyback contract; Refund-dependent demand; Price-dependent demand; Risk-averse; Utility function.

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Chapter 1
Introduction

1.1 Motivation

Recently, the Customer Returns (CR) policy has been widely accepted and used in the retail industries around the world. It has been shown clearly by the surveys of companies about the returned products. The companies in the United States have permitted the customer returned products' proportion up to 20% (Steger *et al.*, 2007) for electronic devices while this rate in the European Union has been between 2% and 9% (Steger *et al.*, 2007). Besides, Toktay *et al.* (2004) has found a high rate that is 35% with the fashion products. According to Stock *et al.*(2002) and Steger *et al.*(2007), the companies in the United States have paid $100 billion per year for the customers while this cost for the electronics industry is $14 billion every year. The policy promotes the customer demand by allowing customers to return the purchased items to retailers if the items do not meet the customer's needs. This leads to increase in customer demand. However, this policy may increase risks to the retailer. By offering a Buyback Contract (BB) to the retailer, the manufacturer is willing to share these risks by buying unsold items from the retailer at the end of a selling season. Thus, both of the CR policy and the BB have been applied in Supply Chain (SC) to stimulate market demand as well as to reduce the SC's risks.

Our research aims at extending the effectiveness of CR policy, in which a buyback agreement will be considered between a risk-averse manufacturer and a risk-averse retailer. The manufacturer and retailer play a leader and a follower roles in the Stackelberg game, respectively. The utilities of total profit of each member and that of the whole SC are developed for both the centralized and the decentralized SCs. In addition, the uncertain demand is constructed by the combination of stochastic and deterministic components, where the deterministic part is represented by a function depending on refund amount or retail price, and the stochastic component captures the natural variation of demands. In summary, the BB between the risk-averse members with CR policy will be analyzed under mean-variance framework following the Stackelberg sequence. After that, some computational experiments are conducted to investigate the effects of risk attitudes and CR and/or retail price parameters on the performance of the SC.

1.2 Purpose of the study

The main purpose of this thesis is to explore a BB between a risk-averse manufacturer and a risk-averse retailer with return policy. The manufacturer plays a leader role in offering a wholesale price (w) and buyback price (b), while the retailer takes a follower role in setting an order quantity (Q) in the Stackelberg Game. The demand in this thesis is discussed over 2 types consisting of (1) refund-dependent and (2) price-dependent demands. Throughout a computational study, we discuss on the effects of risk-attitudes and CR parameters, and/or retail price parameter on the SC's performance.

1.3 Method of thesis

A SC consisting of both risk-averse members is said to be coordinated if the utilities of profits in the decentralized chain is the same as that of the centralized situation. To coordinate the SC, four steps are performed as follows:

Step 1 The utility of profit in the centralized SC is formulated. The maximum value of this utility is considered as a benchmark for the coordination situation.

Step 2 The utilities of profits in the decentralized SC with BB and CR policy following the Stackelberg sequence are developed as follows:

- The manufacturer firstly sets values for wholesale price w and buyback price b.
- The retailer determines order quantity (or z).

Step 3 The optimal values of decision variables are determined through the coordination mechanism.

Step 4 Computational experiments are conducted to illustrate the effects of risk attitudes in the SC and refund amount, and/or retail price on the optimal order decisions when retail price is exogenous.

1.4 Significance of study

This thesis contributes to the SC coordination with risk-averse members when the demands depend on refund amount or retail price, following the Stackelberg sequence for the decentralized system. The retail price is discussed on exogenous type and the customers will be allowed returning their items with partial refund amount (smaller than retail price). The effects of risk attitude parameters, refund amount, and retail price can provide some guidelines to how a manufacturer and a retailer should adjust their decisions so as to increase the SC utilities.

1.5 The scope of thesis

The customers can return their items to the retailer following "partial return policy," but they will receive a unit price, which is smaller than a price that they paid. The manufacturer only offers a BB for all unsold and returned items to the retailer under a Stackelberg sequence. In addition, the BB's items will not be sold by the manufacturer and it is applied for only one kind of item.

1.6 Structure of thesis

The remainder of this thesis is organized as follows. Literature review will be given in Chapter 2. Chapter 3 provides a problem statement and mathematical formulation for the SC in both centralized and decentralized systems. Computational experiments are conducted to analyze the effects of risk attitude parameters and refund amount (or retail price) on the performance of the SC in Chapter 4. The conclusions and recommendations are described in Chapter 5.

Chapter 2
Literature Review

This thesis is closely associated with the Customer Returns (CR) policy, Buyback Contract (BB) between a manufacturer and a retailer following the sequence of the Stackelberg game, Mean-Variance (MV) analysis and uncertainty demand in the Supply Chain (SC). Overview of the relevant literature is provided as follows.

2.1 The customer returns policy

The CR policy motivates the customer demand by allowing customer to return the purchased items back to retailer if the items do not meet the customer's needs. Therefore, the characteristics of this policy have been analyzed in a number of research studies. For instance, Che (1996) has studied the characteristics of CR policies based on the "experience goods" aspect or the "customer's preference" valuation of many items. It has been examined that the return policies permit customers to postpone the purchasing decisions for items when they achieve a few experience targets. After that, the consumers could return the items when the customers realize their preference valuation of each item is smaller than the unit refund amount offered by the retailer. It has been shown that the return policies could bring many benefits for customers. However, there are some limits when these policies are only applied for the customers, whose risk attitudes are highly risk averse or retail costs are high. Besides, Chen and Bell (2009) have investigated impacts of the CR on decisions-making about price and inventory of the retailer, which faces both deterministic and stochastic demand. They have presented analytical solutions about how the firm could limit the negative effects of the CR for the single-period and the multiple-period problems. Meanwhile, Su (2009) has examined how two policies consisting of full and partial returns influence on the SC performances. It has been illustrated that the SC could not reach the optimization situation via the full return policy. Then, he has suggested how to coordinate the SC with partial returns from the customers. Chen and Bell (2012) has investigated the customers return policies under two kinds similar to those in Su (2009) for finding how a company can increase its profit in a dual-channel structure. Besides, they have found the impacts of the CR on decisions-making and profit of a firm in the case where the market is segmented into a dual-channel structure by this return policy. When both the CR policy, product quality, and the pricing strategy have been considered, Li *et al.* (2013) have found that there are the common and the complementary decisions between them under the direct selling model. The endogenous refund amount and retail price have been also included in the demand function. Furthermore, Hematyar *et al.* (2014) have combined the CR policy with two contract forms between a manufacturer and a retailer, which are a linear sale rebate and a target sale rebate contract. They have found that the SC with these contract forms can achieve the perfect coordination situations. Moreover, other studies have focused on the return policies between the supplier and the customer. Yoo (2014) has investigated an additional penalty contract between the suppliers and the customers to control the suppliers' hidden actions. The relationship between decisions-making for product quality and return policy, and shown the conditions for coordinating the SC have been determined. Moreover, Shi and Xiao (2015) have studied the return policies between the manufacturer and the customers while those between the retailer and the customers are still applying. They have found the conditions for the existences of the return policies. Besides, the effects of the integrated SC and the service subsidy rate on the CR policy have been also determined. Then, this result was used to identify the effects of the decentralized system. In brief, the above articles have still discussed around the CR policy.

2.2 The buyback contract

However, the CR policy can increase the risk to the retailer. To solve this problem, the BB is considered. The BB mainly focused on the relationship between the manufacturer/supplier and the retailer in the classical price-taking newsvendor setting with customers' uncertainty demand (Emmons and Gilbert, 1998). They have shown how the manufacturer can increase the profit by using a multiplicative model of demand when the retail price is increased by the uncertainty. Donohue (2000) has developed the model of Emmons and Gilbert (1998) in exploring the effects of the BB on the coordination situation of the SC with the information dissymmetry setting. It has been found that the manufacturer-distributor SC can be coordinated with a contract to increase the channel's profit. Cachon (2003) has also studied BB with newsvendor setting in finding the mechanism for the SC coordination. Besides, the result has illustrated that there is a performance in a BB better than that in a price-only contract. Bose and Anand (2007) have extended the buyback policy in Cachon (2003) by distinguishing between models with an exogenous wholesale price and those with an endogenous price. This price is determined by a dominant member in the SC. They have found the conditions of wholesale price in considering of whether the equilibrium return policy is or is not Pareto-efficient respect to a price-only contract. Furthermore, Yu-ming (2013) has driven the SC coordination following two main aspects of coordination, which are behavior and interest coordinations. The interest coordination problem has been analyzed by three methods. These methods have been an optimization model in the supplier with constraints from the retailer, Nash bargaining method, and a method based on the balance between revenue and risk. The author has shown that the SC profit can be not only maximized but also coordinated. Meanwhile, Wang and Wang (2013) have set demand as a fuzzy variable in investigating the effects of a BB on the SC coordination. They have found that the parameters of the BB can reach optimal values. After that, the manufacturer-retailer SC has been discussed around the retailer's attitude (Ren *et al.* (2015); Zhang *et al.* (2014)). They have explored the impacts of the BB on the SC coordination. It has been illustrated that the SC with a loss-averse retailer can be coordinated with a BB. Other studies have considered the SC with a manufacturer and multiple independent retailers (Ren *et al.*, 2015). The authors have found the mechanism for coordinating the SC. Throughout numerical examples, it has been proved that the SC with multi-retailer can reach both of the coordination situation and effective profit allocation between members.

2.3 The customer returns policy and the buyback contract

In retail industries, both of the CR policy and the BB have been applied in the SC to stimulate market demand as well as to reduce risks for the SC. The CR policy and the BB in SC have been also recognized and attracted attention from many researchers. First, Su (2009) has studied how the full returns policies and partial returns policies can influence on coordinating of the SC. It has been illustrated that the SC can be coordinated with a BB and the CR policies. However, Shen *et al.* (2009) have examined the manufacturer-retailer SC coordination with a partial return policy. It has been shown that the SC could not be coordinated with the BB following their assumption, where the retail price is endogenous. After that, Chen and Grewal (2013) have examined the effects of a new entrant retailer on the performances of the available SC. This retailer has offered a full-refund or a no-refund amount in their return policies. The objective has been to compare the effectiveness between an established retailer and a new retailer. They have shown that the CR policies are considered as the strategies in a competitive market. Meanwhile, Ruiz-Benitez and Muriel (2014) have modeled and analyzed both of a wholesale-price and a BB for coordinating the SC with a manufacturer and a retailer. They have also found the conditions for maintaining the coordination situation. Besides, Xu *et al.* (2015) have considered the CR policies, in which the consumers' valuation is a function of refund amount and return deadline. After that, a new differentiated BB has been presented independent on the return deadline. They have found that the SC with a new contract can be coordinated by the mechanism as similar as that with a traditional BB.

2.4 Stackelberg game

Even though the BB between the manufacturer/supplier and the retailer could decreased risks, caused by uncertain demand for the retailer, the manufacturer's benefit could be also decreased simultaneously. This could create difficulties in determining the benefits which each member gains when this contract is applied. Stackelberg game settings will be an effective sequence for discussing on the BBs. According to basic theories, the Stackelberg game is a one-period game, in which the demand is known for the same kinds of products between two agents. The agents play different roles in the setting. Specifically, one agent plays as a leader role while the rest is a follower. The follower's decisions are based on the leader's decisions. However, the follower in a Stackelberg game is allowed to observe the leader's strategies before selecting his own decisions. Therefore, the advantage is shown that this game helps decreasing unfairness caused by the expected benefits between two agents. Besides, the Stackelberg game is limited when the dynamic pricing is applied. Yao *et al.* (2008) have used computational methods and the Stackelberg game for identifying the BB properties and to analyze the impacts of price-sensitivity factors on the SC's profit. The endogenous retail price has been also assumed in this analysis. After that, Li *et al.* (2012) have studied performances of centralized and decentralized SC's. Then, they have examined optimal policies consisting of pricing and ordering policy in new fashion and off-season product sales. The optimal decisions have been found by a two-step optimization method in the centralized and the decentralized SC's. Meanwhile, Wu (2013) has compared the expected profits between contracts in vertical integration and Manufacturer's Stackelberg (MS) SC. The channel's profit caused by BB is larger than that of the wholesale price contract in these two competing SCs. Chen and Bell (2013) have contributed to the Stackelberg setting by considering MS and Retailer's Stackelberg (RS) channel. They have found that the MS is desired more than the RS when the amount of return products has been very high and vice-versa in no-returns policy. In addition, Jiang and Liu (2014) have used Stackelberg game method for discussing on the supplier-retailer SC coordination with the BB following three conditions. They have found that the SC with BB can be coordinated if the supplier provides sales incentives or adjusts the wholesale price and the sales effort cost caused by the retailer. Wang and Choi (2014) have established a Stackelberg game model, in which the supplier as leader which sets the wholesale price while the retailer decides the retail price and the order quantity. They have illustrated that the SC with BBs under their settings cannot be coordinated. Then, they have also suggested a scheme to achieve Pareto-improvement of the SC. Moreover, Yoo *et al.* (2015) have explored how the wholesale price contract, BB, and quantity discount contract influence on the retailer's decisions in the entire supplier-retailer SC. These decisions is consisted of pricing, return policy, and the profit. They have found the mechanism for coordinating the SC matching for each contract. Furthermore, the demand depending on both of refund and price have been used by Li *et al.* (2013) and Yoo *et al.* (2015). However, the stochastic part of demand has only related to refund amount and retail price.

2.5 Mean-variance framework

In addition, the risk attitudes have also been considered, recently. Generally, risk-neutral and risk-aversion are two popular risk attitudes discussed by many researchers. It is more complicating when the solutions are found in the risk-averse settings than those in the risk-neutral settings. According to the recent articles published, the MV framework has been frequently used for analyzing performances in the SC, which contains risk-averse agents. For example, Markowitz (1959) has offered a concept that MV analysis is a fundamental and an influential theory for risk management in portfolio investment. The advantage of this analysis has been that the solutions are quickly found when there is certain information about the risk. Therefore, its limitations appear when all input information is uncertain. Then, Agrawal and Seshadri (2000) have considered the important roles of intermediaries in the SCs consisting of a risk-averse retailer and a risk-neutral distributor to reduce the financial risk. The risk-averse retailer can receive various contracts by a risk-neutral distributor for optimizing the order quantity and the expected profit in the SC. Choi *et al.* (2008) have conducted a MV analysis of the SC with a return policy. Through numerical examples, the authors have shown how a return policy can be applied in the SC to achieve the coordination situation and control risks effectively. After that, Yongwen and Yongwu (2011) have used the model of Choi *et al.* (2008) to extend the conceptions about the SC coordination with a BB and risk constraints. The MV analysis has been used for developing the functions, which will contribute how the decentralized SC is coordinated. In addition, Zhang and Yao (2014) have used a MV modeling risk approach to make the optimal close-form solutions in dual-channel SC with return policy. The SC is consisted of a risk-averse manufacturer and a retailer. They have found a model that could support decision-making process in the SC when the retailer faces consumer returns and stochastic demand. Moreover, the MV framework has been extended in consideration of the SC including a manufacturer and multiple retailers. In this case, Li *et al.* (2014) have studied a fast fashion SC coordination with BB and return policy. "Negotiated space" approach has also been recommended for making solutions. Then, they have found that the fast fashion SC containing multi-retailer can be coordinated. Meanwhile, Chiu *et al.* (2015) have explored the SC coordination, which consists of the supplier and the multiple heterogeneous retailers with target sales rebate contract. They have illustrated that the supplier could not maximize his expected profit and support the SC to be coordinated simultaneously.

2.6 Stackelberg game and mean-variance framework

Recently, both of the Stackelberg game setting and MV framework have been presented in the SC models. The fact that, these settings have mainly focused on 2-stage

SC, in which the supplier/manufacturer and the retailer are the leader and the follower, respectively. Wei and Choi (2010) have investigated how to use a wholesale pricing and profit sharing scheme for coordinating the SC with the MV framework and Stackelberg game. They have found that a unique equilibrium of the Stackelberg game exists in the SC. Liu and He (2013) have explored the manufacturer-retailer SC coordination with a wholesale price and a BB. The risk attitudes of two agents have been considered when the retailer faces CR. These risk attitudes have consisted of risk-neutral, risk-averse and risk-preference. They have found that the SC with a BB and full refund amount of return policy can be coordinated. After that, Zhao, Choi, Cheng, and Wang (2014) have conducted a mean-risk analysis in a supplier-retailer SC with the Stackelberg game and a wholesale price contract. They have provided a new perspective in realizing the wholesale price contract's performance. Meanwhile, Zhou *et al.* (2014) have found that the SC with a BB model based on Stackelberg game theory under MV analysis can be coordinated.

2.7 The uncertain demand

Finally, the uncertain or stochastic demand is also an important assumption in considering the SC. This assumption has been popular in many articles related to both of the CR policy, BB, and the integration buyback and return policy. To formulate this uncertainty, the researchers have set demand as functions in various types, for example, inventory-level dependent or price-dependent with addictive, multiplicative, and exponential functions. In the newsvendor problem, Petruzzi and Dada (1999) have examined a demand function consisting of a deterministic and a stochastic part under additive and multiplicative types. Besides, the new parameters have been denoted for overstock and understock notations. Meanwhile, Granot and Yin (2005) have studied the SC coordination with BB under the price-dependent multiplicative demand. They have illustrated that the main results established for multiplicative demand setting could not be extended to those for additive demand setting. After that, Song *et al.* (2008) have extended the assumptions about price-dependent demand functions from Granot and Yin (2005) for developing the BB structure. They has also found that the results with multiplicative demand cannot be fitted for that with additive demand setting when the retail price is endogenous. Besides, Zhao, Choi, Cheng, Sethi, *et al.* (2014) have explored a BB over various demand uncertainty levels in a manufacturer-retailer SC based on the main results from Granot and Yin (2005). It has been found that the manufacturer in practice should adjust only the buyback price for the changes of the demand uncertainty levels. When both of buy back contract, CR, and demand uncertainty are appeared in SC, Chen and Bell (2011) have been found that the SC can be coordinated with an integrated contract and the retail price is assumed as a decision variable. This contract have been established from a profit-sharing agreement and a BB. Other researchers have set demand under an inventory-level dependent function or refund dependent function. For instance, Devangan *et al.* (2013) have further examined the supplier-retailer SC coordination when the retailer faces an inventory-level dependent function. They have found the conditions to ensure that BB always supports the decision-making process in the SC effectively. The refund-dependent demand function has been considered by Xiao *et al.* (2010) and Liu *et al.* (2014). Specifically, Xiao *et al.* (2010) have examined the SC coordination with a buyback/markdown money contract when the refund amount is an exogenous. Meanwhile, Liu *et al.* (2014) have extended the main results of Xiao *et al.* (2010) by considering the case where the refund amount is endogenous. Both of them have found that the SC can be coordinated with an exogenous refund amount.

Chapter 3
Mathematical Formulation

3.1 Problem statement

This thesis follows the problem setting in Liu and He (2013) and Liu *et al.* (2014) to formulate risk-averse agents in 2-stage Supply Chain (SC) with Buyback Contract (BB) and Customer Returns (CR) policy. The manufacturer, whose unit production cost is c, sets the unit wholesale price w $(> c)$ for selling the product to the retailer. The retailer would place an order of Q to the manufacturer and sell the products to the end customers with the exogenous unit retail price p $(> w)$. The retailer offers CR with a unit refund amount r $(< p$, set by the retailer). The CR process can incur if and only if r is larger than the customers' unit preference valuation v. Following Che (1996), $v \in [\underline{v}, \overline{v}]$ is assumed to have a Probability Density Function (PDF) $h(v)$ and a Cumulative Distribution Function (CDF) $H(v)$. Therefore, the probability of CR $\alpha = prob(v < r) = \int_{\underline{v}}^{r} h(v)dv = H(r)$ and the probability for the items kept by the customers is $[1 - H(r)]$. For the presence of CR policy, it is implied that $r \geq \underline{v}$ and $\alpha \in (0,1)$. Meanwhile, the retailer is also fined the unit shortage cost g if the customers demand unsatisfied. Moreover, each return will incur a unit handling cost l_r, to the retailer, l_m to the manufacturer, and $l = l_r + l_m$ for the whole SC, respectively, with $l_m \leq l_r < c$.

At the end of selling season, all unsold and returned items kept by the retailer will be bought by the manufacturer following the BB with a unit buyback price b $(< w)$. The manufacturer can then sell the items at the unit salvage value of unsold products s_1 and the unit salvage value of returned product s_2, and $s_2 < s_1 < c$. According to the conditions for the presence of CR policy, it is assumed that $r \geq \underline{v} \geq s_2$.

The customer's demand X is assumed to consist of a deterministic part $D(*)$ and a random variable ε that reflects the stochastic demand faced by the retailer. Specifically, $X = D(*) + \varepsilon$ while ε is assumed to have a mean of zero and defined over $[B, +\infty$) We denote $f(\varepsilon)$ as PDF and $F(\varepsilon)$ as CDF of ε, with $F(\varepsilon)$ being differentiable, invertible and strictly increasing. Similarly, ε is denoted as the stochastic component of Q, i.e., $Q = D(*) + z$ (Petruzzi and Dada, 1999) such that finding an optimal Q to satisfy the demand X becomes finding an optimal z to satisfy ε. In this thesis, deterministic part $D(*)$ will be considered over two types as follows:

- The first type, $D(*)$ is an increasing function of r (namely $D(r)$), thus $\varepsilon \in [-D(0); +\infty)$.
- The rest type, $D(*)$ is a decreasing function of p, namely $D(p) = a_1 - a_2 p$, where a_1 and a_2 are constant, thus $\varepsilon \in [B; +\infty)$ and $B \geq -D(p)$.

In addition, the unsold inventory ($\Lambda(z)$) and the lack of inventory ($\Theta(z)$) are formulated as $\Lambda(z) = \int_B^z (z-\varepsilon) f(\varepsilon) d\varepsilon$ and $\Theta(z) = \int_z^{+\infty} (\varepsilon - z) f(\varepsilon) d\varepsilon$, respectively.

Finally, the level of risk attitudes in the Mean-Variance (MV) utility function are specified by parameters, k_r, k_m and k for the retailer, the manufacturer and the entire SC, respectively, with $k_r, k_m, k \in (0,1)$ and $k = k_r k_m / (k_r + k_m)$.

Specifically, the process is structured as follows:

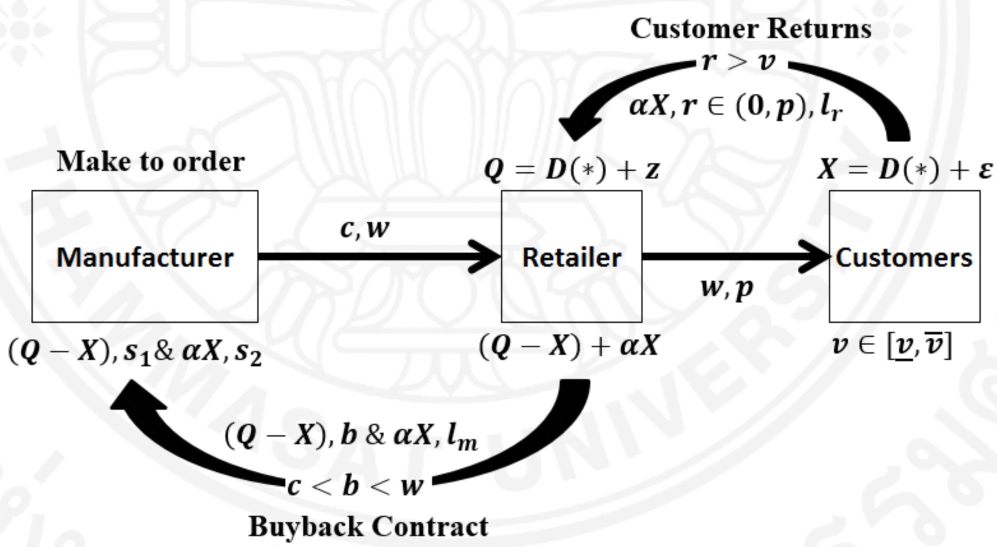

Figure 3.1 The structure of the process.

3.2 Mathematical formulation

The unsold inventory is determined as follows:

$$\Lambda(z) = \int_B^z (z-\varepsilon) f(\varepsilon) d\varepsilon = \int_B^z F(\varepsilon) d\varepsilon \rightarrow \frac{d\Lambda(z)}{dz} = \frac{d \int_B^z F(\varepsilon) d\varepsilon}{dz} = F(z).$$

Let $E[\min(X, Q)]$ denote the expected sales quantity.

The lack of inventory is

$\Theta(z) = E[X - \min(X, Q)] = E[X] - E[\min(X, Q)]..$

$E[\min(X, Q)] = E[X] - \Theta(z) = E[D(*) + \varepsilon] - \Theta(z) = D(*) - \Theta(z).$

Besides, the value of $E[\min(X, Q)]$ can also be identified as follows:

$E[\min(X, Q)] = Q - \Lambda(z) = D(*) + z - \Lambda(z).$

Therefore, $E[\min(X, Q)]$ will be become

$$E[\min(X, Q)] = D(*) + z - \Lambda(z) = D(*) - \Theta(z). \quad (1)$$

From (1), the value of $\Theta(z)$ will become

$\Theta(z) = -z + \Lambda(z) \rightarrow \frac{d\Theta(z)}{dz} = \frac{d(-z+\Lambda(z))}{dz} = -1 + \frac{d\Lambda(z)}{dz} = -1 + F(z).$

$E[\min(X, Q)]^2 = \left(D(*) + z - \Lambda(z)\right)^2 = (D(*) + z)^2 - 2(D(*) + z)\Lambda(z) + \Lambda(z)^2.$

$E[\min(X, Q)^2] = E[\min(D(*) + \varepsilon, D(*) + z)^2] = \int_B^z (D(*) + \varepsilon)^2 f(\varepsilon) d\varepsilon +$
$\int_z^{+\infty} (D(*) + z)^2 f(\varepsilon) d\varepsilon = (D(*) + z)^2 - 2D(*)\Lambda(z) - 2\int_B^z \varepsilon F(\varepsilon) d\varepsilon.$

The variance of the sales quantity is

$$Var[\min(X, Q)] = E[\min(X, Q)^2] - E[\min(X, Q)]^2 = (D(*) + z)^2 - 2D(*)\Lambda(z) - 2\int_B^z \varepsilon F(\varepsilon) d\varepsilon - (D(*) + z)^2 + 2(D(*) + z)\Lambda(z) - \Lambda(z)^2 = 2z\Lambda(z) - 2\int_B^z \varepsilon F(\varepsilon) d\varepsilon - \Lambda(z)^2. \quad (2)$$

The expected demand ($E[X]$) and the variance of demand ($Var[X]$) will be

$$E[X] = E[D(*) + \varepsilon] = E[D(*)] + E[\varepsilon] = D(*) + E[\varepsilon] = D(*). \quad (3)$$

$E[X^2] = E[(D(*) + \varepsilon)^2] = D(*)^2 + 2D(*)E[\varepsilon] + E[\varepsilon^2] = D(*)^2 + E[\varepsilon^2].$

$$Var[X] = E[X^2] - E[X]^2 = E[\varepsilon^2] = \int_B^{+\infty} \varepsilon^2 f(\varepsilon) d\varepsilon \geq 0. \quad (4)$$

The expected order quantity ($E[Q]$) and the variance of order quantity ($Var[Q]$) will become as.

$E[Q] = E[D(*) + z] = \int_B^z (D(*) + z) f(\varepsilon) d\varepsilon + \int_z^{+\infty} (D(*) + z) f(\varepsilon) d\varepsilon = (D(*) + z) \int_B^{+\infty} f(\varepsilon) d\varepsilon = (D(*) + z) F(\varepsilon) \Big|_B^{+\infty} = (D(*) + z)\left(F(+\infty) - F(B)\right) = (D(*) + z)(1 - 0) = D(*) + z = Q.$

$E[Q^2] = E[(D(*) + z)^2] = \int_B^z (D(*) + z)^2 f(\varepsilon) d\varepsilon + \int_z^{+\infty} (D(*) + z)^2 f(\varepsilon) d\varepsilon = (D(*) + z)^2 \int_B^{+\infty} f(\varepsilon) d\varepsilon = (D(*) + z)^2 F(\varepsilon) \Big|_B^{+\infty} = (D(*) + z)^2 \left(F(+\infty) - F(B)\right) = (D(*) + z)^2 (1 - 0) = (D(*) + z)^2 = Q^2.$

$Var[Q] = E[Q^2] - E[Q]^2 = Q^2 - Q^2 = 0.$

3.2.1 The centralized supply chain

The centralized SC is assumed to be under a single ownership where the retailer makes optimal purchase quantity to maximize the total profit of the whole SC.

The channel's profit = the revenue from the products which are sold + the manufacturer's revenue from the customer's returned units which are bought back from the retailer + the manufacturer's revenue from the unsold units which are bought back from the retailer - the manufacturer's production cost - the channel's cost for the returns units – the shortage cost if demand cannot be met.

The profit of the entire SC ($\Pi_C(z)$) is expressed as.

$$\Pi_c(z) = p\min(X,Q) + \alpha s_2 \min(X,Q) + s_1[Q - \min(X,Q)] - cQ - \alpha(r+l)\min(X,Q) - g[X - \min(X,Q)] = (p + \alpha(s_2 - r - l) - s_1 + g)\min(X,Q) - (c - s_1)Q - gX. \quad (5)$$

The expected profit of the entire SC ($E[\Pi_C(z)]$) will be determined as follows:

$$E[\Pi_c(z)] = E[(p + \alpha(s_2 - r - l) - s_1 + g)\min(X,Q) - (c - s_1)Q - gX] = (p + \alpha(s_2 - r - l) - s_1 + g)E[\min(X,Q)] - (c - s_1)Q - gE[X]. \quad (6)$$

Substituting (1) and (3) into (6), the value of $E[\Pi_C(z)]$ will become

$$E[\Pi_c(z)] = (p + \alpha(s_2 - r - l) - s_1 + g)\big(D(*) - \Theta(z)\big) - (c - s_1)\big(D(*) + \Lambda(z) - \Theta(z)\big) - gD(*) = (p + \alpha(s_2 - r - l) - c)\big(D(*) - \Theta(z)\big) - (c - s_1)\Lambda(z) - g\Theta(z). \quad (7)$$

According to Liu *et al.* (2014), the condition $p + \alpha(s_2 - r - l) - c > 0$ is required for getting positive marginal expected profit of the centralized SC.

From (5) and (6), the respective functions can be derived as follows:

$$E[\Pi_c(z)^2] = E\Big[\big((p + \alpha(s_2 - r - l) - s_1 + g)\min(X,Q) - (c - s_1)Q - gX\big)^2\Big] = (p + \alpha(s_2 - r - l) - s_1 +$$

$g)^2 E[\min(X,Q)^2] - 2(p+\alpha(s_2-r-l)-s_1+g)(c-$

$s_1)QE[\min(X,Q)] - 2(p+\alpha(s_2-r-l)-s_1+$

$g)gE[\min(X,Q)]E[X] + (c-s_1)^2Q^2 + 2(c-s_1)gQE[X] +$

$g^2E[X^2]$.

$E[\Pi_c(z)]^2 = \big((p+\alpha(s_2-r-l)-s_1+g)E[\min(X,Q)] - (c-$

$s_1)Q - gE[X]\big)^2 = (p+\alpha(s_2-r-l)-s_1+$

$g)^2E[\min(X,Q)]^2 - 2(p+\alpha(s_2-r-l)-s_1+$

$g)E[\min(X,Q)](c-s_1)Q - 2(p+\alpha(s_2-r-l)-s_1+$

$g)gE[\min(X,Q)]E[X] + (c-s_1)^2Q^2 + 2(c-s_1)QgE[X] +$

$g^2E[X]^2$.

The variance of the profit in the entire SC ($Var[\Pi_c(z)]$) is determined as follows:

$Var[\Pi_c(z)] = E[\Pi_c(z)^2] - E[\Pi_c(z)]^2 = (p+\alpha(s_2-r-l)-s_1+$

$g)^2E[\min(X,Q)^2] + g^2E[X^2] - (p+\alpha(s_2-r-l)-s_1+$

$g)^2E[\min(X,Q)]^2 - g^2E[X]^2 = (p+\alpha(s_2-r-l)-s_1+$

$g)^2Var[\min(X,Q)] + g^2Var[X].$ (8)

From (2) and (4), the variance $Var[\Pi_c(z)]$ in (8) is become as.

$Var[\Pi_c(z)] = (p+\alpha(s_2-r-l)-s_1+g)^2Var[\min(X,Q)] +$

$g^2Var[X] = (p+\alpha(s_2-r-l)-s_1+g)^2\big(2z\Lambda(z) -$

$2\int_B^z \varepsilon F(\varepsilon)d\varepsilon - \Lambda(z)^2\big) + g^2\int_B^{+\infty}\varepsilon^2 f(\varepsilon)d\varepsilon.$ (9)

Under the MV framework, the utility of profit in the entire SC, ($U[\Pi_c(z)]$) is determined as follows:

$U[\Pi_c(z)] = E[\Pi_c(z)] - kVar[\Pi_c(z)] = [p+\alpha(s_2-r-l)-$

$c][D(*) - \Theta(z)] - (c-s_1)\Lambda(z) - g\Theta(z) - k\big[(p+\alpha(s_2-$

$r-l)-s_1+g)^2\big(2z\Lambda(z) - 2\int_B^z \varepsilon F(\varepsilon)d\varepsilon - \Lambda(z)^2\big) +$

$g^2\int_B^{+\infty}\varepsilon^2 f(\varepsilon)d\varepsilon\big].$ (10)

Lemma 1: Following the approach in Liu and He (2013), the optimal value (z_C^*) in the centralized SC, when the utility of profit in the entire SC, $U[\Pi_c(z)]$, is maximized, is determined by solving $dU[\Pi_c(z)]/dz = 0$ as follows:

$(p+\alpha(s_2-r-l)-c+g)-F(z_C^*)(p+\alpha(s_2-r-l)-s_1+g)-$

$$2k(p+\alpha(s_2-r-l)-s_1+g)^2\big(1-F(z_C^*)\big)\Lambda(z_C^*)=0. \quad (11)$$

where $z_C^* \in [B,+\infty)$

Finally, the optimal Q for the centralized SC is given as $Q_C^* = D(*)+z_C^*$.

Proof. See Appendix A.

Lemma 1 implies how to determine the optimal order quantity (or z_C^*), where both of agents of the SC are risk-averse.

3.2.2 The decentralized supply chain with buyback contract

In this section, the manufacturer provides BB to the retailer following the Stackelberg Game's sequence as follows:

- The manufacturer firstly set values for wholesale price w and buyback price b.
- The retailer determines order quantity (or z) based on w and b from the manufacturer.

3.2.2.1 The retailer

The retailer's profit = the retailer's revenue from the products which are sold + the retailer's revenue from the customers' returns units which are returned to the manufacturer + the retailer's revenue from the leftover units which are returned to the manufacturer - the retailer's procurement cost - the cost for the returns products at the retailer – the shortage cost if demand cannot be met.

The profit of the retailer is $\Pi_R(z)$, which is expressed as:

$$\Pi_R(z) = p\min(X,Q)+\alpha b\min(X,Q)+b[Q-\min(X,Q)]-wQ-\alpha(r+l_r)\min(X,Q)-g[X-\min(X,Q)] = (p+\alpha(b-r-l_r)-b+g)\min(X,Q)-(w-b)Q-gX. \quad (12)$$

The retailer's expected profit ($E_D[\Pi_R(z)]$) is determined as follows:

$$E_D[\Pi_R(z)] = E[(p+\alpha(b-r-l_r)-b+g)\min(X,Q)-(w-b)Q-gX] = (p+\alpha(b-r-l_r)-b+g)E[\min(X,Q)]-(w-b)Q-gE[X]. \quad (13)$$

Substituting (1) and (3) into (13), the value of $E_D[\Pi_R(z)]$ will become

$$E_D[\Pi_R(z)] = (p + \alpha(b - l_r - r) - w)\big(D(*) - \Theta(z)\big) - (w - b)\Lambda(z) - g\Theta(z). \quad (14)$$

Likewise, the condition $p + \alpha(b - r - l_r) - w > 0$ is required for getting positive marginal expected profit of the retailer.

From (12) and (13), the respective functions can be derived as follows:

$$E_D[\Pi_R(z)^2] = E\left[\big((p + \alpha(b - r - l_r) - b + g)\min(X, Q) - (w - b)Q - gX\big)^2\right] = (p + \alpha(b - r - l_r) - b + g)^2 E[\min(X, Q)^2] - 2(p + \alpha(b - r - l_r) - b + g)(w - b)QE[\min(X, Q)] - 2(p + \alpha(b - r - l_r) - b + g)gE[X]E[\min(X, Q)] + (w - b)^2 Q^2 + 2(w - b)QgE[X] + g^2 E[X^2].$$

$$E_D[\Pi_R(z)]^2 = \big((p + \alpha(b - r - l_r) - b + g)E[\min(X, Q)] - (w - b)Q - gE[X]\big)^2 = (p + \alpha(b - r - l_r) - b + g)^2 E[\min(X, Q)]^2 - 2(p + \alpha(b - r - l_r) - b + g)E[\min(X, Q)](w - b)Q - 2(p + \alpha(b - r - l_r) - b + g)gE[X]E[\min(X, Q)] + (w - b)^2 Q^2 + 2(w - b)QgE[X] + g^2 E[X]^2.$$

The variance of profit in the retailer ($Var_D[\Pi_R(z)]$) is determined as follows:

$$Var_D[\Pi_R(z)] = E_D[\Pi_R(z)^2] - E_D[\Pi_R(z)]^2 = (p + \alpha(b - r - l_r) - b + g)^2 Var[\min(X, Q)] + g^2 Var[X]. \quad (15)$$

From (2) and (4), the variance $Var_D[\Pi_R(z)]$ in (15) is become as.

$$Var_D[\Pi_R(z)] = (p + \alpha(b - r - l_r) - b + g)^2 Var[\min(X, Q)] + g^2 Var[X] = (p + \alpha(b - r - l_r) - b + g)^2\big(2z\Lambda(z) - 2\int_B^z \varepsilon F(\varepsilon)d\varepsilon - \Lambda(z)^2\big) + g^2 \int_B^{+\infty} \varepsilon^2 f(\varepsilon)d\varepsilon. \quad (16)$$

Under the MV framework, the utility of profit in the retailer ($U_D[\Pi_R(z)]$) is determined as follows:

$$U_D[\Pi_R(z)] = E_D[\Pi_R(z)] - k_r Var_D[\Pi_R(z)] = (p + \alpha(b - l_r - r) - w)\big(D(*) - \Theta(z)\big) - (w - b)\Lambda(z) - g\Theta(z) - k_r\big[(p + \alpha(b - r - l_r) - b + g)^2\big(2z\Lambda(z) - 2\int_B^z \varepsilon F(\varepsilon)d\varepsilon - \Lambda(z)^2\big) + g^2\int_B^{+\infty} \varepsilon^2 f(\varepsilon)d\varepsilon\big]. \quad (17)$$

Lemma 2: Let z_R^* denote the optimal value of z when the utility of profit in the retailer ($U_D[\Pi_R(z)]$) is maximized in the decentralized SC. Therefore, z_R^* is identified by the equation $dU_D[\Pi_R(z)]/dz = 0$.

$$(p + \alpha(b - l_r - r) - w + g) - F(z_R^*)(p + \alpha(b - l_r - r) - b + g) - k_r 2(p + \alpha(b - r - l_r) - b + g)^2\big(1 - F(z_R^*)\big)\Lambda(z_R^*) = 0. \quad (18)$$

where $z_R^* \in [B, +\infty)$

Then, the optimal order quantity (Q_R^*) for the maximum value of $U_D[\Pi_R(z)]$ is determined as $Q_R^* = D(*) + z_R^*$.

Proof. See Appendix B.

Lemma 2 implies how to find the optimal order quantity (or z_R^*), where all agents of the decentralized SC are risk-averse.

3.2.2.2 The manufacturer

The manufacturer's profit = the manufacturer's revenue from selling the products to retailer + the manufacturer's revenue from the unsold products which are bought from the retailer + the manufacturer's revenue from the customers' returns products which are bought from the retailer – the manufacturer's production cost - the lost value from buying back the retailer's unsold product - the lost value from buying back the customers' returned products.

The profit of the manufacturer is $\Pi_M(z)$, which is expressed as

$$\Pi_M(z) = wQ + s_1[Q - \min(X, Q)] + \alpha s_2 \min(X, Q) - cQ - b[Q - \min(X, Q)] - (b + l_m)\alpha \min(X, Q) = (w - c - b + s_1)Q + (b - \alpha(b + l_m - s_2) - s_1)\min(X, Q). \quad (19)$$

The manufacturer expected profit ($E_D[\Pi_M(z)]$) will be determined as follows:

$$E_D[\Pi_M(z)] = E[(w - c - b + s_1)Q + (b - \alpha(b + l_m - s_2) - s_1)\min(X, Q)] = (w - c - b + s_1)Q + (b - \alpha(b + l_m - s_2) - s_1)E[\min(X, Q)]. \quad (20)$$

Substituting (1) and (3) into (20), $E[\Pi_M(z)]$ will become

$$E_D[\Pi_M(z)] = (w - c - b + s_1)[D(*) + z] + (b - \alpha(b + l_m - s_2) - s_1)[D(*) + z - \Lambda(z)] = \left(w - c - \alpha(b + l_m - s_2)\right)[D(*) + z] - (b - \alpha(b + l_m - s_2) - s_1)\Lambda(z). \quad (21)$$

Likewise, the condition $w - c - \alpha(b + l_m - s_2) > 0$ is required for getting positive marginal expected profit of the manufacturer.

From (19) and (20), the respective functions can be derived as follows:

$$E_D[\Pi_M(z)^2] = E\left[\left((w - c - b + s_1)Q + (b - \alpha(b + l_m - s_2) - s_1)\, min(X, Q)\right)^2\right] = (w - c - b + s_1)^2 Q^2 + 2(w - c - b + s_1)(b - \alpha(b + l_m - s_2) - s_1)QE[\min(X, Q)] + (b - \alpha(b + l_m - s_2) - s_1)^2 E[\min(X, Q)^2].$$

$$E_D[\Pi_M(z)]^2 = \left((w - c - b + s_1)Q + (b - \alpha(b + l_m - s_2) - s_1)E[\min(X, Q)]\right)^2 = (w - c - b + s_1)^2 Q^2 + 2(w - c - b + s_1)(b - \alpha(b + l_m - s_2) - s_1)QE[\min(X, Q)] + (b - \alpha(b + l_m - s_2) - s_1)^2 E[\min(X, Q)]^2.$$

The variance of profit in the manufacturer ($Var_D[\Pi_M(z)]$) will be determined as follows:

$$Var_D[\Pi_M(z)] = E_D[\Pi_M(z)^2] - E_D[\Pi_M(z)]^2 = (b - \alpha(b + l_m - s_2) - s_1)^2 Var[\min(X, Q)]. \quad (22)$$

From (2), the variance $Var_D[\Pi_M(z)]$ in (22) is become as:

$$Var_D[\Pi_M(z)] = (b - \alpha(b + l_m - s_2) - s_1)^2 Var[\min(X, Q)] = (b - \alpha(b + l_m - s_2) - s_1)^2\left(2z\Lambda(z) - 2\int_B^z \varepsilon F(\varepsilon)d\varepsilon - \Lambda(z)^2\right). \quad (23)$$

Under the MV framework, the utility of profit in the manufacturer ($U_D[\Pi_M(z)]$) will be determined as follows:

$$U_D[\Pi_M(z)] = E_D[\Pi_M(z)] - k_m Var_D[\Pi_M(z)] = \big(w - c - \alpha(b + l_m - s_2)\big)(D(*) + z) - \big(b - s_1 - \alpha(b + l_m - s_2)\big)\Lambda(z) - k_m(b - \alpha(b + l_m - s_2) - s_1)^2\big(2z\Lambda(z) - 2\int_B^z \varepsilon F(\varepsilon)d\varepsilon - \Lambda(z)^2\big). \quad (24)$$

3.2.3 Coordination mechanism via buyback contract

<u>Proposition 1:</u> When the partial refund amount is offered, the optimal buyback price and wholesale price cannot be found to coordinate the SC.

Proof. See Appendix C.

Proposition 1 indicates that there is no solution for coordinating the SC when the unit shortage cost is presented in our assumption, where the retail price is exogenous.

When the unit shortage cost g for unsatisfying the customer demand is not considered ($g = 0$). A result will be shown in the Proposition 2.

<u>Proposition 2:</u> When the unit shortage cost g for unsatisfying the customer demand is not considered ($g = 0$). The SC can be coordinated with the value of the optimal wholesale price (w_c) and the optimal buyback price (b_c) as (25) and (26) below.

$$w_c = -\alpha(s_2 - l_m - b_c)\big(1 - F(z^*)\big) + c + F(z^*)[b_c - s_1] + 2\big(1 - F(z^*)\big)(k[p + \alpha(s_2 - r - l_r - l_m) - s_1]^2 - k_r[p + \alpha(b_c - l_r - r) - b_c]^2)\Lambda(z^*). \quad (25)$$

$$b_c = \big[[\alpha(l_m - s_2) + s_1]k_m + [p - \alpha(l_r + r)]k_r\big]/[(k_m + k_r)(1 - \alpha)]. \quad (26)$$

Proof. See Appendix D.

Proposition 2 provides the conditions for coordinating the SC, where the retail price is exogenous. The results will be analyzed through a computational study. According to Proposition 2, some results are shown as follows:

- The optimal buyback price (b_c) is the increasing functions of the retailer's risk attitude (k_r), refund amount (r), and retail price (p), and a decreasing function of the manufacturer's risk attitude (k_m).
- The optimal wholesale price (w_c) is an increasing function of b_c.

Proof. See Appendix E.

3.2.3.1 The results in case the refund-dependent demand

According to the results in Proposition 2, the optimal buyback price (b_c) and optimal wholesale price (w_c) are decreased by increasing k_m. Besides, it is also illustrated that increasing k_m leads to decreases in the optimal order quantity (Q^*), the optimal expected profit and the utility of profit in the entire SC ($E[\Pi_c(z^*)]$ and $[\Pi_c(z^*)]$) when k_r and r are fixed. Meanwhile, when r and k_m are fixed, increasing k_r leads to increases in b_c and w_c (see Proposition 2), and decreases in Q^*, $E[\Pi_c(z^*)]$, and $U[\Pi_c(z^*)]$. The SC will lose higher expected profit and utility of profit when the risk, that the retailer or/and the manufacturer face(s), is (are) higher.

In addition, from Proposition 2, b_c, w_c and Q^* are increased when increasing refund amount (r) and fixing k_r and k_m. In this kind, the increases of $E[\Pi_c(z^*)]$, and $U[\Pi_c(z^*)]$ are shown by a computational study. Increasing refund amount leads to increase in customers demand. Therefore, the SC will increase the optimal order quantity, which helps to increase both the optimal expected profit and the utility of profit in the entire SC.

3.2.3.2 The results in case the price-dependent demand

According to the results in Proposition 2, the optimal buyback price (b_c) and optimal wholesale price (w_c) are decreased by increasing k_m. Besides, increasing k_m leads to decreases in the optimal order quantity (Q^*), the optimal expected profit and the utility of profit in the entire SC ($E[\Pi_c(z^*)]$ and $[\Pi_c(z^*)]$) when k_r, p, and r. are fixed. Meanwhile, as r, p, and k_m are fixed, increasing k_r leads to increases in b_c and w_c (see Proposition 2), and decreases in Q^*,

$E[\Pi_c(z^*)]$, and $U[\Pi_c(z^*)]$. The SC will be lost higher expected profit and utility of profit when the risk, that the retailer or/and the manufacturer face(s), is (are) higher.

In addition, from Proposition 2, b_c, w_c and Q^* are increased when increasing retail price (p) and fixing r, k_r, and k_m. It means that the effects of k_m and k_r on b_c, w_c, and Q^* are similar trend when increasing p. However, the effects of k_m and k_r on the expected profits and the utilities of profits, in which p is not fixed, will be discussed in the numerical examples. Besides, it is also discussed on the effects of r on the optimal parameters b_c, w_c, Q^*, and both of the expected profits and the utilities of profits in the SC via these numerical examples.

Chapter 4
Computational Study

4.1 Refund-dependent demand

In this computational study, similar to Liu *et al.* (2014), the parameters in the model are chosen as: $p = 10$, $c = 2$, $l_m = 1$, $l_r = 1$, $l = l_r + l_m = 2$, $s_1 = 1.5$, and $s_2 = 1$. The customers' unit preference valuation is assumed to follow a uniform distribution $v \sim U[1, p]$. This leads to the probability of a customer returning items $\alpha = prob(v < r) = \int_{\underline{v}}^{r} h(v)dv = H(r) = (r-1)/(p-1)$.

The demand consists of [1] deterministic component $D(r) = 10 + 20r \rightarrow B = -D(0) = -10$ and [2] stochastic component that follows a uniform distribution $\varepsilon \sim U[-D(0), \overline{B}]$, where $\overline{B}$ is the upper bound of ε. This leads to $f(\varepsilon) = 1/(\overline{B} + D(0))$ and $F(\varepsilon) = (\varepsilon + D(0))/(\overline{B} + D(0))$. It is implied that the stochastic component of the order quantity, z, also follows the same distribution, $z \sim U\left[-D(0), \overline{B}\right]$, $f(z) = 1/(\overline{B} + D(0))$ and $F(z) = (z + D(0))/(\overline{B} + D(0))$. From $B = -D(0) = -10$, setting $\overline{B} = 10$ lead to $\varepsilon \sim U[-10,10]$ and $z \sim U[-10,10]$. It follows that the overstock amount, $\Lambda(z) = \int_{-10}^{z} (z - \varepsilon)\, 1/20 d\,\varepsilon = 1/40(z + 10)^{2}$, the understock amount, $\Theta(z) = \int_{z}^{10} (\varepsilon - z)\, 1/20 d\,\varepsilon = 1/40(z - 10)^{2}$, and $Var[\min(X, Q)] = -((3z - 50)(z + 10)^3)/4\,800$.

The purpose of the computational study is to check the effects of the refund amount r and risk attitude parameters, k_r and k_m, on the optimal order quantity, the expected profits and the utility values, respectively. To perform a thorough experiment, many combinations of (r, k_r, k_m) values are tested as follows: $r \in [0,1,2,3,4,5\ ]$; k_r and $k_m \in [0.025, 0.05, \ldots, 0.975]$. There are six, 39, and 39 possible values for these parameters, respectively; thus, the total number of combinations is 9,126.

From (11), the optimal value z_C^* in the centralized Supply Chain (SC) can be derived from $(z_C^* + 10)^2 = 0$. Hence, the optimal order Q_C^* in the centralized SC is given as $Q_C^* = 10 + 20r + z_C^*$.

Therefore, from (10), the utility function of total profit $U[\Pi_c(z)]$ of the centralized SC can be determined as

$U[\Pi_c(z_C^*)] = E[\Pi_c(z_C^*)] - kVar[\Pi_c(z_C^*)] = [p + \alpha(s_2 - r - l) - c][10 + 20r - 1/40(z_C^* - 10)^2] - (c - s_1)1/40(z_C^* + 10)^2 - k[(p + \alpha(s_2 - r - l) - s_1)^2 - (3z_C^* - 50)(z_C^* + 10)^3/48\,00].$

Likely, from (18), the optimal value z_R^* in the decentralized SC can be derived from $(z_R^* + 10)^2 = 0$. Hence, the optimal order Q_R^* in the centralized SC is given as $Q_R^* = 10 + 20r + z_R^*$.

Therefore, from (10), the utility function of total profit $U[\Pi_c(z)]$ of the retailer and the manufacturer in the decentralized SC can be determined respectively.

$U_D[\Pi_R(z_R^*)] = E_D[\Pi_R(z_R^*)] - k_r Var_D[\Pi_R(z_R^*)] = (p + \alpha(b - l_r - r) - w)(10 + 20r - 1/40\,(z_R^* - 10)^2) - (w - b)1/40(z_R^* + 10)^2 - k_r[(p + \alpha(b - r - l_r) - b)^2(3z_R^* - 50)(z_R^* + 10)^3/48\,00].$

$U_D[\Pi_M(z_R^*)] = E_D[\Pi_M(z_R^*)] - k_m Var_D[\Pi_M(z_R^*)] = \left(w - c - \alpha(b + l_m - s_2)\right)(10 + 20r + z_R^*) - \left(b - s_1 - \alpha(b + l_m - s_2)\right)1/40(z_R^* + 10)^2 - k_m(b - \alpha(b + l_m - s_2) - s_1)^2(3z_R^* - 50)(z_R^* + 10)^3/48\,00.$

There are two scenarios developed in this computational study for the coordinated and non-coordinated SC's.

4.1.1 Scenario 1: The supply chain is coordinated

When the SC is coordinated:

- According to (26), the optimal buyback price (b_c) is a decreasing function of k_m and increasing functions of k_r and r.
- Following (25), the optimal wholesale price (w_c) is an increasing function of b_c. Therefore, w_c is also a decreasing function of k_m and increasing functions of k_r and r.
- The utilities of profit of the manufacturer and the retailer in the decentralized SC are combined to be equal to that of the centralized SC.

In this scenario, the impacts of k_m, k_r and r on the optimal decisions w_c, b_c, Q_c, and optimal SC performance $E[\Pi_R]$, $E[\Pi_M]$, $E[\Pi_c]$, $U[\Pi_R]$, $U[\Pi_M]$, and $U[\Pi_c]$ are examined through numerical examples. Note that the finding in all 9,126 cases follow the same trends shown in the given numerical examples.

4.1.1.1 The impacts of the manufacturer's risk-attitude

An example of results to demonstrate the impacts of k_m, where $k_r = 0.15$, $r = 3$ and varying $k_m \in [0.025, 0.975]$, can be found in Table 4.1 (only partial results are shown to save space), and in Figures 4.1 - 4.4 as follows:

Table 4.1 The impacts of k_m on the optimal values in the supply chain coordination.

k_m	Q_c	b_c	w_c	$E[\Pi_M]$	$E[\Pi_R]$	$E[\Pi_C]$	$U[\Pi_M]$	$U[\Pi_R]$	$U[\Pi_C]$
0.025	-	-	-	-	-	-	-	-	-
0.050	67.90	9.27	9.39	353.24	117.75	470.98	343.81	114.60	458.41
0.075	66.88	8.45	8.62	311.05	155.52	466.57	303.28	151.64	454.92
0.100	66.29	7.80	8.00	278.33	185.55	463.88	271.71	181.14	452.85
0.125	65.91	7.27	7.49	252.03	210.02	462.05	246.26	205.21	451.47
0.150	65.64	6.82	7.07	230.36	230.36	460.73	225.24	225.24	450.48
0.175	65.44	6.45	6.71	212.18	247.54	459.72	207.57	242.16	449.73
0.200	65.28	6.12	6.41	196.68	262.24	458.93	192.49	256.65	449.14
0.225	65.16	5.84	6.14	183.31	274.97	458.29	179.47	269.20	448.67
0.250	65.06	5.60	5.91	171.66	286.10	457.76	168.10	280.17	448.28
0.275	64.97	5.38	5.71	161.40	295.91	457.31	158.10	289.85	447.95
0.300	64.90	5.19	5.52	152.31	304.62	456.93	149.22	298.45	447.67
0.325	64.84	5.02	5.36	144.19	312.41	456.61	141.29	306.14	447.43
0.350	64.78	4.86	5.21	136.90	319.42	456.32	134.17	313.06	447.22
0.375	64.73	4.72	5.08	130.31	325.76	456.07	127.73	319.31	447.04
0.400	64.69	4.60	4.96	124.32	331.52	455.85	121.88	325.00	446.88
0.425	64.66	4.48	4.85	118.86	336.78	455.65	116.54	330.19	446.73
0.450	64.62	4.38	4.75	113.87	341.60	455.47	111.65	334.95	446.60
0.475	64.59	4.28	4.66	109.27	346.03	455.31	107.16	339.33	446.49
0.500	64.56	4.19	4.57	105.04	350.12	455.16	103.01	343.37	446.38
0.525	64.54	4.10	4.49	101.12	353.91	455.03	99.17	347.11	446.28
0.550	64.52	4.03	4.42	97.48	357.43	454.91	95.61	350.58	446.19
0.575	64.50	3.95	4.35	94.10	360.70	454.79	92.30	353.81	446.11
0.600	64.48	3.89	4.29	90.94	363.75	454.69	89.21	356.83	446.04
0.625	64.46	3.82	4.23	87.99	366.61	454.59	86.32	359.65	445.97
0.650	64.44	3.76	4.17	85.22	369.29	454.51	83.61	362.30	445.90
0.675	64.43	3.71	4.12	82.62	371.80	454.42	81.06	364.78	445.84

k_m	Q_c	b_c	w_c	$E[\Pi_M]$	$E[\Pi_R]$	$E[\Pi_C]$	$U[\Pi_M]$	$U[\Pi_R]$	$U[\Pi_C]$
0.700	64.41	3.66	4.07	80.18	374.17	454.35	78.67	367.12	445.79
0.725	64.40	3.61	4.02	77.88	376.40	454.27	76.41	369.32	445.74
0.750	64.39	3.56	3.98	75.70	378.50	454.21	74.28	371.41	445.69
0.775	64.38	3.52	3.93	73.64	380.50	454.14	72.27	373.38	445.64
0.800	64.37	3.47	3.89	71.70	382.39	454.08	70.36	375.24	445.60
0.825	64.35	3.43	3.86	69.85	384.18	454.03	68.55	377.01	445.56
0.850	64.35	3.40	3.82	68.10	385.88	453.97	66.83	378.69	445.52
0.875	64.34	3.36	3.79	66.43	387.49	453.92	65.19	380.29	445.48
0.900	64.33	3.33	3.76	64.84	389.04	453.87	63.64	381.81	445.45
0.925	64.32	3.29	3.72	63.33	390.50	453.83	62.15	383.27	445.42
0.950	64.31	3.26	3.69	61.88	391.91	453.79	60.73	384.65	445.39
0.975	64.30	3.23	3.67	60.50	393.25	453.75	59.38	385.98	445.36

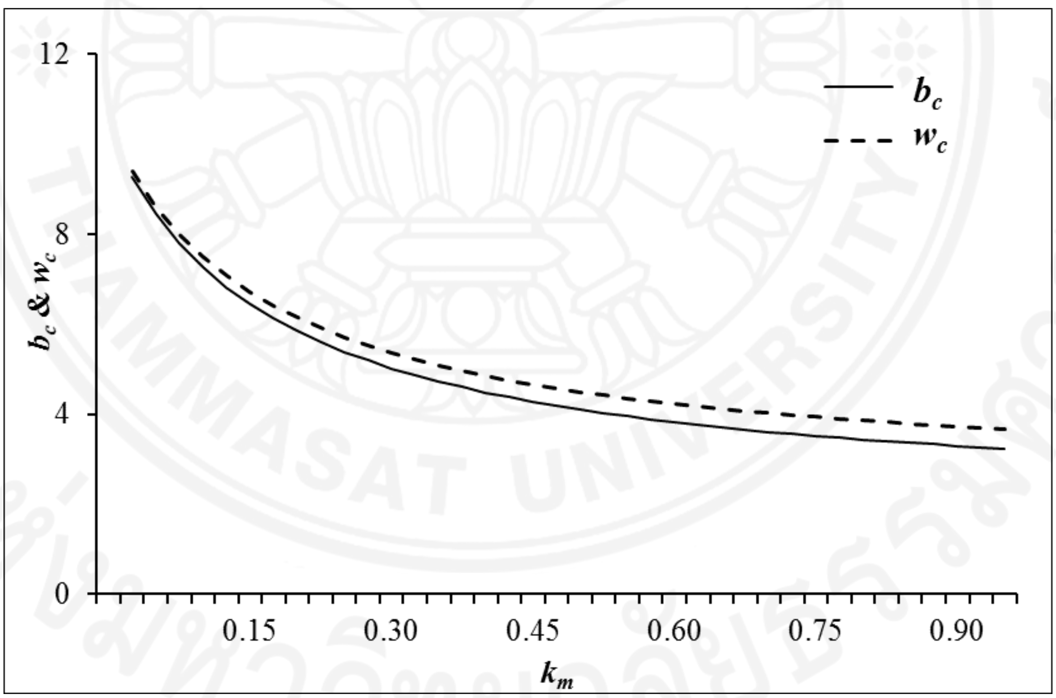

Figure 4.1 The impacts of k_m on w_c, b_c.

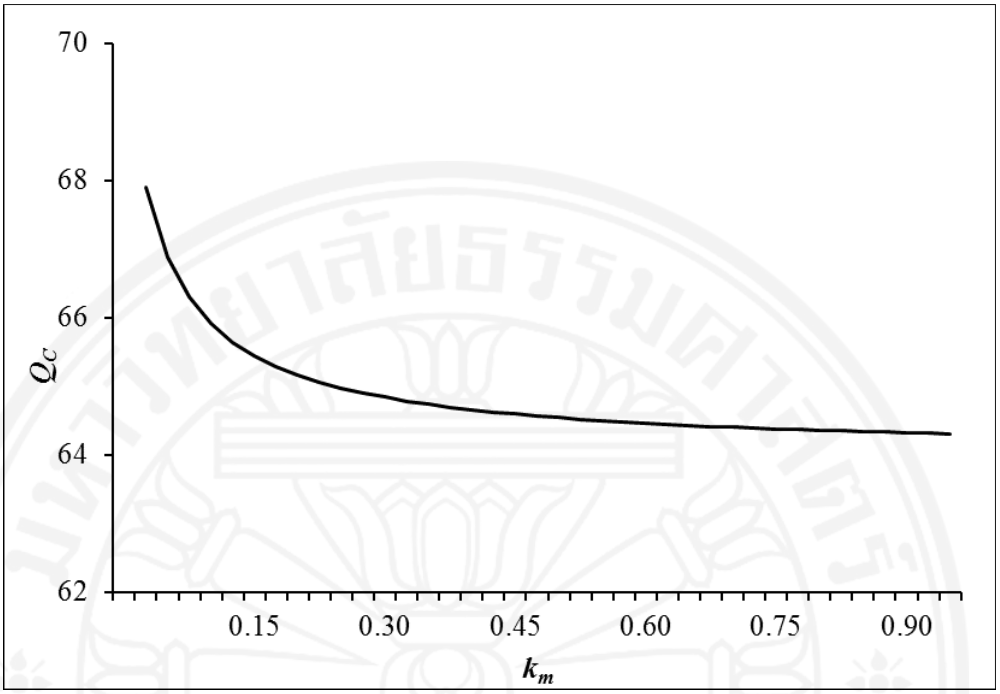

Figure 4.2 The impacts of k_m on Q_c.

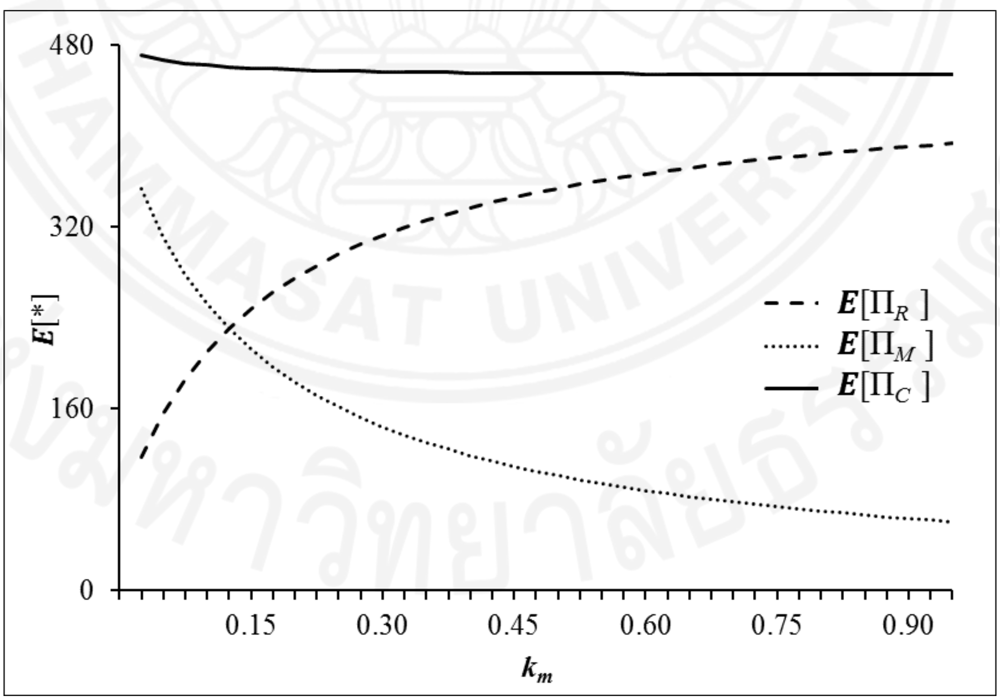

Figure 4.3 The impacts of k_m on the optimal expected profits $E[\Pi_R]$, $E[\Pi_M]$, $E[\Pi_c]$.

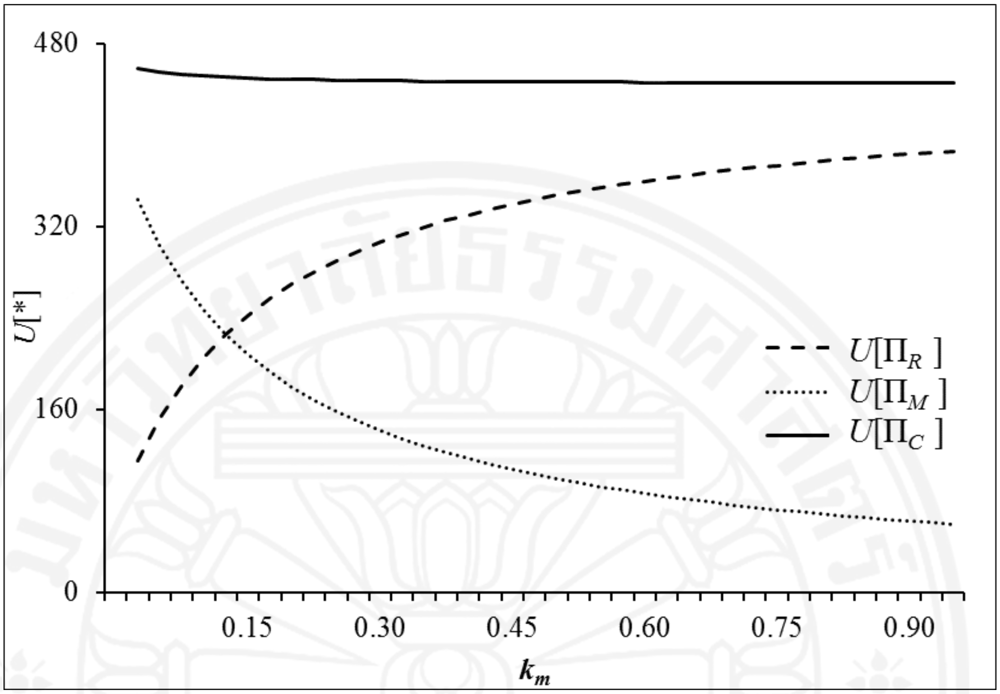

Figure 4.4 The impacts of k_m on the optimal utilities of profits $U[\Pi_R]$, $U[\Pi_M]$, $U[\Pi_c]$.

When k_r and r are fixed, increasing k_m, which indicates that the manufacturer decides to put more emphasis on risk in their decision making, leads to decreases in both the optimal buyback price (b_c) and wholesale price (w_c). As the manufacturer increases k_m to protect themselves more against risks, it follows that the rate of decreases in b_c is steeper than the rate of decrease in w_c, see Figure 4.1. As a consequence, it leads to a decrease in the optimal order quantity (Q_c) from the retailer, see Figure 4.2.

Besides, increasing k_m leads to decreases in both the optimal expected profits and the utilities of profits for the manufacturer ($E[\Pi_M]$ and $U[\Pi_M]$) and the entire SC ($E[\Pi_C]$ and $U[\Pi_c]$). Because the decreasing rates in both $E[\Pi_M]$ and $U[\Pi_M]$ are larger than the decreasing rate in $E[\Pi_C]$ and $U[\Pi_c]$, both the optimal expected profits and the utilities of profits in the retailer ($E[\Pi_R]$ and $U[\Pi_R]$) show increasing trends, see Figures 4.3 – 4.4. This indicates that as the manufacturer becomes more aware of risk, and thus, raising their risk attitude, the optimal decision would benefit the retailer more than the manufacturer. This is due to the condition that the SC is always coordinated.

4.1.1.2 The impacts of the retailer's risk-attitude

Using a similar approach, k_m and r are fixed ($k_m = 0.2$, $r = 3$), while varying $k_r \in [0.025, 0.975]$, the results are shown in Table 4.2 and Figures 4.5 - 4.8.

Table 4.2 The impacts of k_r on the optimal values in the supply chain coordination.

k_r	Q_c	b_c	w_c	$E[\Pi_M]$	$E[\Pi_R]$	$E[\Pi_C]$	$U[\Pi_M]$	$U[\Pi_R]$	$U[\Pi_C]$
0.025	70.12	3.02	3.46	53.24	425.92	479.16	51.71	413.64	465.35
0.050	67.66	3.89	4.29	94.00	375.98	469.98	91.52	366.08	457.60
0.075	66.59	4.60	4.96	126.89	338.38	465.27	123.80	330.12	453.92
0.100	65.98	5.19	5.52	154.12	308.25	462.37	150.57	301.14	451.71
0.125	65.57	5.69	6.00	177.07	283.31	460.38	173.16	277.06	450.22
0.150	65.28	6.12	6.41	196.68	262.24	458.93	192.49	256.65	449.14
0.175	65.07	6.50	6.76	213.65	244.17	457.81	209.22	239.10	448.32
0.200	64.90	6.82	7.07	228.47	228.47	456.93	223.84	223.84	447.67
0.225	64.76	7.11	7.34	241.53	214.69	456.22	236.72	210.42	447.15
0.250	64.65	7.37	7.59	253.12	202.50	455.62	248.17	198.54	446.71
0.275	64.56	7.59	7.80	263.49	191.63	455.12	258.41	187.94	446.35
0.300	64.48	7.80	8.00	272.81	181.88	454.69	267.62	178.42	446.04
0.325	64.41	7.99	8.18	281.24	173.07	454.32	275.95	169.82	445.77
0.350	64.35	8.16	8.34	288.91	165.09	453.99	283.52	162.01	445.53
0.375	64.30	8.31	8.48	295.90	157.81	453.71	290.43	154.90	445.33
0.400	64.25	8.45	8.62	302.30	151.15	453.45	296.76	148.38	445.14
0.425	64.21	8.58	8.74	308.19	145.03	453.23	302.59	142.39	444.98
0.450	64.17	8.70	8.86	313.63	139.39	453.02	307.96	136.87	444.83
0.475	64.14	8.81	8.96	318.66	134.17	452.84	312.94	131.76	444.70
0.500	64.11	8.92	9.06	323.33	129.33	452.67	317.56	127.02	444.58
0.525	64.08	9.01	9.15	327.68	124.83	452.51	321.86	122.61	444.47
0.550	64.05	9.10	9.24	331.74	120.63	452.37	325.87	118.50	444.37
0.575	64.03	9.19	9.32	335.53	116.71	452.24	329.62	114.65	444.27
0.600	64.01	9.27	9.39	339.09	113.03	452.12	333.14	111.05	444.19
0.625	63.99	9.34	9.46	342.43	109.58	452.01	336.45	107.66	444.11

k_r	Q_c	b_c	w_c	$E[\Pi_M]$	$E[\Pi_R]$	$E[\Pi_C]$	$U[\Pi_M]$	$U[\Pi_R]$	$U[\Pi_C]$
0.650	63.97	9.41	9.53	345.58	106.33	451.91	339.56	104.48	444.04
0.675	63.95	9.48	9.59	348.54	103.27	451.81	342.49	101.48	443.97
0.700	63.94	9.54	9.65	351.34	100.38	451.72	345.26	98.64	443.90
0.725	63.92	9.60	9.71	353.99	97.65	451.64	347.88	95.97	443.84
0.750	63.91	9.65	9.76	356.49	95.07	451.56	350.36	93.43	443.79
0.775	63.90	9.71	9.81	358.87	92.61	451.48	352.71	91.02	443.73
0.800	63.88	9.76	9.86	361.13	90.28	451.42	354.95	88.74	443.68
0.825	63.87	9.80	9.90	363.28	88.07	451.35	357.07	86.56	443.64
0.850	63.86	9.85	9.95	365.33	85.96	451.29	359.10	84.49	443.59
0.875	63.85	9.89	9.99	367.28	83.95	451.23	361.03	82.52	443.55
0.900	-	-	-	-	-	-	-	-	-

From Table 4.2, the impacts of k_r on the optimal values when the SC is coordinated will be shown specifically through Figures 4.5 - 4.8 as follows:

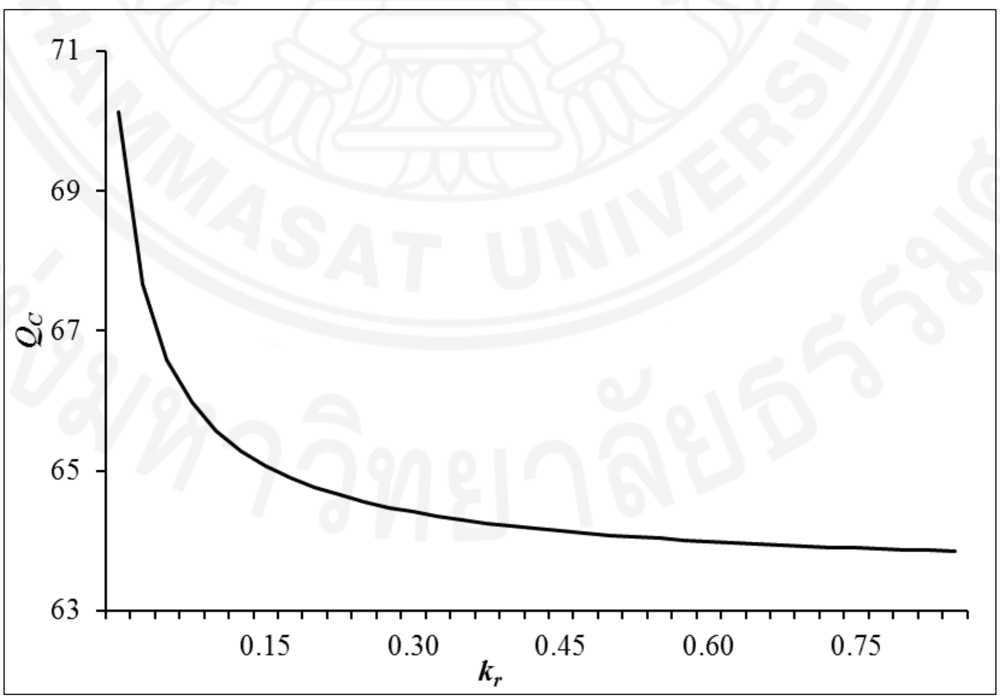

Figure 4.5 The impacts of k_r on Q_C.

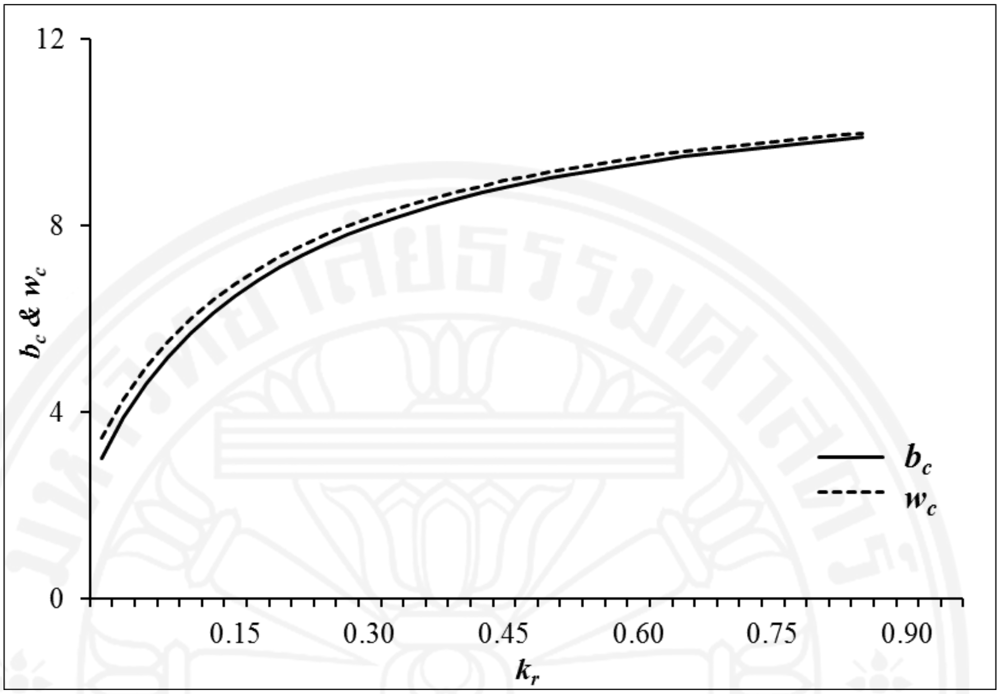

Figure 4.6 The impacts of k_r on w_c, b_c.

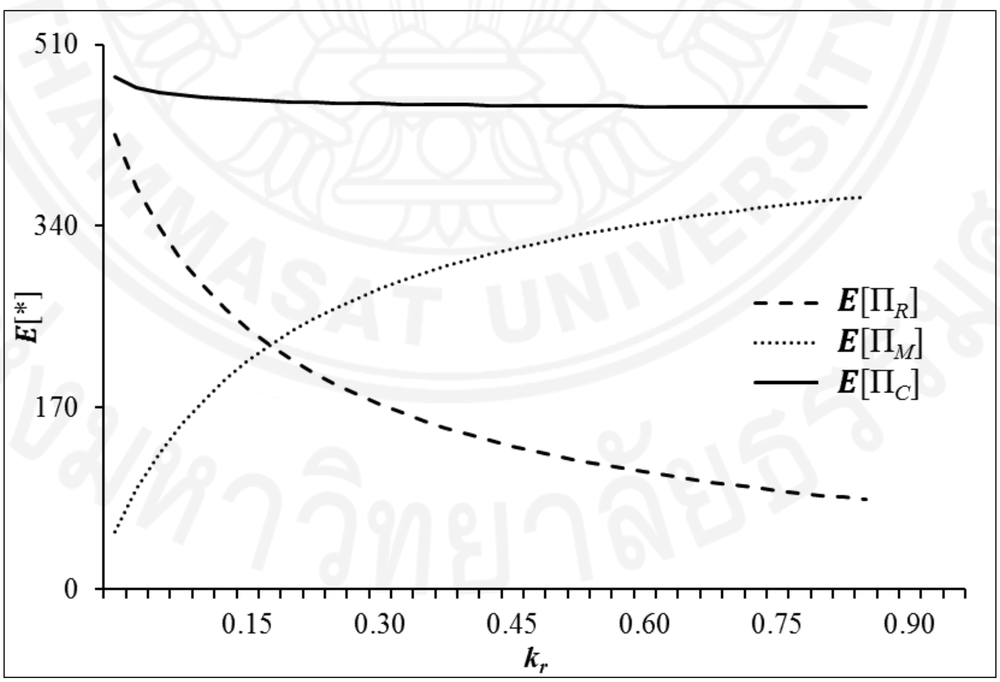

Figure 4.7 The impacts of k_r on the optimal expected profits $E[\Pi_R]$, $E[\Pi_M]$, $E[\Pi_c]$.

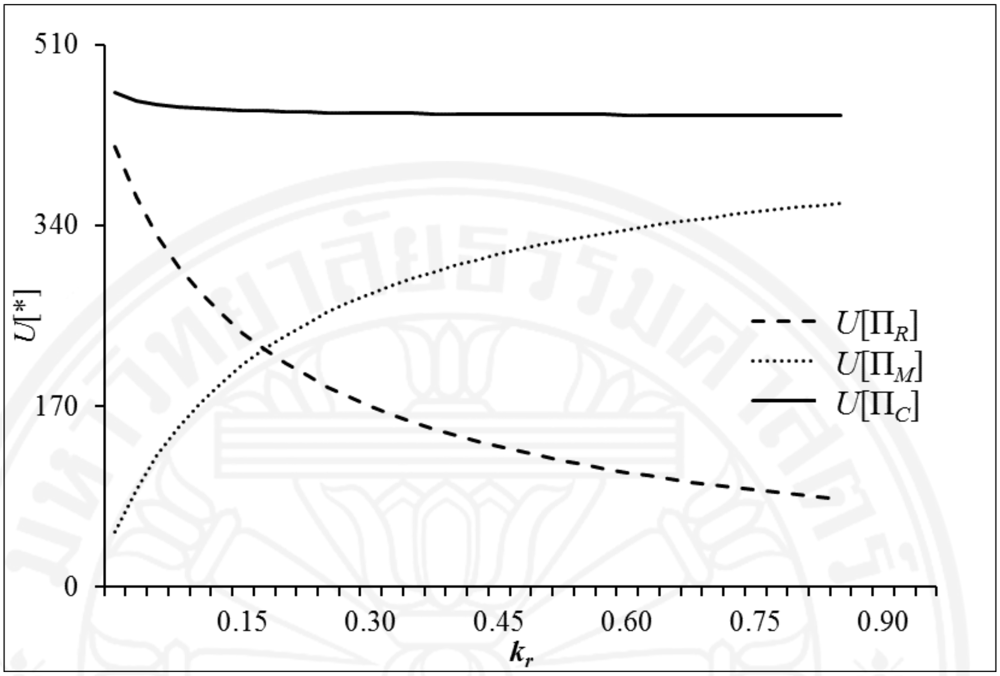

Figure 4.8 The impacts of k_r on the optimal utilities of profits $U[\Pi_R]$, $U[\Pi_M]$, $U[\Pi_c]$.

As the retailer increases k_r, which reduces their risk, it would result in the retailer reducing their optimal order quantity (Q_c), see Figure 4.5. The manufacturer would react by increasing their optimal buyback price (b_c) to encourage the retailer to order more. Also, the optimal wholesale price (w_c) becomes increasing to reflect the increase in the optimal buyback price. The important point is that the gap between the two prices becomes smaller as k_r increases, which indicates that the trend would remain encouraging for the retailer to order more, see Figure 4.6.

Besides, increasing k_r leads to decreases in both the optimal expected profits and the utilities of profits for the retailer ($E[\Pi_R]$ and $U[\Pi_R]$) and the entire SC ($E[\Pi_C]$ and $U[\Pi_c]$). Because the decreasing rate of $E[\Pi_R]$ and $U[\Pi_R]$ are larger than those of $E[\Pi_c]$ and $U[\Pi_c]$, both the optimal expected profits and the utilities of profits for the manufacturer $E[\Pi_M]$ and $U[\Pi_M]$ show an increasing trend. This is similar to the previous result that as an agent, i.e. the retailer in this case, becomes more aware of risk, and thus, raising their risk attitude, the optimal decision would benefit the other agent more, i.e. the manufacturer, whose risk attitude remain the same. This is again due to the condition that the SC is always coordinated.

4.1.1.3 The impacts of the refund-amount

Following the same approach, a numerical example of result with $k_m = 0.2$, $k_r = 0.15$, and varying $r \in [0,5]$ can be found in Table 4.3 and Figures 4.9 – 4.12.

Table 4.3 The impacts of r on the optimal values in the supply chain coordination.

r	Q_c	b_c	w_c	$E[\Pi_M]$	$E[\Pi_R]$	$E[\Pi_C]$	$U[\Pi_M]$	$U[\Pi_R]$	$U[\Pi_C]$
0	5.03	5.14	5.43	14.94	19.92	34.86	10.37	13.83	24.21
1	25.03	5.14	5.43	83.51	111.35	194.86	78.95	105.26	184.21
2	45.12	5.63	5.91	145.96	194.61	340.57	141.53	188.71	330.24
3	65.28	6.12	6.41	196.68	262.24	458.93	192.49	256.65	449.14
4	85.54	6.64	6.93	229.93	306.58	536.51	226.08	301.44	527.52
5	105.93	7.20	7.49	239.92	319.90	559.82	236.55	315.40	551.95

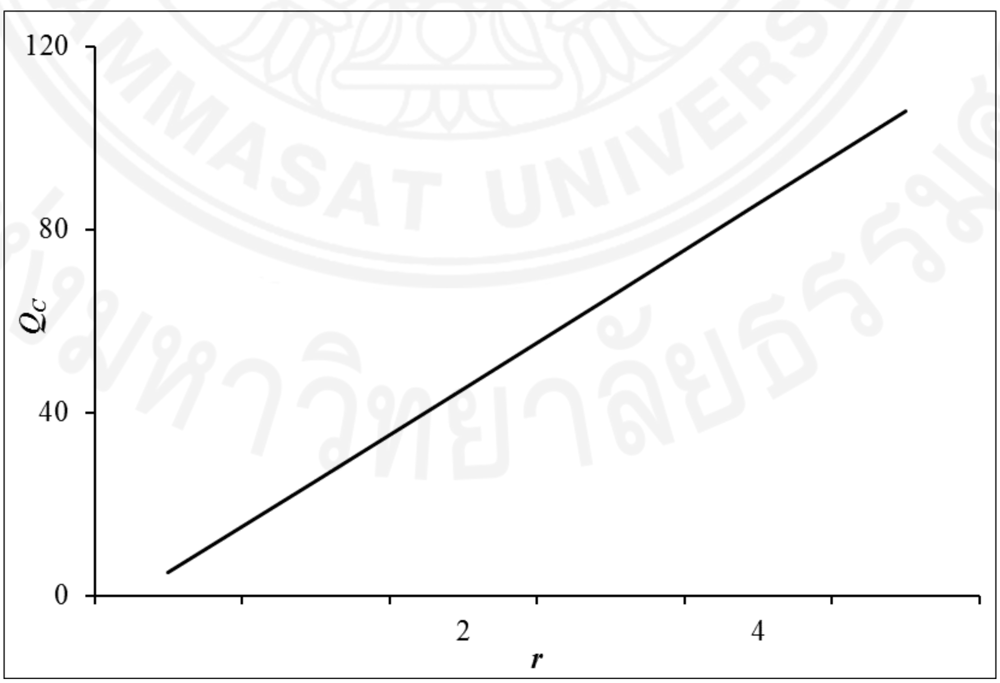

Figure 4.9 The impacts of r on Q_C.

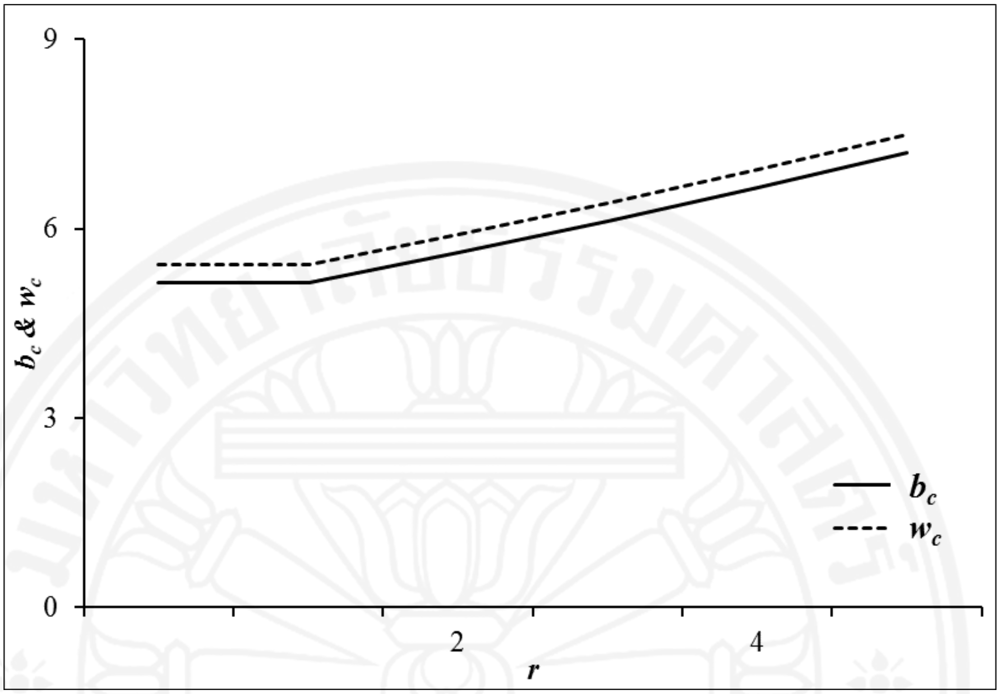

Figure 4.10 The impacts of r on w_c, b_c.

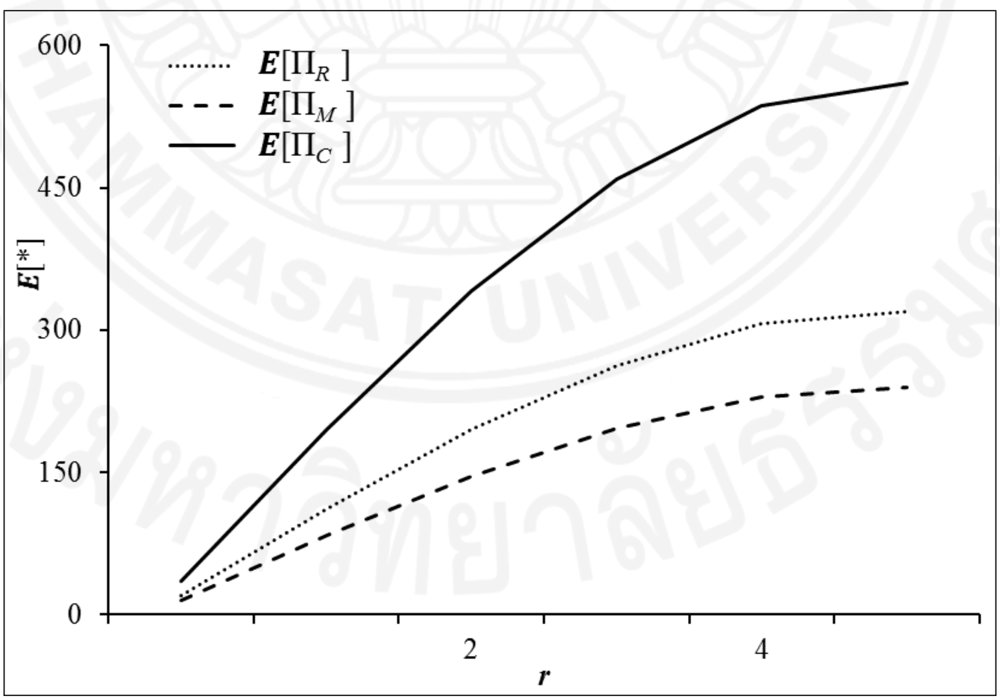

Figure 4.11 The impacts of r on the optimal expected profits $E[\Pi_R]$, $E[\Pi_M]$, $E[\Pi_C]$.

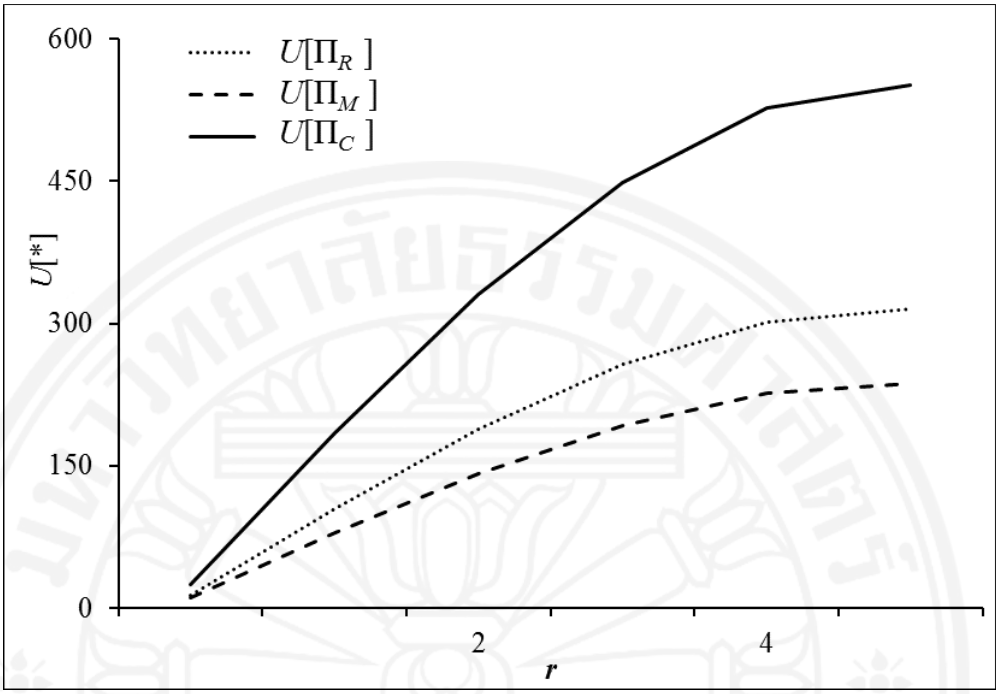

Figure 4.12 The impacts of r on the optimal utilities of profits $U[\Pi_R]$, $U[\Pi_M]$, $U[\Pi_c]$.

From the results, increasing r leads to increases in the customer demand, which results in larger retailer's optimal order quantity (Q_C) to satisfy the demand, see Figure 4.9. As a response, the manufacturer increases the optimal buyback price (b_c) to support larger order from the retailer, and simultaneously increases the wholesale price (w_c) at the same rate (see Figure 4.10) to balance the risk from increasing b_C. In addition, increasing r leads to decreases in the optimal expected profits and the utilities of profits for both agents and for the entire SC (see Figures 4.11 – 4.12).

4.1.2 Scenario 2: The supply chain is not coordinated

In this scenario, the impacts of r, k_m and k_r on the losses in the expected profit (ΔE) and in the utility of profit (ΔU) due to the lack of coordination between the two agents are investigated. The values ΔE and ΔU are determined as follows:

$$\Delta E = E[\Pi_C(z)] - (E_D[\Pi_M(z)] + E_D[\Pi_R(z)]).$$

$$\Delta U = U[\Pi_C(z)] - (U[\Pi_M(z)] + U[\Pi_R(z)]) = \Delta E - \{k.Var[\Pi_C(z)] - (k_r Var[\Pi_R(z)] + k_m Var[\Pi_M(z)])\}.$$

ΔE is positive, $(E_D[\Pi_M(z)] + E_D[\Pi_R(z)])$ has been smaller than $E[\Pi_C(z)]$. The SC's desire for coordinating position is very high in charge of enriching the expected profit of each members as well as whole SC.

ΔU is positive, $(U[\Pi_M(z)] + U[\Pi_R(z)])$ has been smaller than $U[\Pi_C(z)]$. The SC's desire for coordinating position is very high in charge of enriching the utility value of each members as well as whole SC.

Without loss of generality, one agent is chosen arbitrarily (the manufacturer in this scenario) to be the one who makes off-optimal decision(s), that is, the SC is uncoordinated. Eight cases of changes in (w_m, b_m) from the optimal values are tested as shown in Table 4.4. In each case, many combinations of (w_m, b_m) are tested within the range(s) at which the two parameters can vary.

Table 4.4 The cases of the pairs (w_m, b_m).

$(\boldsymbol{w_m},\boldsymbol{b_m})$	$\boldsymbol{b_m = b_C}$	$\boldsymbol{c < b_m < b_C}$	$\boldsymbol{b_C < b_m < w_C}$
$\boldsymbol{w_m = w_C}$	Coordinated SC	Case 3	Case 6
$\boldsymbol{b_C < w_m < w_C}$	Case 1	Case 4	Case 7
$\boldsymbol{w_C < w_m < p}$	Case 2	Case 5	Case 8

For each combination of (w_m, b_m), all 9,126 possible sets of values from varying k_m, k_r and r similar to that of the scenario 1 are tested. The results are summarized in two parts as follows. Part 1 provides general values of ΔE and ΔU, and part 2 discusses the impacts of k_m, k_r, and r on the trends of ΔE and ΔU.

4.1.2.1 Effects of (w_m, b_m) on the values of ΔE and ΔU

The values of ΔE and ΔU from all eight cases can be summarized into two common results:

1. There are losses in both the utility and expected profit of the uncoordinated SC ($\Delta U > 0$ and $\Delta E > 0$). These results occur when the

manufacturer makes decision(s) mainly to increase their profit by increasing w_m (case 2), reducing b_m (case 3), doing both (case 5), reducing b_m more than reducing w_m (some of case 4), or increasing w_m more than increasing b_m (some of case 8). As expected, both the expected profit and expected utility of profit in the uncoordinated SC, from the manufacturer's off-optimal decision(s), are always lower than those of the coordinated SC.

2. There is a loss in the utility of profit ($\Delta U > 0$), but there is a gain in expected profit ($\Delta E < 0$) for the uncoordinated SC. These results occur in situations where the manufacturer makes decision(s) to stimulate demand from the retailer by decreasing w_m (case 1), increasing b_m (case 6), doing both (case 7), reducing w_m more than reducing b_m (some of case 4), or increasing b_m more than increasing w_m (some of case 8). For these cases, uncoordinated SC yields higher expected profit than that of the coordinated SC. This is possible because, in this article, the SC is coordinated through utility function, not the expected profit. Hence, SC coordination can only improve the utility of profit. A closer looks reveal that the loss in the expected profit from coordinating the SC is more than offset by the benefit from reduction in the variance of profit, i.e., gain in the utility of profit.

To summarize, when an agent makes off-optimal decision(s) in favor of their own benefit, then SC coordination can improve both the expected value and utility value of the profit. However, when an agent makes decision(s) in favor of the other agent, then SC coordination can only improve the utility value of the profit.

An example to illustrate these situations use $k_m = 0.2$, $k_r = 0.15$, and $r = 3$. The coordinated SC gives optimal $b_C = 6.12$, $w_C = 6.41$, $E[\Pi_C] = 45893$, and $U[\Pi_C] = 44914$. Changing the values of (w_m, b_m) according to the eight cases above gives the results in Table 4.5.

Table 4.5 The impacts of changes in (w_m, b_m) on ΔE and ΔU.

Cases	b_m	w_m	ΔE	$Var[\Pi_C]$	$Var_D[\Pi_R]$	$Var_D[\Pi_M]$	ΔU
Coordinated	6.12	6.41	0	114.21	37.29	20.98	0
1	6.12	6.28	-0.58	114.21	39.54	22.24	0.01
2	6.12	6.54	0.59	114.21	35.11	19.75	0.01
3	6.00	6.41	0.80	114.21	35.85	18.20	0.03
4	5.88	6.15	0.46	114.21	38.75	17.74	0.03
4	6.00	6.15	-0.33	114.21	40.26	20.44	0.01
5	6.00	6.66	1.96	114.21	31.70	16.09	0.15
6	6.24	6.41	-0.84	114.21	38.82	24.18	0.03
7	6.24	6.28	-1.43	114.21	41.16	25.63	0.09
8	6.37	6.54	-1.10	114.21	38.09	26.25	0.07
8	6.37	6.79	0.15	114.21	33.58	23.15	0.03

4.1.2.2 Effects of k_m, k_r and r on the trends of ΔE and ΔU

The results from Section 4.1.2.1 are further analyzed to examine the trends of ΔE and ΔU from varying k_m, k_r, and r. The trends can be categorized in common patterns as shown in Table 4.6. Examples of these trends are shown in Figures 4.13 – 4.18.

Table 4.6 Effects of k_m, k_r and r on the trends of ΔE and ΔU.

Value		Trends		Parameter		
ΔU	ΔE	ΔU	ΔE	k_m	k_r	r
>0	>0	downward	downward	increase	fixed	fixed
		upward	upward	fixed	increase	fixed
		upward	upward	fixed	fixed	increase
>0	<0	downward	upward	increase	fixed	fixed
		upward	downward	fixed	increase	fixed
		upward	downward	fixed	fixed	Increase

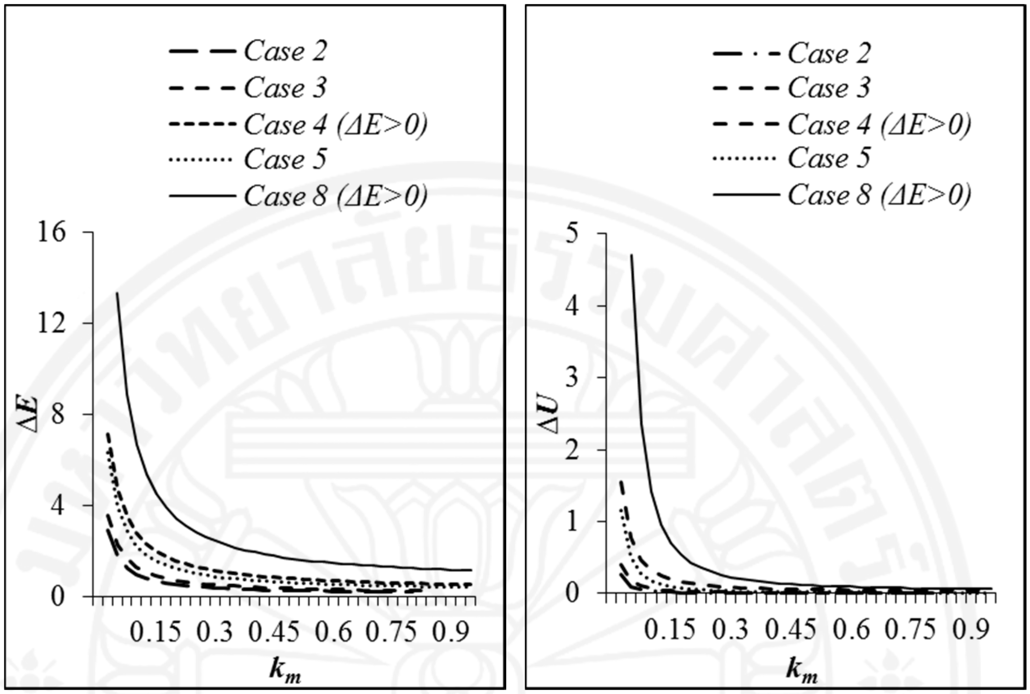

Figure 4.13 The effects of k_m on ΔE (>0) and ΔU.

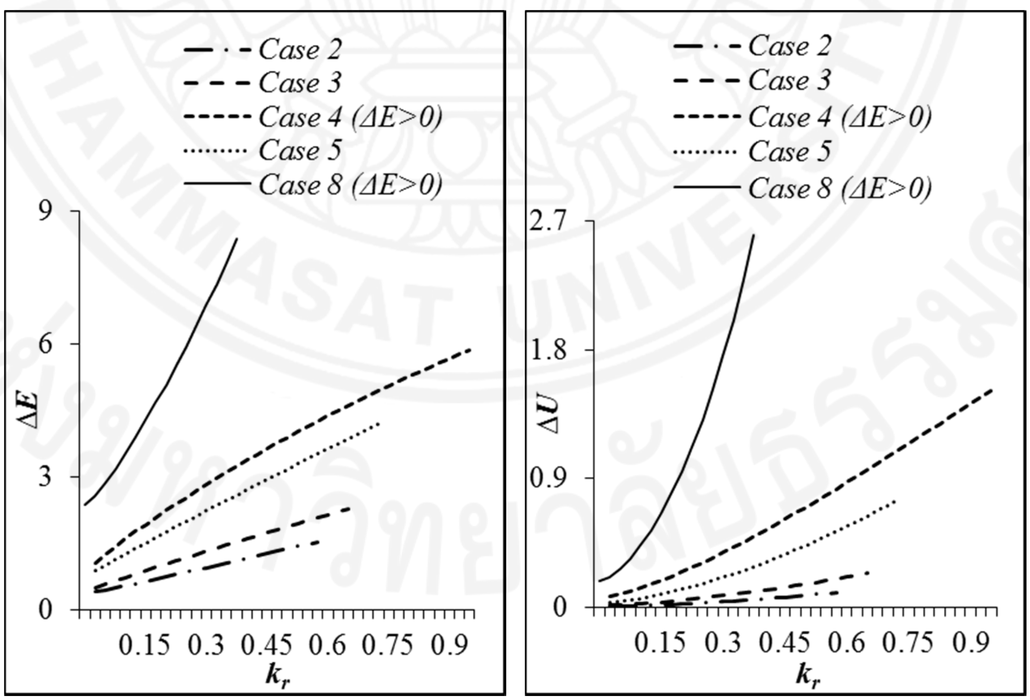

Figure 4.14 The effects of k_r on ΔE (>0) and ΔU.

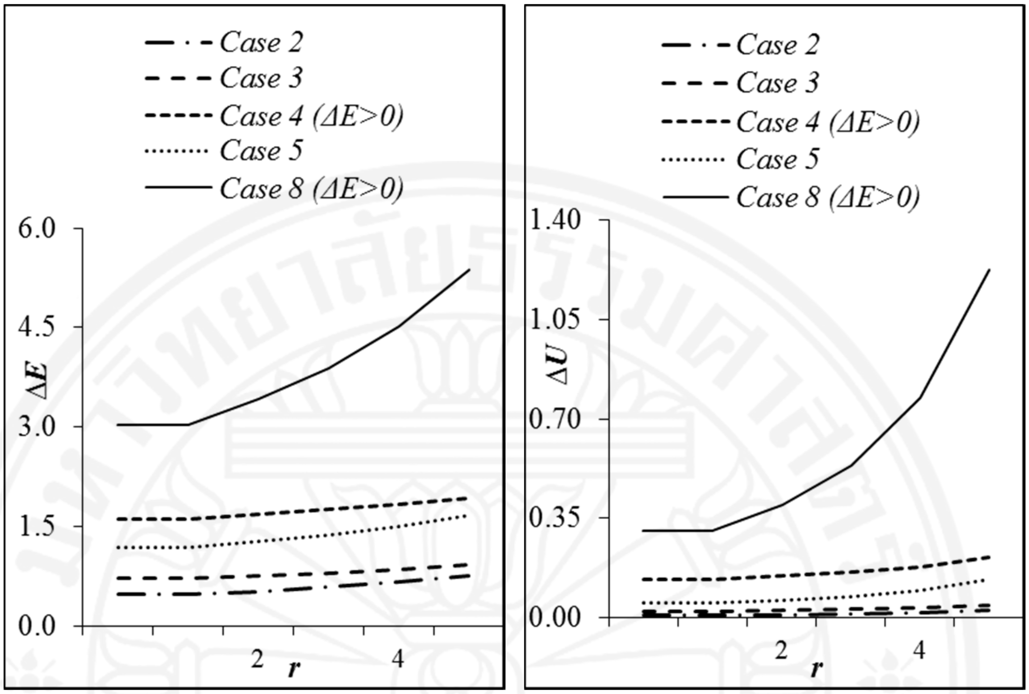

Figure 4.15 The effects of r on ΔE (>0) and ΔU.

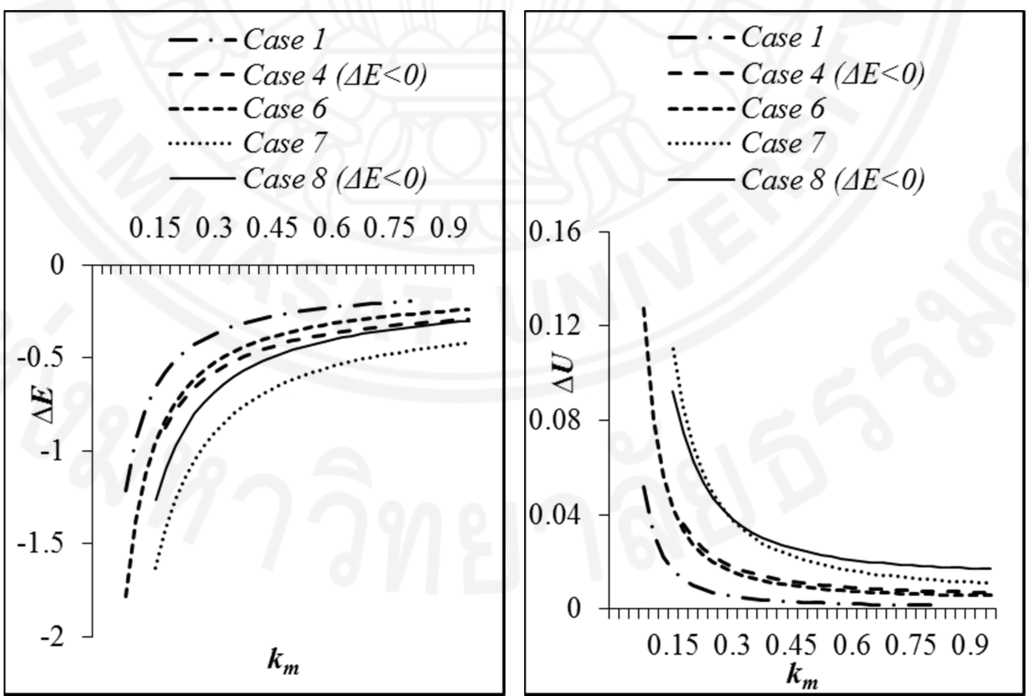

Figure 4.16 The impacts of k_m on ΔE (<0) and ΔU.

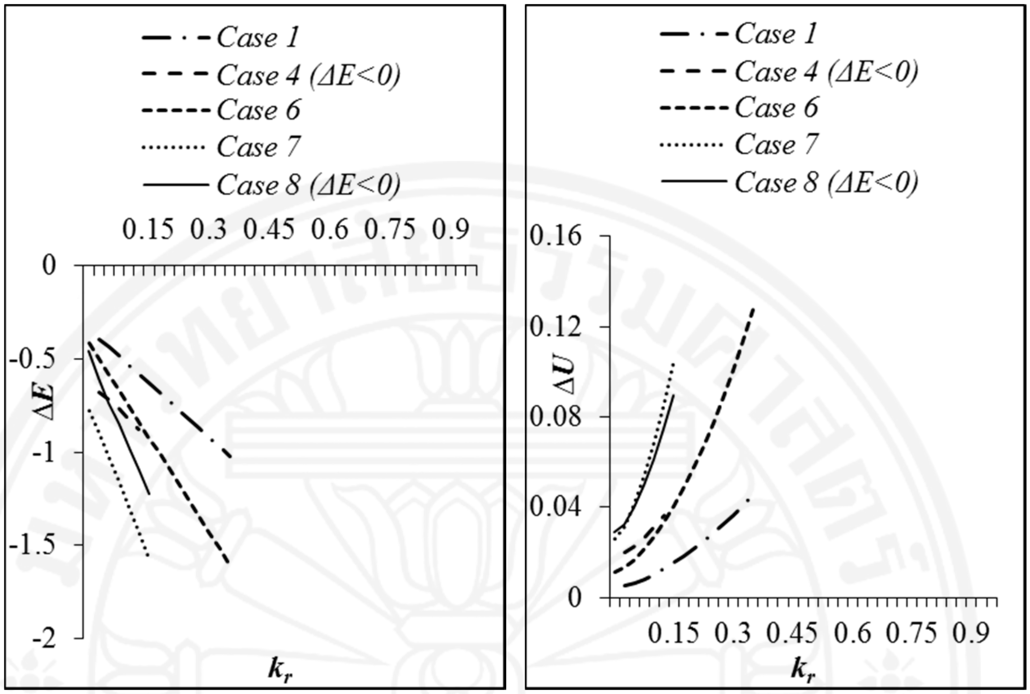

Figure 4.17 The impacts of k_r on ΔE (<0) and ΔU.

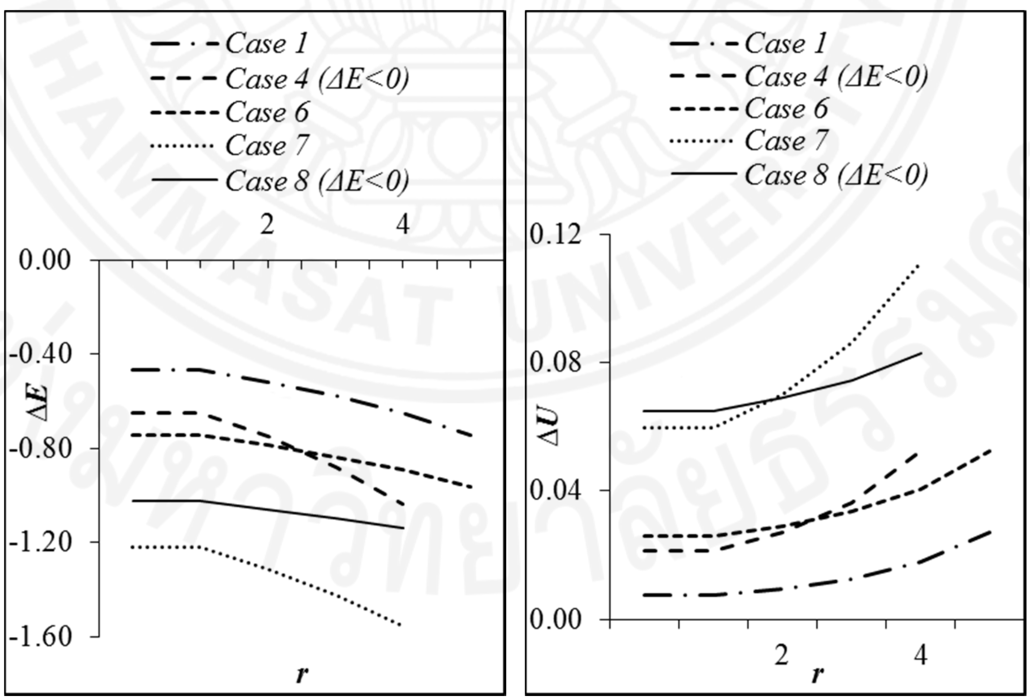

Figure 4.18 The impacts of r on ΔE (<0) and ΔU.

- From Figure 4.13 ($k_r = 0.15$, $r = 3$, $k_m \in [0.025, 0.975]$), when the manufacturer makes off-optimal decision(s) to increase their profit, the losses on both the expected profit and utility of profit for the entire SC decrease as the manufacturer's risk attitude increases.

- In Figure 4.14 ($k_m = 0.2$, $r = 3$, $k_r \in [0.025, 0.975]$, and Figure 4.15 ($k_r = 0.15$, $k_m = 0.2$, $r \in [0,5]$), when the manufacturer makes off-optimal decision(s) for their benefit, the losses on both ΔE and ΔU worsen, as the retailer's risk attitude increases (equivalently, the retailer's order quantity decreases), or as the refund amount (to stimulate customer demands) increases, respectively.

- However, when the manufacturer makes decision(s) to stimulate retailer's demand, the loss in the utility is rather small and decreasing, while the gain in the expected profit diminishes quickly as the manufacturer's risk attitude (to balance the effect of their decisions) increases. Figure 4.16 shows an example with $k_r = 0.15$, $r = 3$, and $k_m \in [0.025, 0.975]$.

- The opposites can be seen in Figure 4.17 ($k_m = 0.2$, $r = 3$, and $k_r \in [0.025, 0.975]$) and Figure 4.18 ($k_r = 0.15$, $k_m = 0.2$, and $r \in [0,5]$. When the manufacturer attempts to stimulate the retailer's demand, the loss in the utility of profit for the whole SC becomes worsen, while the gain in the expected profit is higher, as the retailer's risk-attitude increases (i.e. as the retailer's order quantity decreases), or as refund amount increases.

According to the results above, the SC with lower k_m, higher k_r, or higher r have more motivations to reach the coordination situation. In addition, for the SC that is coordinated through utility function, it is natural that the utility values would be less sensitive to changes in k_m, k_r and r than the expected profit value, i.e. both the magnitudes of ΔE are larger than those of ΔU, and the slopes of the trends of ΔE are steeper than those of the trends of ΔU in Figures 4.13 – 4.18.

4.2 Price-dependent demand

In this computational study, similar to Liu *et al*. (2014), the parameters in the model are chosen as $c = 2$, $s_1 = 1.5$, $s_2 = 1$, $l_m = 1$, $l_r = 1$, $l = l_r + l_m = 2$, $v \sim U[1, p]$. The probability of returned items of customer is $\alpha = prob(v < r) = \int_{\underline{v}}^{r} h(v)dv = H(r) = (r-1)/(p-1)$.

Let $D(p) = 15 - 0.25p$; and $\varepsilon \sim U[-10,10] \rightarrow X \in [D(p) - 10, D(p) + 10]$, Since $X \geq 0 \rightarrow D(p) - 10 \geq 0 \leftrightarrow 15 - 0.25p - 10 \geq 0 \rightarrow p \leq 20$ In addition, $f(\varepsilon) = 1/20$ and $F(\varepsilon) = (\varepsilon + 10)/20$. It means $z \sim U[-10,10]$, $f(z) = 1/20$, and $F(z) = (z + 10)/20$. Hence, the overstock amount, $\Lambda(z) = \int_{-10}^{z}(z-\varepsilon)1/20d\varepsilon = 1/40(z+10)^2$, and understock amount, $\Theta(z) = \int_{z}^{10}(\varepsilon - z)1/20d\varepsilon = 1/40(z-10)^2$, and $Var[\min(X, Q)] = -(3z - 50)(z + 10)^3/4800$.

The purpose of the computational study is to investigate the impact of the retail price p, refund amount r, and risk attitude parameters k_r and k_m on the optimal order quantity, the expected profits and the utility values, respectively. To perform a thorough experiment, many combinations of (r, p, k_m, and k_r) values are tested as follows: $p \in [8,12,16,20]$; k_r and $k_m \in [0.025, 0.05, 0.075, \ldots, 0.975]$, and $r \in [1, 2, \ldots, p-1]$ with the total number of combinations from 10,647 to 28,899 for each p.

From (11), the optimal value z_C^* in the centralized SC can be derived from $(z_C^* + 10)^2 = 0$. Hence, the optimal order Q_C^* in the centralized SC is given as $Q_C^* = 15 - 0.25p + z_C^*$.

Therefore, from (10), the utility function of total profit $U[\Pi_c(z)]$ of the centralized SC can be determined as,

$$U[\Pi_c(z_C^*)] = E[\Pi_c(z_C^*)] - kVar[\Pi_c(z_C^*)] = [p + \alpha(s_2 - r - l) - c][15 - 0.25p - 1/40(z_C^* - 10)^2] - (c - s_1)1/40(z_C^* + 10)^2 - k[(p + \alpha(s_2 - r - l) - s_1)^2 - (3z_C^* - 50)(z_C^* + 10)^3/4800].$$

Similarly, from (18), the optimal value z_R^* in the decentralized SC can be derived from $(z_R^* + 10)^2 = 0$. Hence, the optimal order Q_R^* in the centralized SC is given as $Q_R^* = 15 - 0.25p + z_R^*$.

Therefore, from (10), the utility function of total profit $U[\Pi_c(z)]$ of the retailer and the manufacturer in the decentralized SC can be determined respectively as

$U_D[\Pi_R(z_R^*)] = E_D[\Pi_R(z_R^*)] - k_r Var_D[\Pi_R(z_R^*)] = (p + \alpha(b - l_r - r) - w)(15 - 0.25p - 1/40\,(z_R^* - 10)^2) - (w - b)1/40(z_R^* + 10)^2 - k_r[(p + \alpha(b - r - l_r) - b)^2(3z_R^* - 50)(z_R^* + 10)^3/4800]$.

$U_D[\Pi_M(z_R^*)] = E_D[\Pi_M(z_R^*)] - k_m Var_D[\Pi_M(z_R^*)] = \big(w - c - \alpha(b + l_m - s_2)\big)(15 - 0.25p + z_R^*) - \big(b - s_1 - \alpha(b + l_m - s_2)\big)1/40\,(z_R^* + 10)^2 - k_m(b - \alpha(b + l_m - s_2) - s_1)^2(3z_R^* - 50)(z_R^* + 10)^3/4800$.

There are two scenarios developed in this computational study for the coordinated and uncoordinated SCs.

4.2.1 Scenario 1: The supply chain is coordinated

When the SC is coordinated:

- According to (26), the optimal buyback price (b_c) is a decreasing function of k_m and increasing functions of k_r and p.
- Following (25), the optimal wholesale price (w_c) is an increasing function of b_c. Therefore, w_c is also a decreasing function of k_m and increasing functions of k_r and p.
- The total of utilities of profit in the decentralized SC is equal to that in the centralized SC.

In this scenario, the impacts of k_m, k_r, and p on the optimal decisions w_c ,b_c, Q_c, and optimal SC measures of performance $E[\Pi_R]$, $E[\Pi_M]$, $E[\Pi_c]$, $U[\Pi_R]$, $U[\Pi_M]$, and $U[\Pi_c]$ are examined through numerical examples. Note that the findings in all 10,647 to 28,899 cases follow the same trends shown in the given numerical examples.

4.2.1.1 The impacts of k_m.

An example of results to demonstrate the impact of k_m, where $k_r = 0.15$, $r = 3$, and varying $k_m \in [0.025, 0.975]$, can be found for all $p \in [8,10,12,14,16,18,20]$ in Figures 4.19 – 4.23 as follows:

In this part, when k_r and r are fixed, increasing k_m with the smallest value of p, the trends of both the optimal buyback price (b_c) and wholesale price (w_c), and the optimal order quantity (Q_c) are similar to the cases in the refund-dependent demand above.

Specifically, there are decreases in the values of b_c, w_c, and the optimal order quantity (Q_c) when the manufacturer increases k_m. These results are the same for all values of p. In addition, increasing p with a fixed k_m, the optimal values b_C and w_C increases while Q_C decreases, see Figures 4.19 – 4.21.

Besides, increasing k_m with a fixed p, leads to decreases in both the optimal expected profits and the utilities of profits for the manufacturer ($E[\Pi_M]$ and $U[\Pi_M]$) and the entire SC ($E[\Pi_C]$ and $U[\Pi_c]$). Because the decreasing rates in both $E[\Pi_M]$ and $U[\Pi_M]$ are larger than the decreasing rate in $E[\Pi_C]$ and $U[\Pi_c]$, both the optimal expected profits and the utilities of profits in the retailer ($E[\Pi_R]$ and $U[\Pi_R]$) show increasing trends. This is due to the condition that the SC is always coordinated.

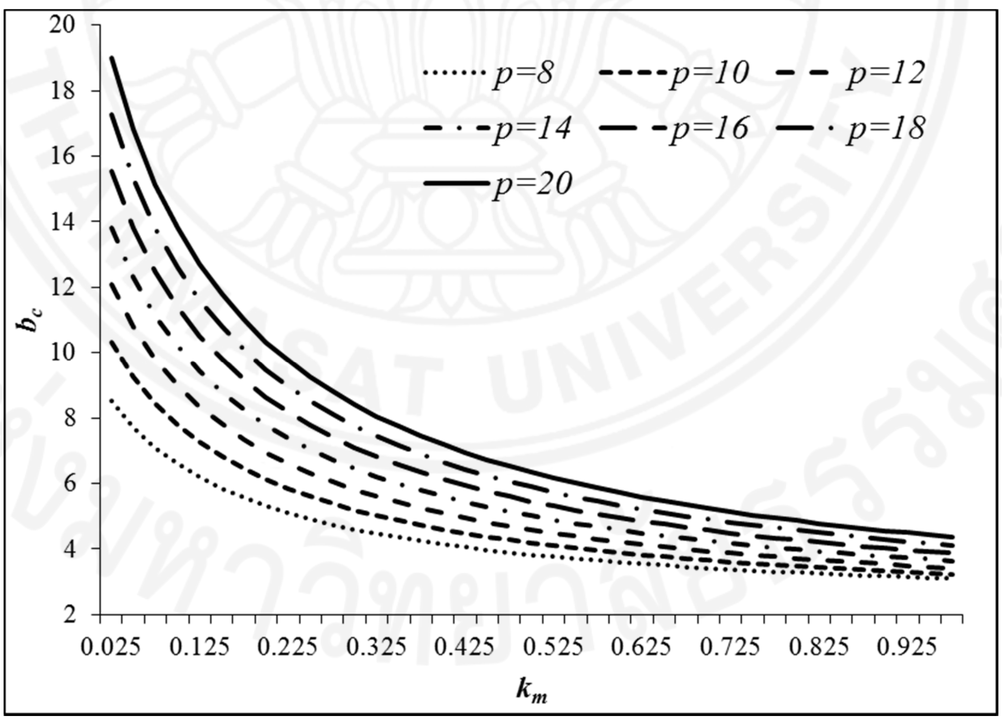

Figure 4.19 The impacts of k_m on b_c.

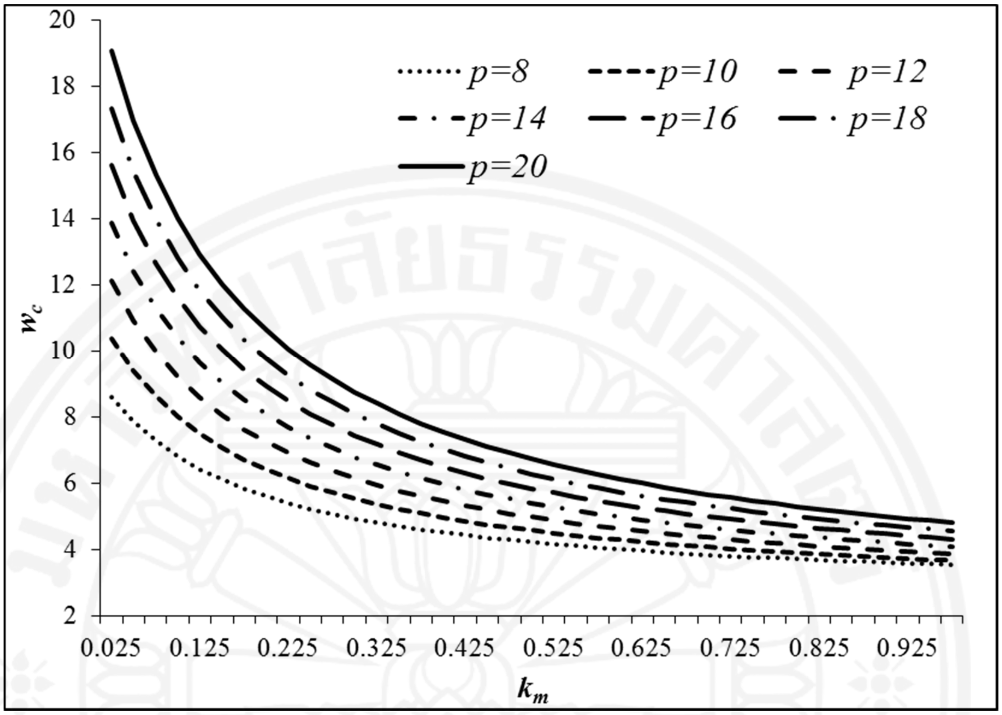

Figure 4.20 The impacts of k_m on w_c.

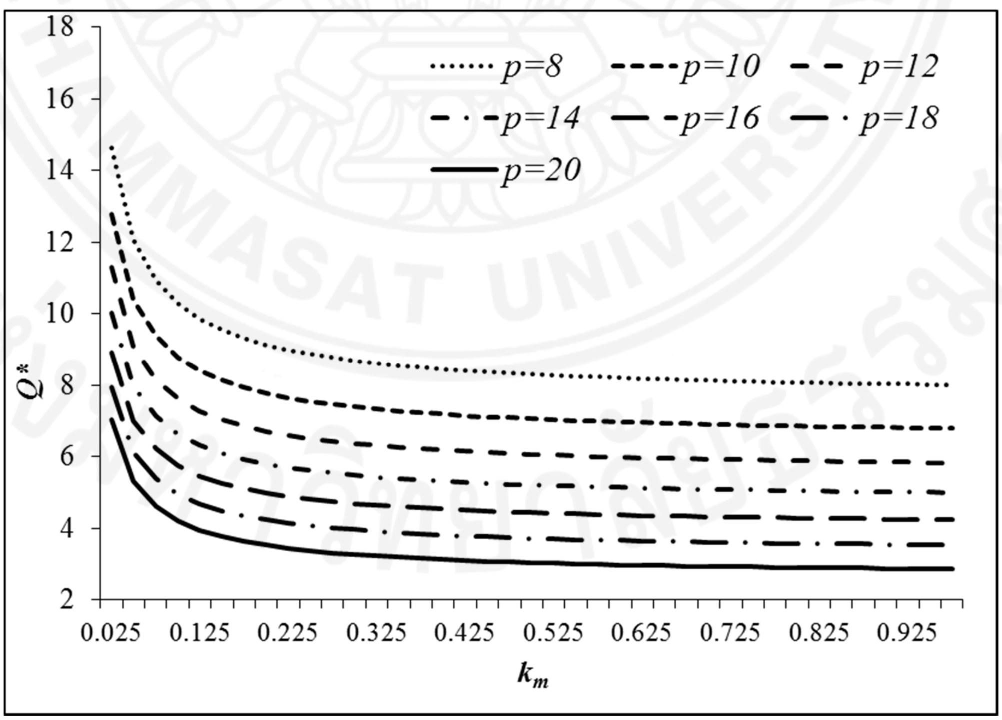

Figure 4.21 The impacts of k_m on Q_c.

Moreover, when retail price (p) increases, there are break-even points of $E[\Pi_C]$ and $U[\Pi_c]$ between different prices, as k_m increases. First, when k_m is smaller than a break-even point, the manufacturer should choose a higher retail price p to gain higher values in $E[\Pi_M]$ and $U[\Pi_M]$, which leads to higher values in $E[\Pi_C]$ and $U[\Pi_c]$. According to discussions above, $E[\Pi_R]$ and $U[\Pi_R]$ are higher with the manufacturer's decision in this option. However, when k_m is higher than the break-even point, the manufacturer should choose a lower price p to gain higher expected profits and utilities of the SC. Therefore, the manufacturer can adjust the retail price p against the decreases in the values ($E[\Pi_M]$ and $U[\Pi_M]$) as well as ($E[\Pi_C]$ and $U[\Pi_c]$) as k_m increases, see Figures 4.22 – 4.23 and Tables 4.7 – 4.8.

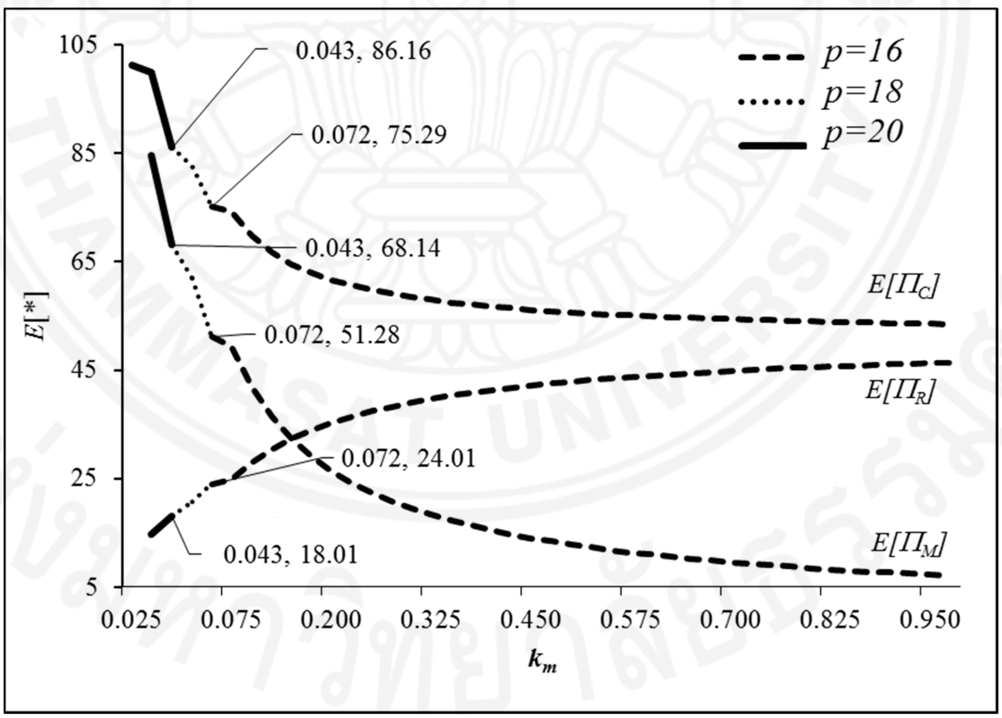

Figure 4.22 The impacts of k_m on the optimal expected profits $E[\Pi_R]$, $E[\Pi_M]$, $E[\Pi_c]$.

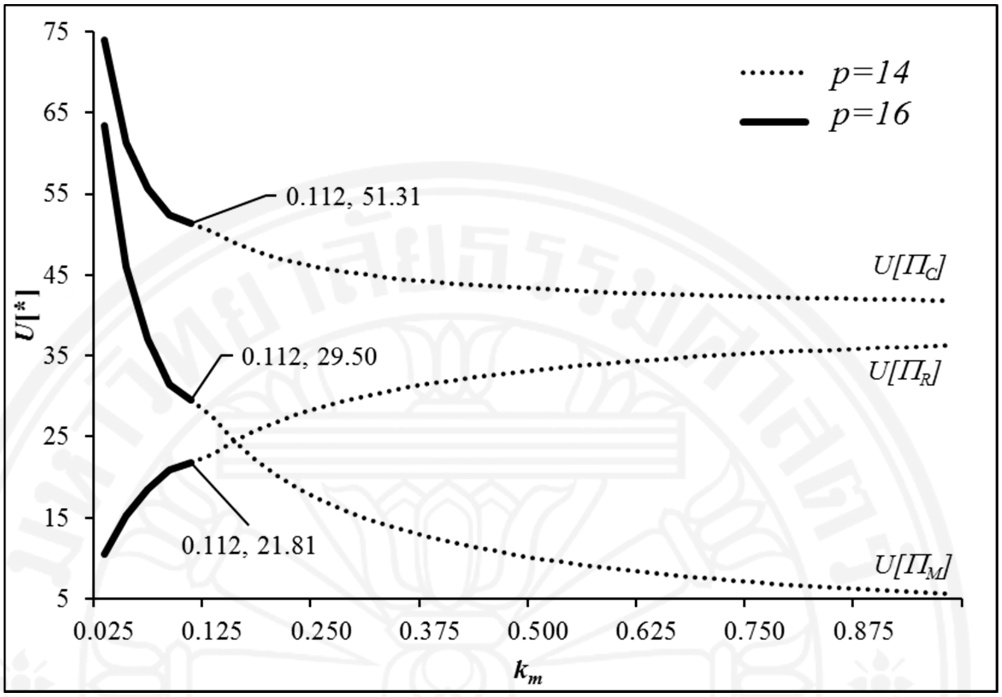

Figure 4.23 The impacts of k_m on the optimal utilities of profits $U[\Pi_R]$, $U[\Pi_M]$, $U[\Pi_c]$.

From Figure 4.22, the effects of k_m and p on the expected profits in the SC are summarized to show the optimal expected profits in the supply chain with each respective manufacturer's risk attitude. By setting the retail price as the manufacturer's risk-attitude increases, the SC can reach the optimal expected profits in each member or the entire SC, see Table 4.7.

Table 4.7 The relationship between k_m and p for the optimal expected profits in the entire supply chain.

$\boldsymbol{k_m}$	$\boldsymbol{p}$
$k_m < 0.043$	20
$0.043 < k_m < 0.072$	18
$k_m > 0.072$	16

Table 4.7 provides a guideline of how to choose the values of retail price p as the manufacturer's risk attitude k_m changes so as to maximize the expected profits for the entire SC.

In addition, using a same approach, Figure 4.23 and Table 4.8 provide some results in reaching the optimal utilities of profits in the entire SC by setting the retail price as the manufacturer's risk-attitude increases.

Table 4.8 The relationship between k_m and p for the optimal utilities of profits in the entire supply chain.

$\boldsymbol{k_m}$	$\boldsymbol{p}$
< 0.112	16
> 0.112	14

4.2.1.2 The impacts of $\boldsymbol{k_r}$

Using a similar approach, k_m and r are fixed ($k_m = 0.2$, $r = 3$), while varying $k_r \in [0.025, 0.975]$ and $p \in [8, 10, 12, 14, 16, 18, 20]$, the results are shown in Figures 4.24 – 4.28. There are the same trends in the results as those of the refund-dependent demand above with a fixed p, in which increasing k_r leads to a decrease in the optimal order quantity (Q_c), and the increases in the optimal buyback price (b_c) and the optimal wholesale price (w_c). These trends are similar for all considered retail prices (p). Besides, increasing p with a fixed k_r leads to the increases in the optimal values b_C and w_C and the decreases in Q_C, see Figures 4.24 – 4.26.

In addition, when p is fixed, increasing k_r leads to decreases in both the optimal expected profits and the utilities of profits for the retailer ($E[\Pi_R]$ and $U[\Pi_R]$) and the entire SC ($E[\Pi_C]$ and $U[\Pi_c]$) and increases in both the optimal expected profits and the utilities of profits for the manufacturer $E[\Pi_M]$ and $U[\Pi_M]$. This is similar to the previous result that as an agent, i.e. the retailer in this case, becomes more aware of risk, thus, raising their risk attitude, the optimal decision would benefit the other agent more, i.e. the manufacturer, whose risk attitude remain the same. This is again due to the condition that the SC is always coordinated.

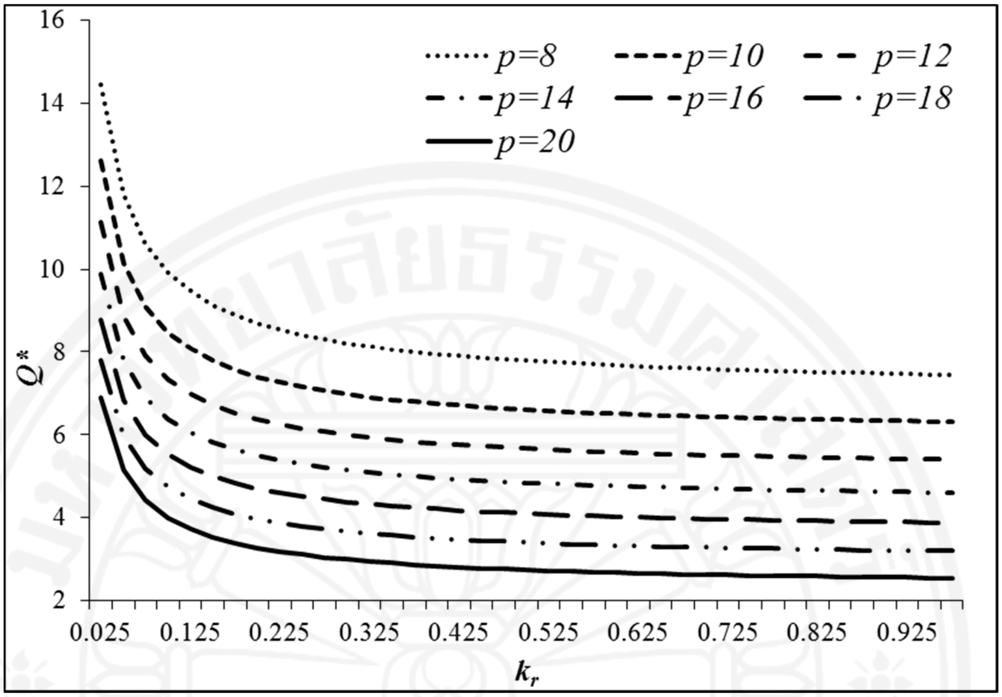

Figure 4.24 The impacts of k_r on Q_C.

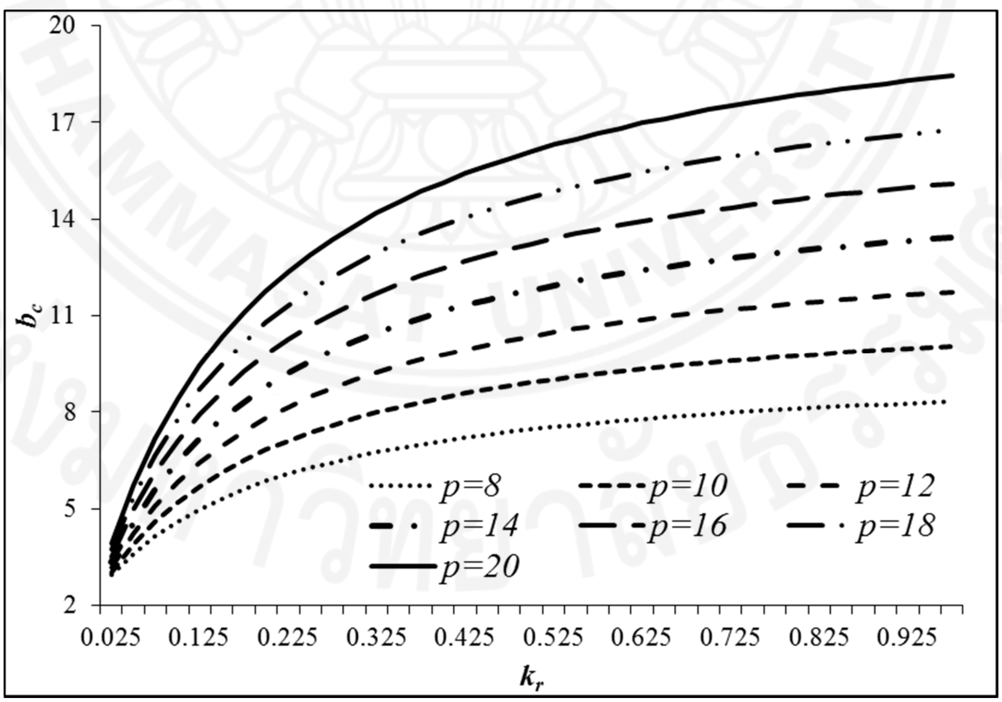

Figure 4.25 The impacts of k_r on b_c.

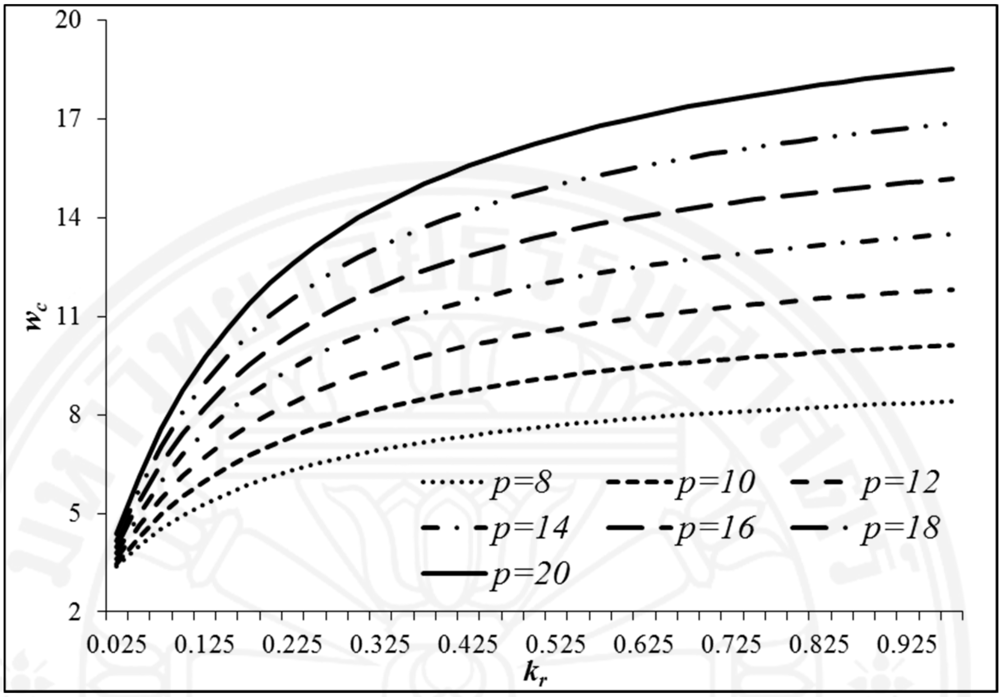

Figure 4.26 The impacts of k_r on w_c.

Moreover, when retail price p is increased, there are break-even points of $E[\Pi_C]$ and $U[\Pi_c]$ at different values of p, as k_r increases. When k_r is smaller than a break-even point, the manufacturer should choose a higher retail price p to gain higher values in $E[\Pi_M]$ and $U[\Pi_M]$, which leads to higher values in $E[\Pi_C]$ and $U[\Pi_c]$. According to discussions above, $E[\Pi_R]$ and $U[\Pi_R]$ are higher with the manufacturer's decision at these values of k_r (less than the break-even point). However, when k_r is higher than the break-even point, the manufacturer should choose lower p to obtain higher expected profits and utilities in the SC. Therefore, the manufacturer can adjust the retail price p to help the retailer and the entire SC against the decreases in the values ($E[\Pi_R]$ and $U[\Pi_R]$) as well as ($E[\Pi_C]$ and $U[\Pi_c]$) as k_r increases, see Figures 4.27 – 4.28 and Tables 4.9 – 4.10.

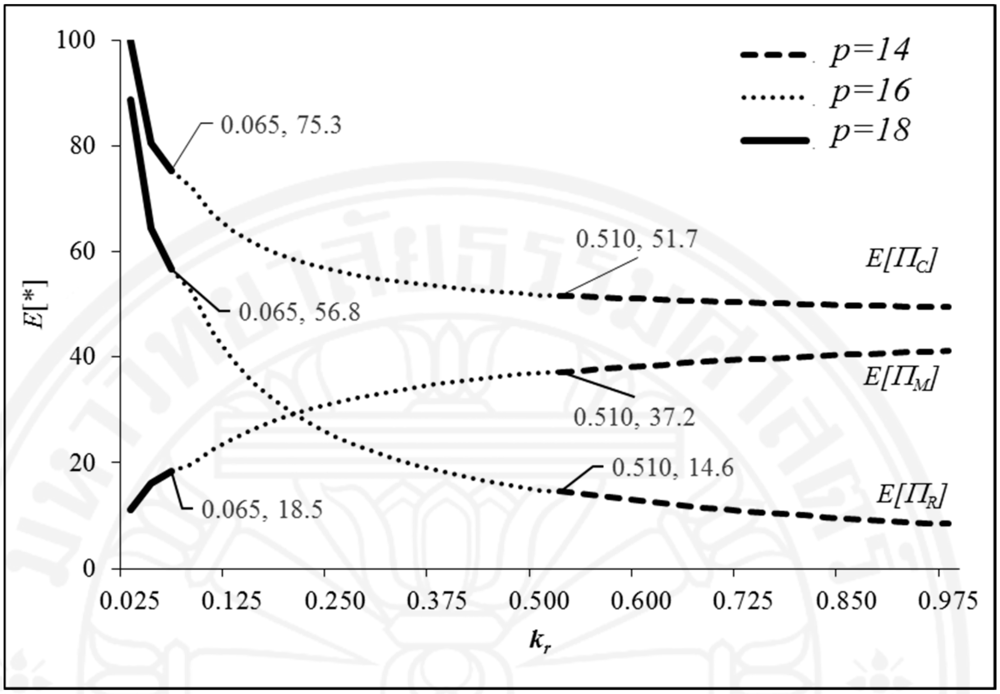

Figure 4.27 The impacts of k_r on the optimal expected profits $E[\Pi_R]$, $E[\Pi_M]$, $E[\Pi_c]$.

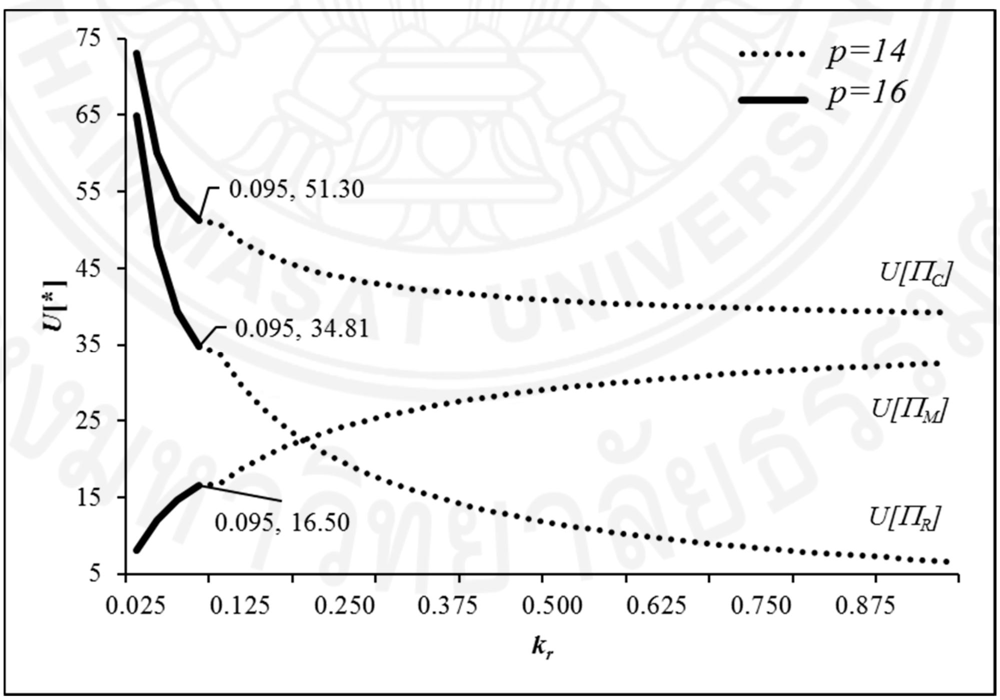

Figure 4.28 The impacts of k_r on the optimal utilities of profits $U[\Pi_R]$, $U[\Pi_M]$, $U[\Pi_c]$.

According to the results in Figure 4.27, the effects of k_r and p on the expected profits in the SC are summarized to show the optimal expected profits in the supply chain with each respective retailer's risk attitude. These results are shown in Table 4.9 as follows:

Table 4.9 The relationship between k_r and p for the optimal expected profits in the entire supply chain.

k_r	p
$k_r < 0.065$	18
$0.065 < k_r < 0.51$	16
$k_r > 0.51$	14

In addition, Figure 4.28 and Table 4.10 provide some results in reaching the optimal utilities of profits in the entire SC by setting the retail price as the retailer's risk-attitudes increases.

Table 4.10 The relationship between k_r and p for the optimal utilities of profits in the entire supply chain.

k_r	p
< 0.095	16
> 0.095	14

4.2.1.3 The impacts of r

Following the same approach, a numerical example of result with $k_r = 0.15$, $k_m = 0.2$, and varying $p \in [8,10,12,14,16,18,20]$ with $r \in [1,2, \dots, p-1]$ can be found in Figures 4.29 – 4.33.

From the results, increasing r with a fixed p, leads to increases in the customer demand, which results in larger retailer's optimal order quantity (Q_C) to satisfy the demand. As a response, the manufacturer increases the optimal buyback price (b_C) to support larger order from the retailer, and simultaneously increases the wholesale price (w_C) at the same rate to balance the risk from increasing b_C. These trends are applicable to all considered retail prices (p). Besides, increasing p with a fixed r, the optimal values b_C and w_C increase while Q_C decreases, see Figures 4.29 – 4.31.

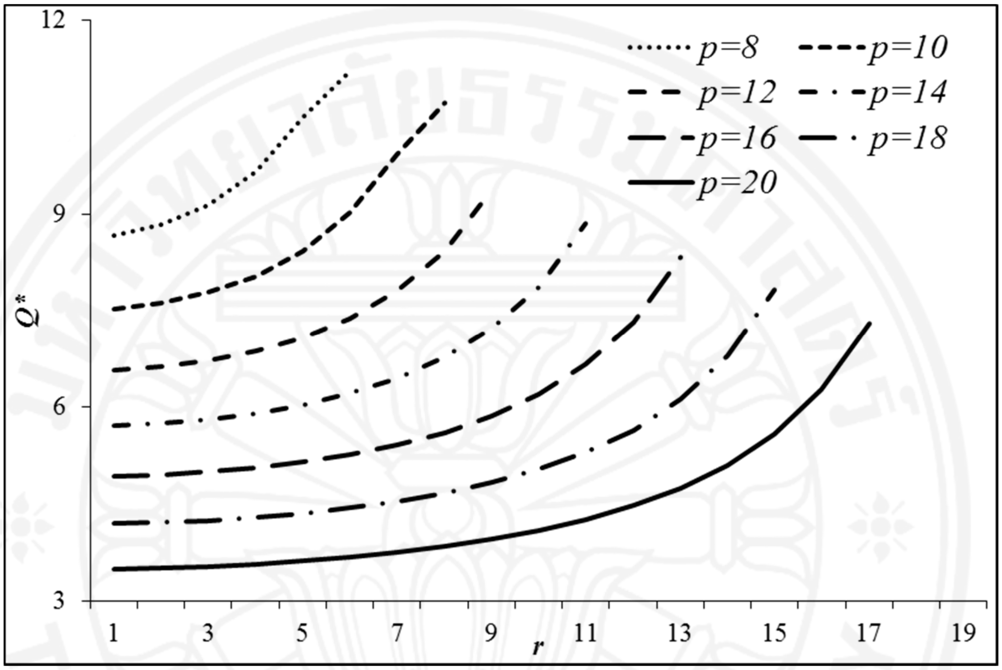

Figure 4.29 The impacts of r on Q_c.

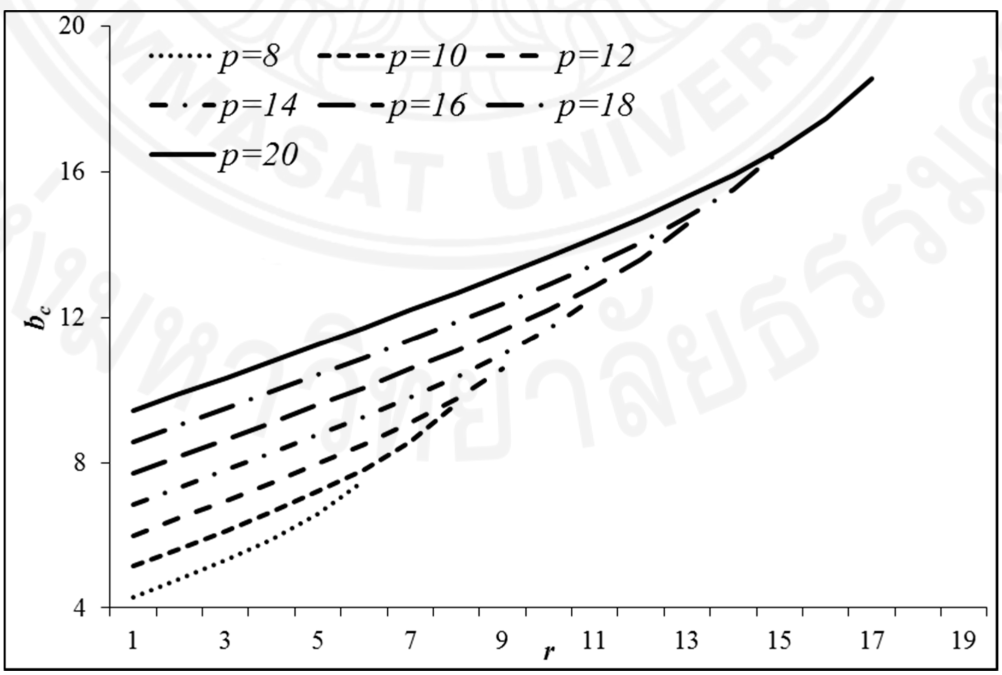

Figure 4.30 The impacts of r on b_c.

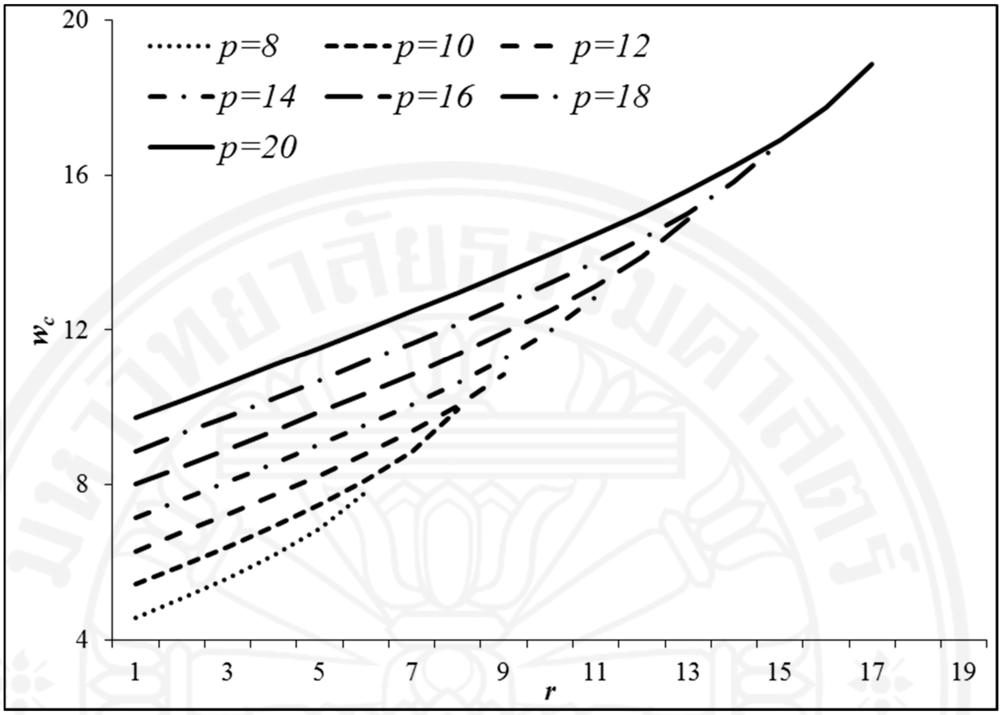

Figure 4.31 The impacts of r on w_c.

In addition, when p is fixed, increasing r leads to decreases in the optimal expected profits and the utilities of profits for both agents and for the entire SC. From the break-even points for setting p as r increases: When r is smaller than a break-even point, the manufacturer would set higher retail price to gain higher expected profits and utilities of profits in the SC. However, when r is larger than the break-even point, the manufacturer would choose the lower p to obtain higher expected profits and utilities of profits in the SC. Therefore, the SC can adjust the gap between p and r to get the optimal performance against the decreases in the values ($E[\Pi_R]$ and $U[\Pi_R]$), ($E[\Pi_M]$ and $U[\Pi_M]$), and ($E[\Pi_C]$ and $U[\Pi_c]$) as r increases, see Figures 4.32 – 4.33 and Tables 4.11 – 4.12.

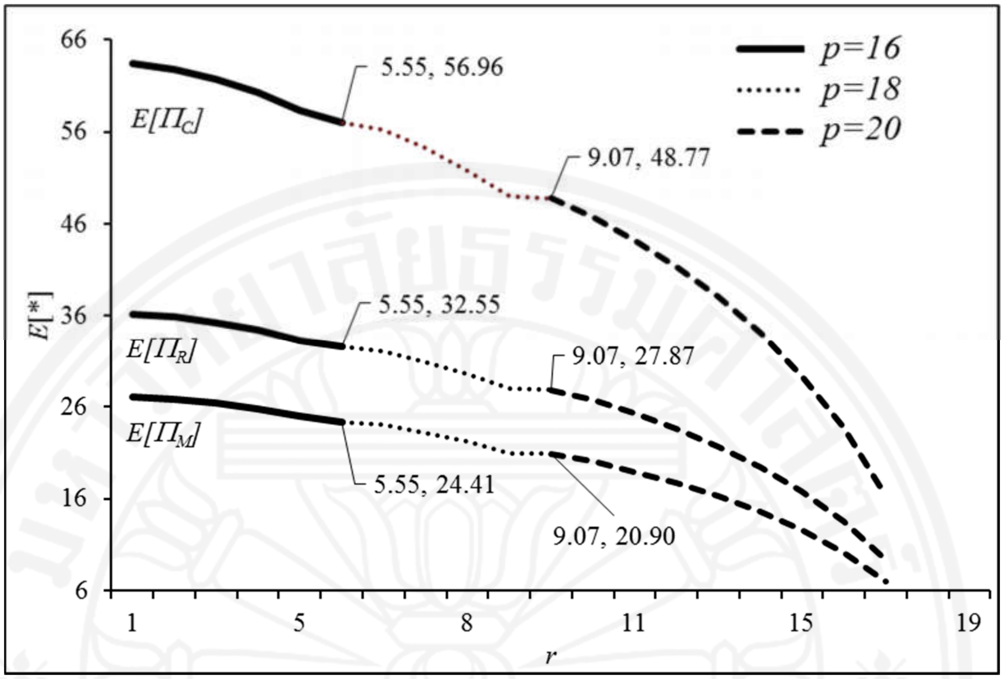

Figure 4.32 The impacts of r on the optimal expected profits $E[\Pi_R]$, $E[\Pi_M]$, $E[\Pi_c]$.

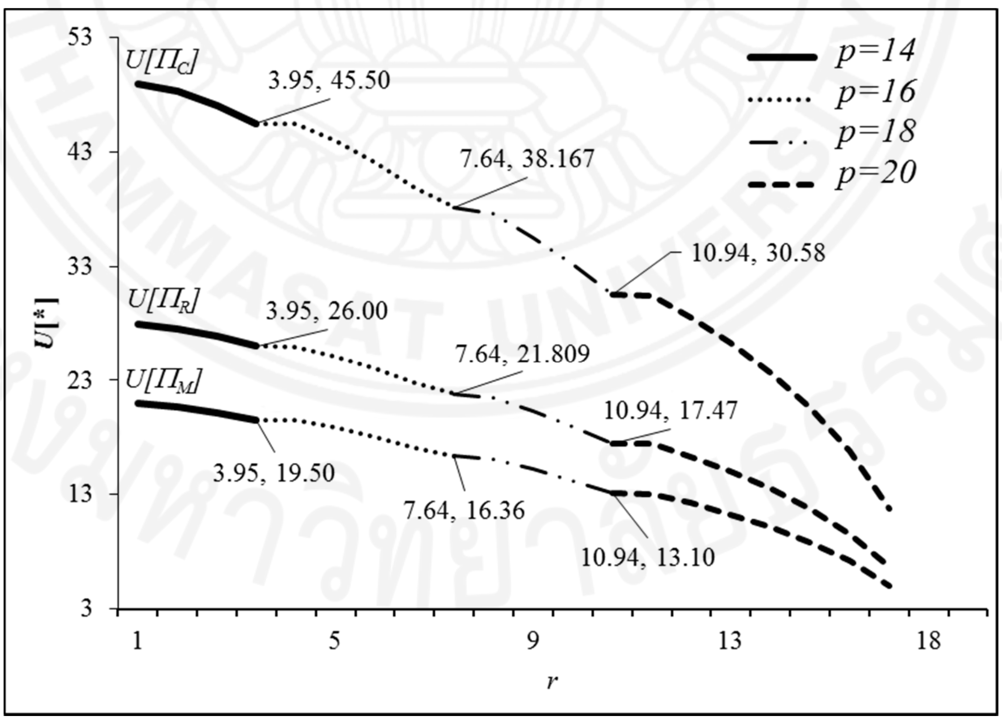

Figure 4.33 The impacts of r on the optimal utilities of profits $U[\Pi_R]$, $U[\Pi_M]$, $U[\Pi_c]$.

Figure 4.32 shows the effects of r and p on the expected profits in the SC. By choosing appropriate retail price, the SC can reach the optimal expected profits for each member or the entire SC, see Table 4.11.

Table 4.11 The relationship between r and p for the optimal expected profits in the entire supply chain.

$\boldsymbol{r}$	$\boldsymbol{p}$
$\boldsymbol{r} < 5.55$	16
$5.55 < \boldsymbol{r} < 9.07$	18
$\boldsymbol{r} > 9.07$	20

Using a same approach, Figure 4.33 and Table 4.12 provide some results in reaching the optimal utilities of profits in the entire SC by setting the retail price as the refund amount increases.

Table 4.12 The relationship between r and p for the optimal utilities of profits in the entire supply chain.

$\boldsymbol{r}$	$\boldsymbol{p}$
$\boldsymbol{r} < 3.95$	14
$3.95 < \boldsymbol{r} < 7.64$	16
$7.64 < \boldsymbol{r} < 10.94$	18
$\boldsymbol{r} > 10.94$	20

4.2.2 Scenario 2: The supply chain is not coordinated

According to discussions in Scenario 2 for refund-dependent demand style, in this scenario, the impacts of p, k_m, and k_r on the losses in the expected profit (ΔE) and in the utility of profit (ΔU) due to the lack of coordination between the two agents are investigated. The values ΔE and ΔU are determined as Scenario 2 from the case of refund-dependent demand above (Section 4.1.2). In addition, eight cases of changes in (w_m, b_m) from the optimal values are tested as shown in Table 4.4 and the results are also summarized in two parts as follows. Part 1 provides general values of ΔE and ΔU, and part 2 discusses the impacts of k_m and k_r on the trends of ΔE and ΔU. However, part 2 still provides the results with each fixed p.

4.2.2.1 Effects of (w_m, b_m) on the values of ΔE and ΔU

The values of ΔE and ΔU from all eight cases can be summarized into two common results the same as the refund-dependent demand case above, which consist of (1) $\Delta U > 0$ and $\Delta E > 0$, and (2) $\Delta U > 0$ and $\Delta E < 0$. It also means when an agent makes off-optimal decision(s) in favor of their own benefit, then SC coordination can improve both the expected value and utility value of the profit. However, when an agent makes decision(s) in favor of the other agent, then SC coordination can only improve the utility value of the profit.

An example to illustrate these situations use $k_m = 0.3$, $k_r = 0.1$, $p = 16$, and $r = 5$. The coordinated SC gives optimal $b_C = 6.44$, $w_C = 6.82$, $E[\Pi_C] = 61.02$, and $U[\Pi_C] = 45.92$. Changing the values of (w_m, b_m) according to eight cases gives the results in Table 4.13.

Table 4.13 The impacts of changes in (w_m, b_m) on ΔE and ΔU.

Cases	b_m	w_m	ΔE	$Var[\Pi_C]$	$Var_D[\Pi_R]$	$Var_D[\Pi_M]$	ΔU
Coordinated	6.44	6.82	0.00	201.32	113.24	12.58	0.00
1	6.44	6.68	-0.40	201.32	116.26	12.92	0.00
2	6.44	6.95	0.40	201.32	110.26	12.25	0.00
3	6.31	6.82	0.57	201.32	111.19	11.42	0.01
4	6.31	6.68	0.17	201.32	114.16	11.72	0.00
4	6.31	6.55	-0.22	201.32	117.17	12.03	0.01
5	6.31	6.95	0.96	201.32	108.26	11.12	0.03
6	6.57	6.82	-0.58	201.32	115.35	13.85	0.01
7	6.57	6.68	-0.98	201.32	118.42	14.22	0.03
8	6.57	7.09	0.23	201.32	109.33	13.12	0.01
8	6.57	6.95	-0.17	201.32	112.32	13.48	0.01

4.2.2.2 Effects of k_m, k_r and r on the trends of ΔE and ΔU

The results from Section 4.2.2.1 are further analyzed to examine the trends of ΔE and ΔU from varying k_m, k_r, r, and p. However, differing from the previous section, the values will be grouped with each p value. The trends can be categorized in common patterns as shown in Table 4.14. Examples of these trends are shown in Figures 4.34 – 4.39.

Table 4.14 Effects of k_m, k_r and r on the trends of ΔE and ΔU with each fixed p.

Value		Trends		Parameter		
ΔU	ΔE	ΔU	ΔE	k_m	k_r	r
>0	>0	downward	downward	increase	fixed	fixed
		upward	upward	fixed	increase	fixed
		upward	upward	fixed	fixed	increase
>0	<0	downward	upward	increase	fixed	fixed
		upward	downward	fixed	increase	fixed
		upward	downward	fixed	fixed	increase

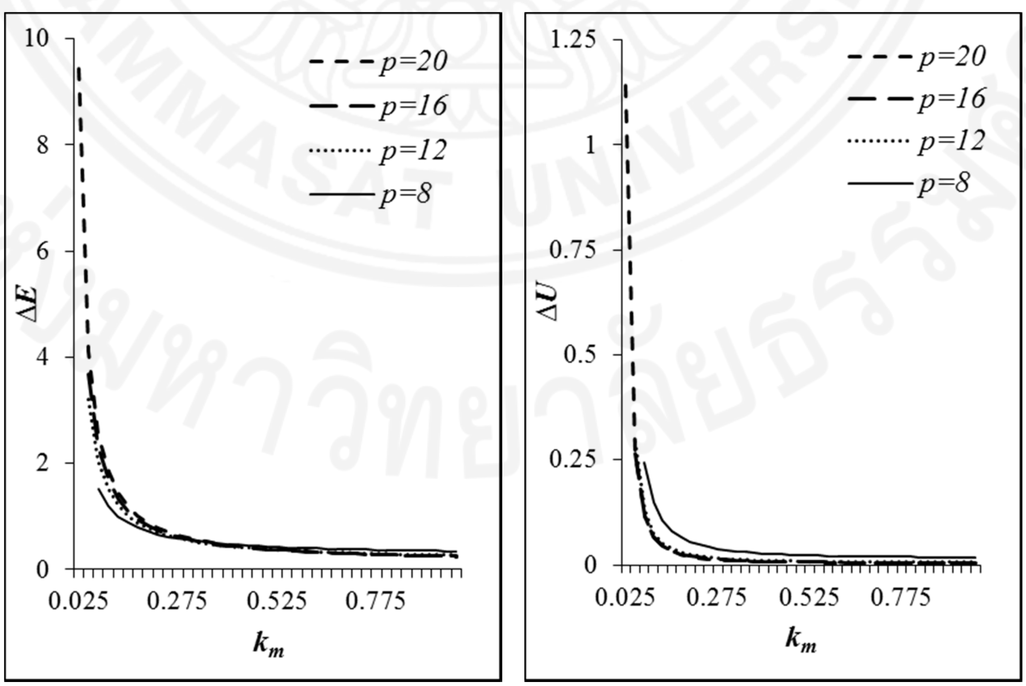

Figure 4.34 The effects of k_m on ΔE (>0) and ΔU.

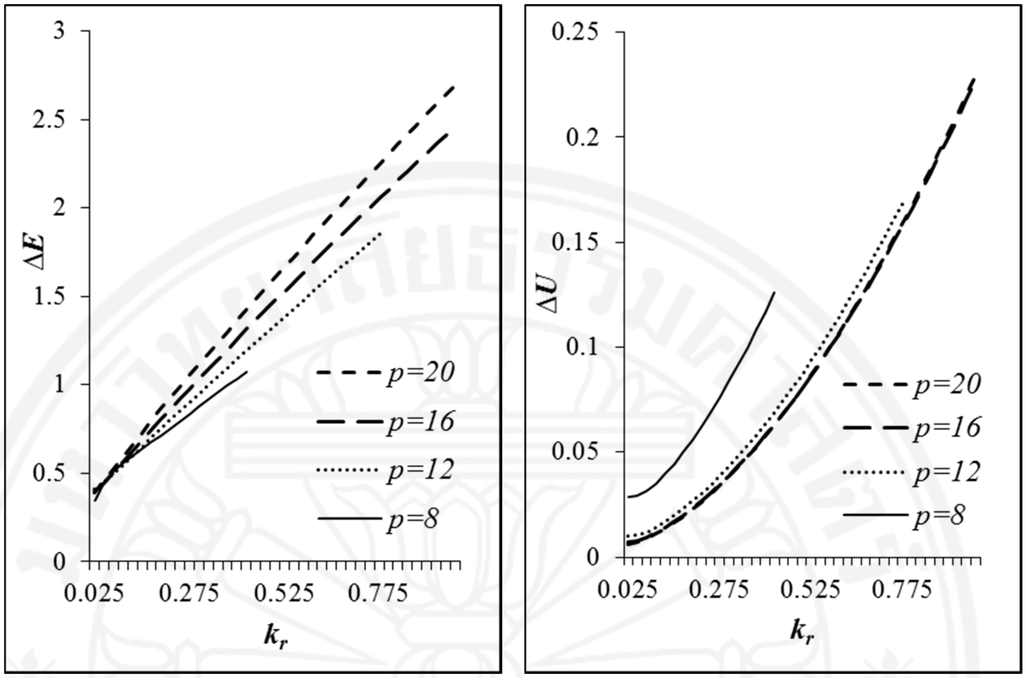

Figure 4.35 The effects of k_r on ΔE (>0) and ΔU.

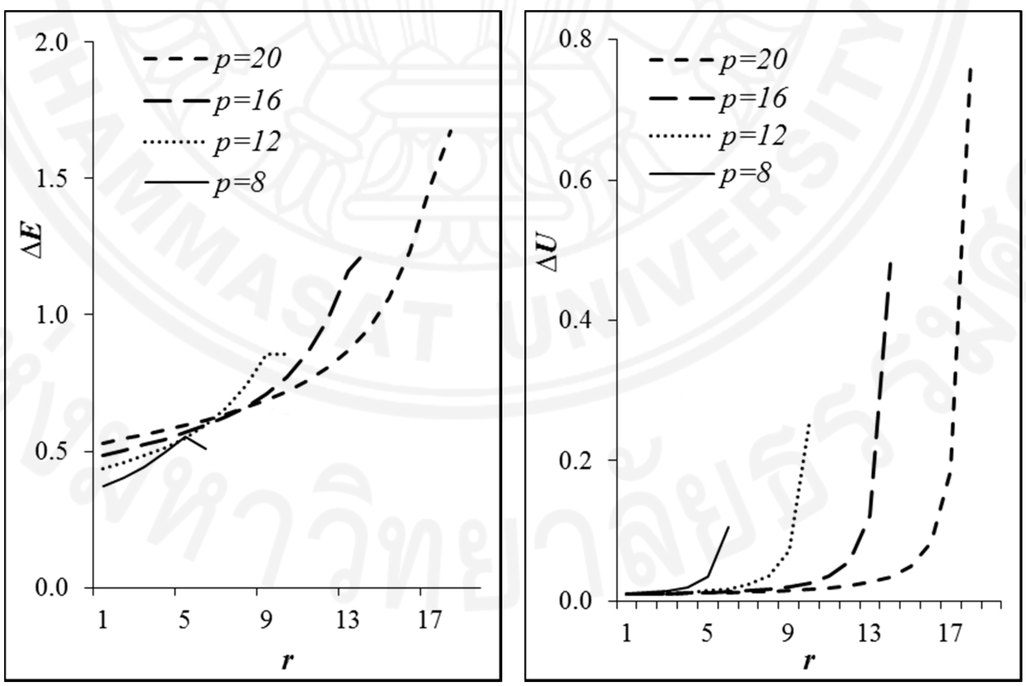

Figure 4.36 The effects of r on ΔE (>0) and ΔU.

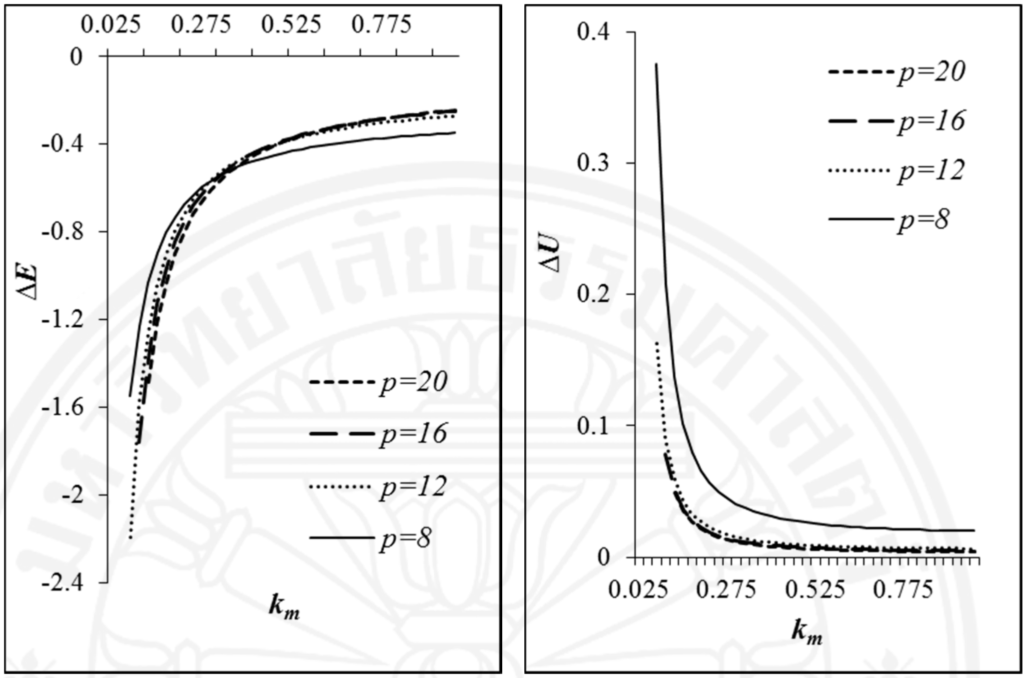

Figure 4.37 The impacts of k_m on ΔE (<0) and ΔU.

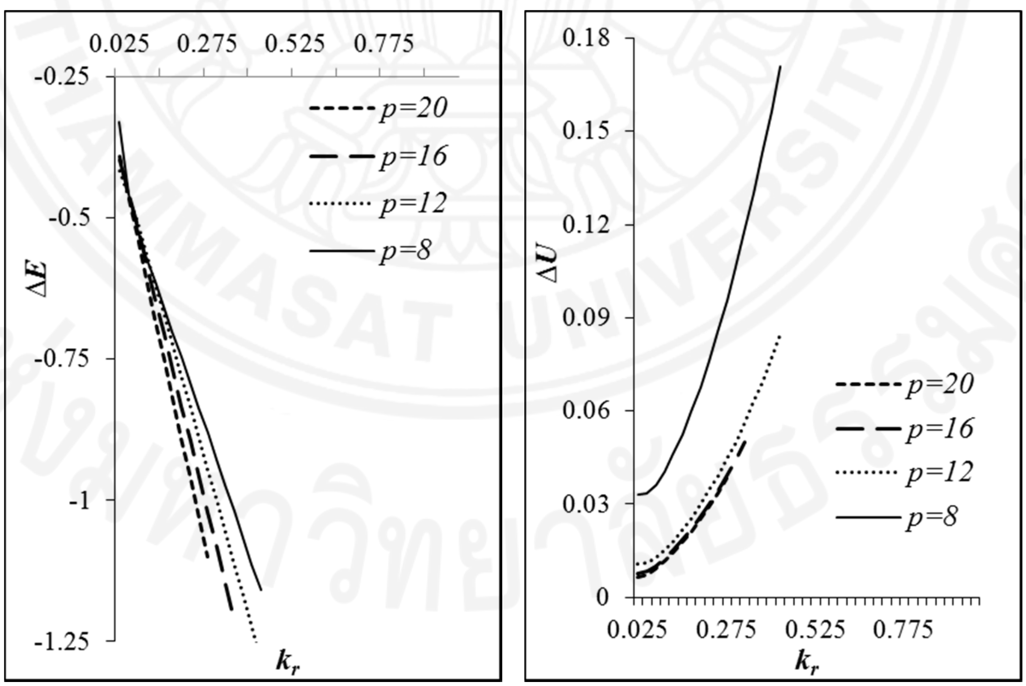

Figure 4.38 The impacts of k_r on ΔE (<0) and ΔU.

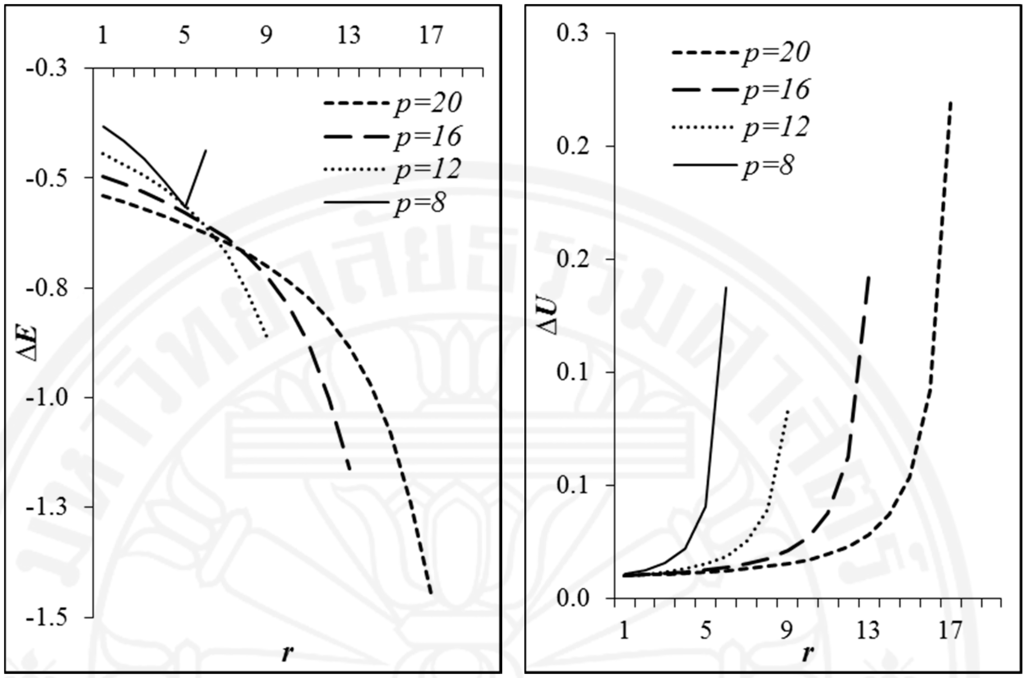

Figure 4.39 The impacts of r on ΔE (<0) and ΔU.

- From Figure 4.34 ($k_r = 0.1$, $r = 5$, $k_m \in [0.025, 0.975]$, and $p \in [8, 12, 16, 20]$), when the manufacturer makes off-optimal decision(s) to increase their profit, the losses on both the expected profit and utility of profit for the entire SC decrease as the manufacturer's risk attitude increases with a fixed p. These trends will be similar to many p.

- In Figure 4.35 ($k_m = 0.3$, $r = 5$, $k_r \in [0.025, 0.975]$, $p \in [8, 12, 16, 20]$) and Figure 4.36 ($k_r = 0.1$, $k_m = 0.3$, $p \in [8, 12, 16, 20]$ and $r \in [1, 2, \ldots, p - 1]$), when the manufacturer makes off-optimal decision(s) for their benefit, the losses on both ΔE and ΔU worsen, as the retailer's risk attitude increases (equivalently, the retailer's order quantity decreases), or as the refund amount (to stimulate customer demands) increases, respectively. However, ΔE starts to decrease if the retailer increases r to $(p - 2)$. These results are only found with a fixed p.

- However, at a fixed retail price, when the manufacturer makes decision(s) to stimulate retailer's demand, the loss in the utility is rather small and decreasing, while the gain in the expected profit diminishes quickly as the manufacturer's risk attitude (to balance the effect of their decisions) increases. Figure 4.37 shows an example with $k_r = 0.1$, $r = 5$, $k_m \in [0.025, 0.975]$, and each $p \in [8, 12, 16, 20]$).

- The opposites can be seen in Figure 4.38 ($k_m = 0.3$, $r = 5$, and $k_r \in [0.025, 0.975]$) and Figure 4.39 ($k_r = 0.1$, $k_m = 0.3$, $p \in [8, 12, 16, 20]$ and $r \in [1, 2, \ldots, p-1]$). When the manufacturer attempts to stimulate the retailer's demand, the loss in the utility of profit for the whole SC becomes worsen, while the gain in the expected profit is higher, as the retailer's risk attitude increases (i.e. as the retailer's order quantity decreases) and a fixed retail price, or as refund amount increases with a fixed retail price. However, ΔE starts to decrease if the retailer increases r to $(p-2)$.

According to the results above, the SC with lower k_m, higher k_r, or higher r have more motivations to reach the coordination situation. In addition, for the SC that is coordinated through utility function, it is natural that the utility values would be less sensitive to changes in k_m, k_r, and r than the expected profit value, i.e. both the magnitudes of ΔE are larger than those of ΔU, and the slopes of the trends of ΔE are steeper than those of the trends of ΔU in Figures 4.34 – 4.39. However, when the retailer increases the refund amount closed to the retail price, the SC in these positions will tend to improve the utilities of profits in reaching the coordination situation in Figures 4.36 and 4.39. Furthermore, even though the trend lines of ΔE and ΔU will be similar for all of p, the effects of p on both ΔE and ΔU are not shown clearly due to the multiple-change of b_m and w_m.

Chapter 5
Conclusions and Recommendations

5.1 Conclusions

In this thesis, a two-stage supply chain with buyback contract and end customer returns policy is considered under two patterns of demand: refund-dependent demand and price-dependent demand. When the retail price is treated as exogenously given, the supply chain cannot be coordinated through a utility function with the unit shortage cost g in the mathematical formulation for both cases. In other words, the supply chain can be coordinated without considering g. Extensive computational experiments are conducted to investigate the effects of risk attitude parameters and customer return parameter or/and retail price parameter on the performance of the supply chain.

For refund-dependent demand, in the coordinated supply chain, increasing risk attitude for either agent would result in lower optimal order quantity from the retailer, which leads to losses in the utility and expected value of profit for the whole supply chain. In addition, the losses that occur would apply to the agent who decides to raise their risk attitude, while the other agent would gain some benefit. Furthermore, increasing the refund amount would always lead to benefits in the utility and expected profit to both agents. For uncoordinated supply chain, when an agent makes off-optimal decision(s) for their own benefit, it always leads to losses in both utility and expected profit of the entire supply chain, with utility value being the more robust performance. However, if off-optimal decision(s) are made in favor of the other agent, then it would incur small loss to the utility of profit, while gain larger expected profit for the entire supply chain.

For price-dependent demand, the effects of risk attitude parameters, refund amount, and retail price on the optimal decisions and measure of performance for both coordinated and uncoordinated supply chains are relatively similar to the case of refund-dependent demand. In addition, an analysis of the break-even points of the expected profit and utility of profit in the coordinated supply chain is performed. The results provide some guidelines of how retail price should be chosen for different values of risk-attitudes and refund amount in order to gain higher expected profit and utility of profit.

5.2 Recommendations

In this thesis, uncertain demand is considered as refund-dependent demand or price-dependent demand. Therefore, an interesting future research could be built for the coordination of the supply chain with both refund and price dependent demand in one function.

References

Agrawal, V., & Seshadri, S. (2000). Risk intermediation in supply chains. *IIE Transactions, 32*(9), 819-831.

Bose, I., & Anand, P. (2007). On returns policies with exogenous price. *European Journal of Operational Research, 178*(3), 782-788.

Cachon, G. P. (2003). Supply chain coordination with contracts. *Handbooks in operations research and management science, 11*, 227-339.

Che, Y.-K. (1996). Customer return policies for experience goods. *The Journal of Industrial Economics*, 17-24.

Chen, J., & Bell, P. C. (2009). The impact of customer returns on pricing and order decisions. *European Journal of Operational Research, 195*(1), 280-295.

Chen, J., & Bell, P. C. (2011). Coordinating a decentralized supply chain with customer returns and price-dependent stochastic demand using a buyback policy. *European Journal of Operational Research, 212*(2), 293-300.

Chen, J., & Bell, P. C. (2012). Implementing market segmentation using full-refund and no-refund customer returns policies in a dual-channel supply chain structure. *International Journal of Production Economics, 136*(1), 56-66.

Chen, J., & Bell, P. C. (2013). The impact of customer returns on supply chain decisions under various channel interactions. *Annals of Operations Research, 206*(1), 59-74.

Chen, J., & Grewal, R. (2013). Competing in a supply chain via full-refund and no-refund customer returns policies. *International Journal of Production Economics, 146*(1), 246-258.

Chiu, C.-H., Choi, T.-M., Hao, G., & Li, X. (2015). Innovative Menu of Contracts for Coordinating a Supply Chain with Multiple Mean-Variance Retailers. *European Journal of Operational Research*.

Choi, T.-M., Li, D., & Yan, H. (2008). Mean–variance analysis of a single supplier and retailer supply chain under a returns policy. *European Journal of Operational Research, 184*(1), 356-376.

Devangan, L., Amit, R., Mehta, P., Swami, S., & Shanker, K. (2013). Individually rational buyback contracts with inventory level dependent demand. *International Journal of Production Economics, 142*(2), 381-387.

Donohue, K. L. (2000). Efficient supply contracts for fashion goods with forecast updating and two production modes. *Management science, 46*(11), 1397-1411.

Emmons, H., & Gilbert, S. M. (1998). Note. The role of returns policies in pricing and inventory decisions for catalogue goods. *Management science, 44*(2), 276-283.

Granot, D., & Yin, S. Y. (2005). On the effectiveness of returns policies in the price-dependent newsvendor model. *Naval Research Logistics, 52*(8).

Hematyar, S., Chaharsooghi, K., & Malakafali, P. (2014). *Supply Chain Coordination with Consumer Returns using a Sales Rebate Contract and VMI partnership.* Paper presented at the IIE Annual Conference. Proceedings,(p. 3921).

Jiang, G., & Liu, J. (2014). *Research on the Supply Chain Coordination of the Buyback Contract Based on Sales Effort.* Paper presented at the Proceedings of the Seventh International Conference on Management Science and Engineering Management,(pp. 827-838).

Li, J., Choi, T.-M., & Cheng, T. E. (2014). Mean variance analysis of fast fashion supply chains with returns policy. *Systems, Man, and Cybernetics: Systems, IEEE Transactions on, 44*(4), 422-434.

Li, Y., Wei, C., & Cai, X. (2012). Optimal pricing and order policies with B2B product returns for fashion products. *International Journal of Production Economics, 135*(2), 637-646.

Li, Y., Xu, L., & Li, D. (2013). Examining relationships between the return policy, product quality, and pricing strategy in online direct selling. *International Journal of Production Economics, 144*(2), 451-460.

Liu, J., & He, Y. (2013). Coordinating a Supply Chain with Risk-Averse Agents under Demand and Consumer Returns Uncertainty. *Mathematical Problems in Engineering, 2013*.

Liu, J., Mantin, B., & Wang, H. (2014). Supply chain coordination with customer returns and refund-dependent demand. *International Journal of Production Economics, 148*, 81-89.

Markowitz, H. (1959). Portfolio selection: efficient diversification of investments. *Cowies Foundation Monograph*(16).

Petruzzi, N. C., & Dada, M. (1999). Pricing and the newsvendor problem: A review with extensions. *Operations Research, 47*(2), 183-194.

Ren, J., Xie, H., Liu, Y., Zeng, P., & Tao, Z. (2015). Coordinating a multi-retailer decentralized distribution system with random demand based on buyback and compensation contracts. *Journal of Industrial Engineering and Management, 8*(1), 203-216.

Ruiz-Benitez, R., & Muriel, A. (2014). Consumer returns in a decentralized supply chain. *International Journal of Production Economics, 147*, 573-592.

Shen, C., Zhang, X., & Ma, K. (2009). *Supply chain coordination for perishable products with consumer returns policies.* Paper presented at the Automation and Logistics, 2009. ICAL'09. IEEE International Conference on,(pp. 1062-1065).

Shi, J., & Xiao, T. (2015). Service investment and consumer returns policy in a vendor-managed inventory supply chain. *Journal of Industrial and Management Optimization, 11*(2), 439-459.

Song, Y., Ray, S., & Li, S. (2008). Structural properties of buyback contracts for price-setting newsvendors. *Manufacturing & Service Operations Management, 10*(1), 1-18.

Steger, T., Sprague, B., & Douthit, D. (2007). Big Trouble with No Trouble Found: How Consumer Electronics Firms Confront the High Cost of Customer Returns. *Accenture Communications & High tech, Accenture*.

Stock, J. H., Wright, J. H., & Yogo, M. (2002). A survey of weak instruments and weak identification in generalized method of moments. *Journal of Business & Economic Statistics, 20*(4).

Su, X. (2009). Consumer returns policies and supply chain performance. *Manufacturing & Service Operations Management, 11*(4), 595-612.

Toktay, L. B., van der Laan, E. A., & de Brito, M. P. (2004). *Managing product returns: the role of forecasting*: Springer.

Wang, F., & Choi, I.-C. (2014). Optimal decisions in a single-period supply chain with price-sensitive random demand under a buy-back contract. *Mathematical Problems in Engineering, 2014*.

Wang, X. B., & Wang, Y. H. (2013). *Supply Chain Coordination Based on Buyback Contract under the Imperfect Product Quality and Fuzzy Demand Environments.* Paper presented at the Advanced Materials Research,(pp. 2834-2839).

Wei, Y., & Choi, T.-M. (2010). Mean–variance analysis of supply chains under wholesale pricing and profit sharing schemes. *European Journal of Operational Research, 204*(2), 255-262.

Wu, D. (2013). Coordination of competing supply chains with news-vendor and buyback contract. *International Journal of Production Economics, 144*(1), 1-13.

Xiao, T., Shi, K., & Yang, D. (2010). Coordination of a supply chain with consumer return under demand uncertainty. *International Journal of Production Economics, 124*(1), 171-180.

Xu, L., Li, Y., Govindan, K., & Xu, X. (2015). Consumer returns policies with endogenous deadline and supply chain coordination. *European Journal of Operational Research, 242*(1), 88-99.

Yao, Z., Leung, S. C., & Lai, K. K. (2008). Analysis of the impact of price-sensitivity factors on the returns policy in coordinating supply chain. *European Journal of Operational Research, 187*(1), 275-282.

Yongwen, C., & Yongwu, Z. (2011). Supply chain coordination with buyback contract under risk constraints. *Journal of University of Science and Technology of China, 41*(3), 274-282.

Yoo, S. H. (2014). Product quality and return policy in a supply chain under risk aversion of a supplier. *International Journal of Production Economics, 154*, 146-155.

Yoo, S. H., Kim, D., & Park, M.-S. (2015). Pricing and return policy under various supply contracts in a closed-loop supply chain. *International Journal of Production Research, 53*(1), 106-126.

Yu-ming, X. (2013). *Analysis on supply chain coordination and profit allocation based on buyback contract.* Paper presented at the Management Science and Engineering (ICMSE), 2013 International Conference on,(pp. 585-591).

Zhang, L., & Yao, Z. (2014). Decision Making for a Risk-Averse Dual-Channel Supply Chain with Customer Returns *Decision Support Systems III-Impact of Decision Support Systems for Global Environments* (pp. 118-130): Springer.

Zhang, H., Zhou, Z., & Chen, Y. (2014). *Research on Buy-Back Contract for Supply Chain Coordination with Prospect Theory.* Paper presented at the Enterprise Systems Conference (ES), 2014,(pp. 125-129).

Zhao, Y., Choi, T.-M., Cheng, T., Sethi, S. P., & Wang, S. (2014). Buyback contracts with price-dependent demands: Effects of demand uncertainty. *European Journal of Operational Research, 239*(3), 663-673.

Zhao, Y., Choi, T.-M., Cheng, T., & Wang, S. (2014). Mean-risk analysis of wholesale price contracts with stochastic price-dependent demand. *Annals of Operations Research*, 1-28.

Zhou, Y., Chen, Q., Chen, X., & Wang, Z. (2014). Stackelberg Game of Buyback Policy in Supply Chain with a Risk-Averse Retailer and a Risk-Averse Supplier Based on CVaR.

Appendices

Appendix A

Proof of Lemma 1

$Var[\min(X,Q)] = 2z\Lambda(z) - 2\int_B^z \varepsilon F(\varepsilon)d\varepsilon - \Lambda(z)^2.$ (2)

$E[\Pi_c(z)] = (p+\alpha(s_2-r-l)-c)\big(D(*)-\Theta(z)\big)-(c-s_1)\Lambda(z)-g\Theta(z).$ (7)

$Var[\Pi_c(z)] = [p+\alpha(s_2-r-l)-s_1+g]^2 Var[\min(X,Q)] = [p+\alpha(s_2-r-l)-s_1+g]^2\big(2z\Lambda(z)-2\int_B^z \varepsilon F(\varepsilon)d\varepsilon-\Lambda(z)^2\big).$ (9)

$U[\Pi_c(z)] = E[\Pi_c(z)] - kVar[\Pi_c(z)].$ (10)

$\frac{dE[\Pi_c(z)]}{dz} = (p+\alpha(s_2-r-l)-c+g)-F(z)[p+\alpha(s_2-r-l)-s_1+g].$

$\frac{d^2E[\Pi_c(z)]}{dz^2} = -f(z)[p+\alpha(s_2-r-l)-s_1+g] < 0.$

$\rightarrow$ $E[\Pi_C(z)]$ is a concave function of z $\rightarrow$ $E[\Pi_c(z)]$ will reach maximum value when $z \in [B,+\infty)$

Let $z^*_{C,N}$ assumes for the optimal value of z when $E[\Pi_c(z)]$ will reach maximum value. The results are produced as follows:

- $E[\Pi_c(z)]$ is an increasing function of z when $z \in [B, z^*_{C,N})$ or $\frac{dE[\Pi_c(z)]}{dz} > 0.$
- $E[\Pi_c(z)]$ is a decreasing function of z when $z \in (z^*_{C,N}, +\infty)$ or $\frac{dE[\Pi_c(z)]}{dz} < 0.$
- $\frac{dE[c(z)]}{dz} = 0$ when $z = z^*_{C,N}$.

$\frac{dVar[\Pi_c(z)]}{dz} = 2[p+\alpha(s_2-r-l)-s_1+g]^2(1-F(z))\Lambda(z) \geq 0.$

$\frac{dVar[\Pi_C(z)]}{dz} = 0 \leftrightarrow \left[\begin{array}{l} 1-F(z)=0 \\ \Lambda(z)=0 \end{array}\right. \leftrightarrow \left[\begin{array}{l} F(z)=1 \rightarrow z=F^{-1}(1) \rightarrow z=+\infty \\ \int_B^z (z-\varepsilon)f(\varepsilon)d\varepsilon \rightarrow z=B \end{array}\right.$

$\rightarrow$ $Var[\Pi_c(z)]$ is an increasing function of z when $z \in (B,+\infty)$

Therefore, there are some results as follows:

- When $z = B$ $\rightarrow \begin{cases} \frac{dE[\Pi_c(B)]}{dz} > 0 \\ \frac{dVar[\Pi_c(B)]}{dz} = 0 \end{cases}$ $\rightarrow \frac{dU[c(B)]}{dz} = \frac{dE[\Pi_c(B)]}{dz} - k\frac{dVar[c(B)]}{dz} = \frac{dE[\Pi_c(B)]}{dz} > 0.$

- When $z = z^*_{C,N} \rightarrow \begin{cases} \frac{dE[\Pi_c(z^*_{C,N})]}{dz} = 0 \\ \frac{d\mathrm{Var}[\Pi_c(z^*_{C,N})]}{dz} = 0 \end{cases} \rightarrow \frac{dU[\Pi_c(z^*_{C,N})]}{dz} = \frac{dE[\Pi_c(z^*_{C,N})]}{dz} - k\frac{dVar[\Pi_c(z^*_{C,N})]}{dz} = -k\frac{dVar[\Pi_c(z^*_{C,N})]}{dz} < 0.$

There will exist a value of $z \in [B, z^*_{C,N}]$ for $\frac{dU[c(z)]}{dz} = 0$.

$$\frac{d^2\mathrm{Var}[\Pi_c(z)]}{dz^2} = 2[p + \alpha(s_2 - r - l) - s_1 + g]^2\big((1 - F(z))F(z) - f(z)\Lambda(z)\big) = 2[p + \alpha(s_2 - r - l) - s_1 + g]^2(1 - F(z))F(z) - 2[p + \alpha(s_2 - r - l) - s_1 + g]^2 f(z)\Lambda(z).$$

$$\frac{dU[\Pi_c(z)]}{dz} = \frac{dE[\Pi_c(z)]}{dz} - k\frac{dVar[\Pi_c(z)]}{dz} = [p + \alpha(s_2 - r - l) - c + g] - F(z)[p + \alpha(s_2 - r - l) - s_1 + g] - k\Big(2[p + \alpha(s_2 - r - l) - s_1 + g]^2\big(1 - F(z)\big)\Lambda(z)\Big) = (s_1 - c) + \big(1 - F(z)\big)\big([p + \alpha(s_2 - r - l) - s_1 + g] - 2k[p + \alpha(s_2 - r - l) - s_1 + g]^2\Lambda(z)\big).$$

$$\frac{d^2U[\Pi_c(z)]}{dz^2} = -f(z)[p + \alpha(s_2 - r - l) - s_1 + g] - k\Big(2[p + \alpha(s_2 - r - l) - s_1 + g]^2\big(1 - F(z)\big)F(z) - 2[p + \alpha(s_2 - r - l) - s_1 + g]^2 f(z)\Lambda(z)\Big) = -f(z)\big([p + \alpha(s_2 - r - l) - s_1 + g] - 2k[p + \alpha(s_2 - r - l) - s_1 + g]^2\Lambda(z)\big) - 2k[p + \alpha(s_2 - r - l) - s_1 + g]^2\big(1 - F(z)\big)F(z).$$

Let z^*_C denote the value of z when $\frac{dU[\Pi_c(z)]}{dz} = 0$. In addition, the second derivative of $U[\Pi_c(z)]$ respect to z at z^*_C is determined as follows:

Since $\frac{dU[\Pi_c(z^*_C)]}{dz^*_C} = 0 \leftrightarrow (s_1 - c) + \big(1 - F(z^*_C)\big)\big([p + \alpha(s_2 - r - l) - s_1 + g] - 2k[p + \alpha(s_2 - r - l) - s_1 + g]^2\Lambda(z^*_C)\big) = 0.$

$$\leftrightarrow \big(1 - F(z^*_C)\big)\big([p + \alpha(s_2 - r - l) - s_1 + g] - 2k[p + \alpha(s_2 - r - l) - s_1 + g]^2\Lambda(z^*_C)\big) = c - s_1.$$

Since $c > s_1 \rightarrow \big(1 - F(z^*_C)\big)\big([p + \alpha(s_2 - r - l) - s_1 + g] - 2k[p + \alpha(s_2 - r - l) - s_1 + g]^2\Lambda(z^*_C)\big) > 0 \rightarrow 1 - F(z^*_C) \neq 0.$

The above equation will become:

$$[p + \alpha(s_2 - r - l) - s_1 + g] - 2k[p + \alpha(s_2 - r - l) - s_1 + g]^2\Lambda(z^*_C) = \frac{c - s_1}{1 - F(z^*_C)}.$$

$$\frac{d^2U[\Pi_c(z_C^*)]}{dz_C^{*2}} = -f(z_C^*)\big([p+\alpha(s_2-r-l)-s_1+g] - 2k[p+\alpha(s_2-r-l)-s_1+g]^2\Lambda(z_C^*)\big) - 2k[p+\alpha(s_2-r-l)-s_1+g]^2\big(1-F(z_C^*)\big)F(z_C^*) =$$

$$-f(z_C^*)\left(\frac{c-s_1}{1-F(z_C^*)}\right) - 2k[p+\alpha(s_2-r-l)-s_1+g]^2\big(1-F(z_C^*)\big)F(z_C^*) =$$

$$-\left(f(z_C^*)\left(\frac{c-s_1}{1-F(z_C^*)}\right) + 2k[p+\alpha(s_2-r-l)-s_1+g]^2\big(1-F(z_C^*)\big)F(z_C^*)\right).$$

Since $\left.\begin{cases} \left.\begin{matrix} F(z_C^*) \in [0,1] \\ 1-F(z_C^*) \neq 0 \end{matrix}\right\} \rightarrow\ 1-F(z_C^*) > 0 \\ c > s_1 \end{cases}\right\} \rightarrow \frac{c-s_1}{1-F(z_C^*)} > 0.$

And $\left.\begin{cases} c > s_1 \\ p+\alpha(s_2-r-l)-c+g > 0 \end{cases}\right\} \rightarrow\ p+\alpha(s_2-r-l)-s_1+g > 0.$

From $\begin{cases} f(z_C^*) > 0 \\ F(z_C^*) \in [0,1] \\ 1-F(z_C^*) > 0 \\ k \in (0,1) \\ p+\alpha(s_2-r-l)-s_1+g > 0 \end{cases}$

$$\rightarrow\ f(z_C^*)\left(\frac{c-s_1}{1-F(z_C^*)}\right) + 2k[p+\alpha(s_2-r-l)-s_1+g]^2\big(1-F(z_C^*)\big)F(z_C^*) > 0.$$

$$\frac{d^2U[\Pi_c(z_C^*)]}{dz_C^{*2}} = -\left(f(z_C^*)\left(\frac{c-s_1}{1-F(z_C^*)}\right) + 2k[p+\alpha(s_2-r-l)-s_1+g]^2\big(1-F(z_C^*)\big)F(z_C^*)\right) < 0.$$

Therefore, z_C^* will be the optimal value of z, where $U[\Pi_c(z)]$ reach maximum value and z_C^* is found by solving the equation as follows:

$$\frac{dU[\Pi_c(z_C^*)]}{dz_C^*} = 0.$$

$$[p+\alpha(s_2-r-l)-c+g] - F(z_C^*)[p+\alpha(s_2-r-l)-s_1+g] - k\big(2[p+\alpha(s_2-r-l)-s_1+g]^2[1-F(z_C^*)]\Lambda(z_C^*)\big) = 0. \quad (11)$$

where $z_C^* \in [B, +\infty\ )$

Appendix B

Proof of Lemma 2

$$E_D[\Pi_R(z)] = (p + \alpha(b - l_r - r) - w + g)\big(D(*) - \Theta(z)\big) - (w - b)\Lambda(z) - g\Theta(z). \quad (14)$$

$$Var_D[\Pi_R(z)] = (p + \alpha(b - l_r - r) - b + g)^2 Var[\min(X, Q)] = (p + \alpha(b - l_r - r) - b + g)^2\left[2z\Lambda(z) - 2\int_B^z \varepsilon F(\varepsilon)d\varepsilon - \Lambda(z)^2\right]. \quad (16)$$

$$U_D[\Pi_R(z)] = E_D[\Pi_R(z)] - k_r Var_D[\Pi_R(z)]. \quad (17)$$

According to the illustration in Proof of Lemma 1 above, the similar results are found by using same way for the decentralized SC. Specifically, it can be concluded that $U[\Pi_R(z)]$ is a concave function of z and reaches the maximum value at $z_R^* \in [B, +\infty\ )$in the decentralized SC.

Therefore, $z_{R,A}^*$ is found by solving the equation $\frac{dU_D[\Pi_R(z)]}{dz} = 0$ as follows:

$$\{[p + \alpha(b - l_r - r) - w + g] - F(z_R^*)[p + \alpha(b - l_r - r) - b + g]\} - k_r\{2[p + \alpha(b - l_r - r) - b + g]^2[1 - F(z_R^*)]\Lambda(z_R^*)\} = 0. \quad (18)$$

where $z_R^* \in [B, +\infty\ )$

Appendix C
Proof of Proposition 1

SC coordination mechanism will use the Proposition 1 and Proof of Proposition 1. Therefore, two conditions for coordinating SC are shown as follows:

$$\begin{cases} z_C^* = z_R^* = z^* & \text{(Condition 1)} \\ U[\Pi_c(z_C^*)] - \{U_D[\Pi_M(z_R^*)] + U_D[\Pi_R(z_R^*)]\} = 0 & \text{(Condition 2)} \end{cases}$$

Condition 1:

From $z_C^* = z_R^* = z^*$, (11) and (18) will become

$$\begin{cases} (p + \alpha(s_2 - r - l) - c + g) - F(z^*)(p + \alpha(s_2 - r - l) - s_1 + g) - 2k(p + \alpha(s_2 - r - l) - s_1 + g)^2\big(1 - F(z^*)\big)\Lambda(z^*) = 0. \\ (p + \alpha(b_C - l_r - r) - w_C + g) - F(z^*)(p + \alpha(b_C - l_r - r) - b_C + g) - k_r 2(p + \alpha(b_C - l_r - r) - b_C + g)^2\big(1 - F(z^*)\big)\Lambda(z^*) = 0. \end{cases}$$

$$\rightarrow (p + \alpha(s_2 - r - l) - c + g) - F(z^*)(p + \alpha(s_2 - r - l) - s_1 + g) - 2k(p + \alpha(s_2 - r - l) - s_1 + g)^2\big(1 - F(z^*)\big)\Lambda(z^*) = (p + \alpha(b_c - l_r - r) - w_c + g) - F(z^*)(p + \alpha(b_c - l_r - r) - b_c + g) - k_r 2(p + \alpha(b_C - l_r - r) - b_c + g)^2\big(1 - F(z^*)\big)\Lambda(z^*)$$

$$w_c = (p + \alpha(b_c - l_r - r) + g) - F(z^*)(p + \alpha(b_c - l_r - r) - b_c + g) - k_r 2(p + \alpha(b_c - l_r - r) - b_c + g)^2\big(1 - F(z^*)\big)\Lambda(z^*) - (p + \alpha(s_2 - r - l) - c + g) + F(z^*)(p + \alpha(s_2 - r - l) - s_1 + g) + 2k(p + \alpha(s_2 - r - l) - s_1 + g)^2\big(1 - F(z^*)\big)\Lambda(z^*)$$

$$w_c = (p + \alpha(b_c - l_r - r) + g) - (p + \alpha(s_2 - r - l) - c + g) + F(z^*)(p + \alpha(s_2 - r - l) - s_1 + g) - F(z^*)(p + \alpha(b_c - l_r - r) - b_c + g) + 2k(p + \alpha(s_2 - r - l) - s_1 + g)^2\big(1 - F(z^*)\big)\Lambda(z^*) - k_r 2(p + \alpha(b_c - l_r - r) - b_c + g)^2\big(1 - F(z^*)\big)\Lambda(z^*)$$

$$w_c = (p + \alpha(b_c - l_r - r) + g - p - \alpha(s_2 - r - l) + c - g) + F(z^*)(p + \alpha(s_2 - r - l) - s_1 + g - p - \alpha(b_c - l_r - r) + b_c - g) + 2(k(p + \alpha(s_2 - r - l) - s_1 + g)^2 - k_r(p + \alpha(b_c - l_r - r) - b_c + g)^2)\big(1 - F(z^*)\big)\Lambda(z^*)$$

$$w_c = (\alpha(b_c - l_r - r - s_2 + r + l_r + l_m) + c) + F(z^*)(\alpha(s_2 - r - l_r - l_m - b_c + l_r + r) - s_1 + b_c) + 2(k(p + \alpha(s_2 - r - l_r - l_m) - s_1 + g)^2 - k_r(p + \alpha(b_c - l_r - r) - b_c + g)^2)\big(1 - F(z^*)\big)\Lambda(z^*)$$

$$w_c = (\alpha(b_c - s_2 + l_m) + c) + F(z^*)(\alpha(s_2 - l_m - b_c) - s_1 + b_c) + 2(k(p + \alpha(s_2 - r - l_r - l_m) - s_1 + g)^2 - k_r(p + \alpha(b_c - l_r - r) - b_c + g)^2)\big(1 - F(z^*)\big)\Lambda(z^*)$$

$$w_c = -\alpha(s_2 - l_m - b_c)(1 - F(z^*)) + c + F(z^*)[b_c - s_1] + 2(k(p + \alpha(s_2 - r - l_r - l_m) - s_1 + g)^2 - k_r(p + \alpha(b_c - l_r - r) - b_c + g)^2)\big(1 - F(z^*)\big)\Lambda(z^*). \tag{C1}$$

From (C1), the optimal whole-sale price (w_c) will be determined.

<u>Condition 2</u> will become

$$U[\Pi_c(z^*)] - \{U_D[\Pi_M(z^*)] + U_D[\Pi_R(z^*)]\} = 0$$

$$\leftrightarrow \{[p + \alpha(s_2 - r - l) - c][D(*) - \Theta(z^*)] - (c - s)\Lambda(z^*) - g\Theta(z^*)\} - k\{[p + \alpha(s_2 - r - l) + g - s_1]^2 Var[\min(X, Q^*)] + g^2 Var[\mathrm{X}]\} - \{[w_c - c - \alpha(b_c - s_2 + l_m)][D(*) + z^*] - [b_c - s_1 - \alpha(b_c - s_2 + l_m)]\Lambda(z^*)\} + k_m\{[b_C - \alpha(b_c - s_2 + l_m) - s_1]^2 Var[\min(X, Q^*)]\} - \{[p + \alpha(b_c - l_r - r) - w_c][D(*) - \Theta(z^*)] - (w_c - b_c)\Lambda(z^*) - g\Theta(z^*)\} + k_r\{[p + \alpha(b_c - l_r - r) + g - b_c]^2 Var[\min(X, Q^*)] + g^2 Var[\mathrm{X}]\} = 0.$$

Since $\{[p + \alpha(s_2 - r - l) - c][D(*) - \Theta(z^*)] - (c - s)\Lambda(z^*) - g\Theta(z^*)\} - \{[w_c - c - \alpha(b_c - s_2 + l_m)][D(*) + z^*] - [b_c - s_1 - \alpha(b_c - s_2 + l_m)]\Lambda(z^*)\} - \{[p + \alpha(b_c - l_r - r) - w_c][D(*) - \Theta(z^*)] - (w_c - b_c)\Lambda(z^*) - g\Theta(z^*)\} = 0.$

The above equation will become as follows:

$$-k\{[p + \alpha(s_2 - r - l) + g - s_1]^2 Var[\min(X, Q^*)] + g^2 Var[\mathrm{X}]\} + k_m\{[b_c - \alpha(b_c - s_2 + l_m) - s_1]^2 Var[\min(X, Q^*)]\} + k_r\{[p + \alpha(b_c - l_r - r) + g - b_c]^2 Var[\min(X, Q^*)] + g^2 Var[\mathrm{X}]\} = 0.$$

$$\leftrightarrow \ \frac{k_r k_m}{k_r + k_m}\{[p + \alpha(s_2 - r - l) + g - s_1]^2 Var[\min(X, Q^*)] + g^2 Var[\mathrm{X}]\} + k_m\{[b_c - \alpha(b_c - s_2 + l_m) - s_1]^2 Var[\min(X, Q^*)]\} + k_r\{[p + \alpha(b_c - l_r - r) + g - b_c]^2 Var[\min(X, Q^*)] + g^2 Var[\mathrm{X}]\} = 0.$$

$$\leftrightarrow (k_m + k_r)(1-\alpha)^2 Var[\min(X, Q^*)]{b_c}^2 - \{2(1-\alpha)\big[[\alpha(l_m - s_2) + s_1]k_m + [p - \alpha(l_r + r) + g]k_r\big]Var[\min(X, Q^*)]\}b_c - \frac{k_r k_m}{k_r + k_m}\{[p + \alpha(s_2 - r - l) + g - s_1]^2 Var[\min(X, Q^*)] + g^2 Var[\mathrm{X}]\} + k_m[\alpha(l_m - s_2) + s_1]^2 Var[\min(X, Q^*)] + k_r[p - \alpha(l_r + r) + g]^2 Var[\min(X, Q^*)] + k_r g^2 Var[\mathrm{X}] = 0.$$

Set:

$$\Delta = 4(1-\alpha)^2\big[[\alpha(l_m - s_2) + s_1]k_m + [p - \alpha(l_r + r) + g]k_r\big]^2 Var[\min(X, Q^*)]^2 + 4(k_m + k_r)(1-\alpha)^2 Var[\min(X, Q^*)]\left(\left\{\frac{k_r k_m}{k_r + k_m}[p + \alpha(s_2 - r - l) + g - s_1]^2 - k_m[\alpha(l_m - s_2) + s_1]^2 - k_r[p - \alpha(l_r + r) + g]^2\right\}Var[\min(X, Q^*)] + \left(\frac{k_r k_m}{k_r + k_m} - k_r\right)g^2 Var[\mathrm{X}]\right)$$

$$= 4(1-\alpha)^2\left\{\big[[\alpha(l_m - s_2) + s_1]k_m + [p - \alpha(l_r + r) + g]k_r\big]^2 + [p + \alpha(s_2 - r - l) + g - s_1]^2 k_r k_m - k_m(k_m + k_r)[\alpha(l_m - s_2) + s]^2 - k_r(k_m + k_r)[p - \alpha(l_r + r) + g]^2\right\}Var[\min(X, Q^*)]^2 + 4(1-\alpha)^2(k_m + k_r)\left(\frac{k_r k_m}{k_r + k_m} - k_r\right)g^2 Var[\mathrm{X}]Var[\min(X, Q^*)]$$

$$= 4(1-\alpha)^2\left\{\big[[\alpha(l_m - s_2) + s_1]k_m + [p - \alpha(l_r + r) + g]k_r\big]^2 + [p + \alpha(s_2 - r - l) + g - s_1]^2 k_r k_m - k_m(k_m + k_r)[\alpha(l_m - s_2) + s_1]^2 - k_r(k_m + k_r)[p - \alpha(l_r + r) + g]^2\right\}Var[\min(X, Q^*)]^2 + 4(1-\alpha)^2(k_m + k_r)\left(\frac{k_r k_m}{k_r + k_m} - k_r\right)g^2 Var[\mathrm{X}]Var[\min(X, Q^*)]$$

$$= 4(1-\alpha)^2\{2k_r k_m[\alpha(l_m - s_2) + s_1][p - \alpha(l_r + r) + g] + [p + \alpha(s_2 - r - l) + g - s_1]^2 k_r k_m - k_r k_m[\alpha(l_m - s_2) + s_1]^2 - k_r k_m[p - \alpha(l_r + r) + g]^2\}Var[\min(X, Q^*)]^2 + 4(1-\alpha)^2(k_m + k_r)\left(\frac{k_r k_m}{k_r + k_m} - k_r\right)g^2 Var[\mathrm{X}]Var[\min(X, Q^*)]$$

$$= 4(1-\alpha)^2\{[p + \alpha(s_2 - r - l) + g - s_1]^2 k_r k_m - k_r k_m[\alpha(l_m - s_2) + s_1]^2 - k_r k_m[p - \alpha(l_r + r) + g]^2 + 2k_r k_m[\alpha(l_m - s_2) + s_1][p - \alpha(l_r + r) +$$

$$g]\}Var[\min(X,Q^*)]^2 + 4(1-\alpha)^2(k_m + k_r)\left(\frac{k_r k_m}{k_r + k_m} - k_r\right) g^2 Var[\mathrm{X}]Var[\min(X,Q^*)]$$

$$= 4(1-\alpha)^2\{[p+\alpha(s_2 - r - l) + g - s_1]^2 k_r k_m - k_r k_m([p-\alpha(l_r + r) + g]^2 - 2[\alpha(l_m - s_2) + s_1][p - \alpha(l_r + r) + g] + [\alpha(l_m - s_2) + s_1]^2)\}Var[\min(X,Q^*)]^2 + 4(1-\alpha)^2(k_m + k_r)\left(\frac{k_r k_m}{k_r + k_m} - k_r\right) g^2 Var[\mathrm{X}]Var[\min(X,Q^*)]$$

$$= 4(1-\alpha)^2\{[p+\alpha(s_2 - r - l) + g - s_1]^2 k_r k_m - k_r k_m[p + \alpha(s_2 - r - l) + g - s_1]^2\}Var[\min(X,Q^*)]^2 + 4(1-\alpha)^2(k_m + k_r)\left(\frac{k_r k_m}{k_r + k_m} - k_r\right) g^2 Var[\mathrm{X}]Var[\min(X,Q^*)]$$

$$= 4(1-\alpha)^2(k_m + k_r)\left(\frac{k_r k_m}{k_r + k_m} - k_r\right) g^2 Var[\mathrm{X}]Var[\min(X,Q^*)]$$

$$= 4(1-\alpha)^2\left(-{k_r}^2\right) g^2 Var[\mathrm{X}]Var[\min(X,Q^*)]$$

$$= -4(1-\alpha)^2 {k_r}^2 g^2 Var[\mathrm{X}]Var[\min(X,Q^*)]$$

$$\text{From}\begin{cases} \alpha \in (0,1) \\ k_r \in (0,1) \\ g > 0 \\ Var[\mathrm{X}] > 0 \\ Var[\min(X,Q^*)] \end{cases} \rightarrow \Delta < 0$$

There is no solution for the optimal buyback price.

Appendix D
Proof of Proposition 2

According to Appendix C. Proof of Proposition 1, when the unit shortage cost g for unsatisfying the customer demand and fund return policy is not considered ($g = 0$ and $r < p$). The value of w_C in (C1) will become

$$w_c = -\alpha(s_2 - l_m - b_c)(1 - F(z^*)) + c + F(z^*)[b_c - s_1] + 2(1 - F(z^*))(k[p + \alpha(s_2 - r - l_r - l_m) - s_1]^2 - k_r[p + \alpha(b_c - l_r - r) - b_c]^2)\Lambda(z^*). \tag{25}$$

When $g = 0 \rightarrow \Delta = -4(1-\alpha)^2 {k_r}^2 g^2 Var[\mathrm{X}]Var[\min(X, Q^*)] = 0$. The optimal buyback price (b_c) is determined as follows:

$$b_c = \frac{[[\alpha(l_m - s_2) + s_1]k_m + [p - \alpha(l_r + r)]k_r]}{(k_m + k_r)(1 - \alpha)}. \tag{26}$$

Appendix E

Proof of The results from Proposition 2

Part 1: Refund-dependent demand

Case 1: When k_r, r fixed and k_m increases, there are some results as follows:

1. b_c decreases.

The trend of b_c is found by (26) as follows:

$$b_c = \frac{[[\alpha(l_m - s_2) + s_1]k_m + [p - \alpha(l_r + r)]k_r]}{(k_m + k_r)(1-\alpha)}$$

Set $b_c = y(k_m)$, the first derivative of $y(k_m)$ with respect to k_m *is* determined as follows:

$$\frac{\partial b_c}{\partial k_m} = \frac{\partial y(k_m)}{\partial k_m} = -\frac{[p + \alpha(s_2 - l_r - l_m - r) - s_1]}{(1-\alpha)} x \frac{k_r}{(k_m + k_r)^2}$$

According to the condition for positive value of the SC's expected profit discussed above, the respective function can be derived as follows:

$$\begin{cases} (p + \alpha(s_2 - r - l) - c) > 0. \\ s_1 < c \end{cases} \rightarrow [p - \alpha(l + r - s_2) - s_1] > 0.$$

where $l = l_r + l_m$.

$\rightarrow \frac{\partial y(k_m)}{\partial k_m} < 0 \rightarrow y(k_m)$ is a decreasing function of $k_m \in (0,1)$.

➔When k_m increases, b_C will be decreased.

2. w_c decreases.

When $\begin{cases} z_C^* = z_R^* = z^* \\ g = 0 \\ r < p \end{cases}$, (18) will become:

$$\{[p + \alpha(b_c - l_r - r) - w_c] - F(z^*)[p + \alpha(b_c - l_r - r) - b_c]\} - k_r\{2[p + \alpha(b - l_r - r) - b_c]^2[1 - F(z^*)]\Lambda(z^*)\} = 0.$$

The trend of w_C is found by the above equation as follows:

$$w_C = \{[p + \alpha(b_c - l_r - r)] - F(z^*)[p + \alpha(b_c - l_r - r) - b_c]\} - k_r\{2[p + \alpha(b - l_r - r) - b_c]^2[1 - F(z^*)]\Lambda(z^*)\}.$$

Let denote $w_C = y(b_c)$, the first derivative function of $y(b_c)$ with respect to b_c is determined as follows:

$$\frac{\partial y(b_c)}{\partial b_C} = \{\alpha + F(z^*)[1-\alpha]\} + 4k_r\{[p + \alpha(b_c - l_r - r) - b_c][1 - \alpha][1 - F(z^*)]\Lambda(z^*)\}.$$

From

$$\begin{cases} \begin{cases} (p + \alpha(b_c - l_r - r) - w_c) > 0. \\ b_c < w_c \end{cases} \rightarrow p + \alpha(b_c - l_r - r) - b_c > 0. \\ \alpha \in (0,1) \\ F(z^*) \in [0,1] \\ \Lambda(z^*) \geq 0 \\ k_r \in (0,1) \end{cases}$$

$$\rightarrow \{\alpha + F(z^*)[1-\alpha]\} + 4k_r\{[p + \alpha(b_c - l_r - r) - b_c][1-\alpha][1 - F(z^*)]\Lambda(z^*)\} \geq 0.$$

When $\{\alpha + F(z^*)[1-\alpha]\} + 4k_r\{[p + \alpha(b_c - l_r - r) - b_c][1-\alpha][1 - F(z^*)]\Lambda(z^*)\} = 0.$

$$\begin{cases} \alpha + F(z^*)[1-\alpha] = 0 \leftrightarrow F(z^*)[1-\alpha] \leftrightarrow F(z^*) = 0. \\ 4k_r[p + \alpha(b_c - l_r - r) - b_c][1-\alpha] \\ [1 - F(z^*)]\Lambda(z^*) = 0 \leftrightarrow \left[\begin{array}{l} 1 - F(z^*) = 0 \\ \Lambda(z^*) = 0 \end{array} \right. \end{cases}$$

$$\leftrightarrow \begin{cases} F(z^*) = 0 \\ \left[\begin{array}{l} F(z^*) = 1 \\ \Lambda(z^*) = 0 \end{array} \right. \leftrightarrow \left[\begin{array}{l} \begin{cases} F(z^*) = 1 \\ F(z^*) = 0 \end{cases} \rightarrow \text{No meaning} \\ \begin{cases} F(z^*) = 0 \leftrightarrow z^* = B \leftrightarrow z^* = B \\ \Lambda(z^*) = 0 \leftrightarrow z^* = B \end{cases} \end{array} \right. \end{cases}$$

$$\rightarrow \{\alpha + F(z^*)[1-\alpha]\} + 4k_r\{[p + \alpha(b_c - l_r - r) - b_c][1-\alpha][1 - F(z^*)]\Lambda(z^*)\} > 0, \forall z^* \in (B, +\infty)$$

$\rightarrow \frac{\partial y(b_c)}{\partial b_C} > 0 \rightarrow y(b_c)$ is an increasing function of b_c.

→ When b_C increases, w_C will be increased.

According to the discussion above about the trend of b_c, when k_m increases; b_c decreases.

Therefore, k_m increases, w_c will be decreased.

3. Q^* decreases.

$$U[\Pi_c(z)] = E[\Pi_c(z)] - kVar[\Pi_c(z)]. \quad (10)$$

When the supply chain is coordinated $\begin{cases} z_C^* = z_R^* = z^* \\ g = 0 \\ r < p \end{cases}$, (10) will become

$$U[\Pi_c(z^*)] = E[\Pi_c(z^*)] - kVar[\Pi_c(z^*)].$$

Following the Appendix 1, $z_{C,}^* \in [B, z_{C,N}^*] \rightarrow z^* \in [B, z_{C,N}^*]$.

And $kVar[\Pi_c(z^*)] \geq 0 \rightarrow U[\Pi_c(z^*)] \leq E[\Pi_c(z^*)]$.

- When $kVar[\Pi_c(z^*)] = 0 \leftrightarrow U[\Pi_c(z^*)] = E[\Pi_c(z^*)] \rightarrow z^* = z_{C,N}^*$.
- When $kVar[\Pi_c(z^*)] > 0 \leftrightarrow U[\Pi_c(z^*)] < E[\Pi_c(z^*)] \rightarrow z^* \in [B, z_{C,N}^*)$.

Besides, when the supply chain's risk attitude (k) decreases to 0, $U[\Pi_C(*)]$ will be increased to $E[\Pi_C(*)]$ and z^* increases to $z_{C,N}^*$. Therefore, k decreases, z^* increases.

In addition, $k = k_r.k_m/(k_r + k_m)$ is an increasing function of $k_m \rightarrow$ When k_m decreases, k will be decreased$\rightarrow$ When k_m decreases, z^* increases.

Therefore, k_m increases, z^* decreases. Thus, Q^* also decreases.

4. $\boldsymbol{E[\Pi_C(z^*)]}$ and $\boldsymbol{U[\Pi_C(z^*)]}$ decrease.

When $\begin{cases} z_C^* = z_R^* = z^* \\ g = 0 \\ r < p \end{cases}$, (7) will become:

$$E[\Pi_c(z^*)] = (p + \alpha(s_2 - r - l) - c)\big(D(*) - \Theta(z^*)\big) - (c - s_1)\Lambda(z^*)$$

According to Appendix A. Proof of Lemma 1 above, it can be also proved that $E[\Pi_C(z_C^*)]$ is an increasing function of z_C^* because $z_C^* \in [B, z_{C,N}^*)$.

Therefore, $E[\Pi_c(z^*)]$ is also an increasing function of $z^* \rightarrow z^*$ decreases, $E[\Pi_C(z^*)]$ will be decreased.

According to discussions above for Q^***,** when k_m increases → z^* will be decreased.

When k_m increases, $E[\Pi_c(z^*)]$ is also decreased.

In addition, when $\begin{cases} z_C^* = z_R^* = z^* \\ g = 0 \\ r < p \end{cases}$, (10) will become:

$U[\Pi_c(z^*)] = E[\Pi_c(z^*)] - kVar[\Pi_c(z^*)]$.

→ When k increases, $U[\Pi_c(z^*)]$ will be decreased.

When k_m increases → k also increases. Therefore, when k_m increases, $U[\Pi_c(z^*)]$ will also be decreased.

Case 2: When r, k_m fixed and k_r increases, there are some results as follows:

1. b_c is increased.

The trend of b_c is found by (26) as follows:

$$b_c = \frac{[[\alpha(l_m - s_2) + s_1]k_m + [p - \alpha(l_r + r)]k_r]}{(k_m + k_r)(1-\alpha)}$$

Let denote $b_c = y(k_r)$. The first derivative function of $y(k_r)$ with respect to k_r is determined as follows:

$$\frac{\partial b_c}{\partial k_r} = \frac{\partial y(k_r)}{\partial k_r} = \frac{[p - \alpha(l_r + l_m + r - s_2) - s_1]}{(1-\alpha)} x \frac{k_m}{(k_m + k_r)^2}$$

According to the condition for positive value of the supply chain's expected profit discussed above, it is illustrated as follows:

$$\begin{cases} (p + \alpha(s_2 - r - l) - c) > 0 \\ s_1 < c \end{cases} \rightarrow [p - \alpha(l + r - s_2) - s_1] > 0$$

where $l = l_r + l_m$.

$\rightarrow \frac{\partial y(k_r)}{\partial k_r} > 0 \rightarrow y(k_r)$ is an increasing function of k_r.

When k_r increases, b_c will be increased.

2. w_c is increased.

According to discussions in Case 1 for w_c, it is also resulted that when b_C increases, w_c will be increased.

In addition, when k_r increases, b_c decreases.

Therefore, k_r increases, w_c also decreases.

3. Q^* is decreased.

According to discussions in Case 1 for Q^*, it is also illustrated that when k decreases, z^* increases.

In addition, $k = k_r . k_m/(k_r + k_m)$ is an increasing function of k_r → k_r decreases, k will be decreased→ When k_r decreases, z^* increases.

Therefore, k_r increases, z^* decreases. Thus, Q^* also decreases.

4. $E[\Pi_C(z^*)]$ and $U[\Pi_C(z^*)]$ are decreased.

According to discussions in Case 1 for $E[\Pi_C(z^*)]$, it is also proved that when z^* decreases, $E[\Pi_C(z^*)]$ will be decreased.

Following discussions above for Q^***,** when k_r increases → z^* will be decreased.

When k_r increases, $E[\Pi_C(z^*)]$ will also be decreased

According to discussions in Case 1 for $U[\Pi_C(z^*)]$, the following result is found that when k increases, $U[\Pi_C(z^*)]$ will be decreased. Besides, when k_r increases → k also increases.

Therefore, when k_r increases, $U[\Pi_C(z^*)]$ will be decreased.

Case 3: When k_r, k_m fixed and r increases, there are some results as follows:

1. b_C increases.

The trend of b_c is found by (26) as follows:

$$b_C = \frac{[[\alpha(l_m - s_2) + s_1]k_m + [p - \alpha(l_r + r)]k_r]}{(k_m + k_r)(1-\alpha)}$$

Let denote $b_C = y(r)$.

According to our assumptions above, the returned rate $\alpha = prob(v < r) = \int_v^r h(v)dv = H(r)$. Therefore, the respective function can be derived as follows:

$$y(r) = \frac{[[H(r)(l_m - s_2) + s_1]k_m + [p - H(r)(l_r + r)]k_r]}{(k_m + k_r)(1 - H(r))}$$

$$\frac{\partial y(r)}{\partial r} = \frac{[l_m - s_2 + s_1]h(r)k_m + [p - l_r - r]h(r)k_r}{(k_m + k_r)(1 - H(r))^2}$$

According to our assumptions, $[p - l_r - r] > 0$. Besides, it was assumed $s_1 > s_2 \rightarrow l_m - s_2 + s_1 > 0$.

Therefore, $\frac{\partial y(r)}{\partial r} > 0 \rightarrow y(r)$ is an increasing function of $r \rightarrow$ When r increases the optimal buyback price (b_C) will be increased.

2. w_c increases.

According to discussions in Case 1 for w_C, it is also resulted that when b_C increases, w_C will be increased.

In addition, when r increases, b_C increases.

Therefore, r increases, w_C also increases.

3. Q^* increases.

When $z_C^* = z_R^* = z^*$, the equation of the order quantity is optimized as $Q^* = D(r) + z^*$, where $D(r)$ is an increasing function of r.

Therefore, Q^* is an increasing function of $r \rightarrow$ When r increases, Q^* also increases.

The results in the expected profits and the utilities of profits in the SC will be shown in the numerical examples part.

Part 2: Price-dependent demand

Case 1: When k_r, p, r, fixed and k_m increases.

There are the same results and same ways to illustrate as Part 1. Case 1 in Refund-dependent demand case above for b_c, w_c, and Q_c.

Case 2: When p, r, k_m, fixed and k_r increases.

There are the same results and same ways to illustrate as Part 1. Case 2 in Refund-dependent demand case above for b_c, w_c, and Q_c.

Case 3: When k_r, k_m, p fixed and r increases.

There are the same results and same ways to illustrate as Part 1. Case 3 in Refund-dependent demand case above for b_c, w_c, and Q_c.

These above patterns from Cases 1-3 will be the same trends between varying p in our assumptions. This result is shown clearly via the numerical example part.

Case 4: When $\boldsymbol{k_r}$, $\boldsymbol{k_m}$, $\boldsymbol{r}$ fixed and $\boldsymbol{p}$ increases, there are some results as follows:

1. $\boldsymbol{b_C}$ increases.

The trend of b_c is found by (26) as follows:

$$b_c = \frac{[[\alpha(l_m - s_2) + s_1]k_m + [p - \alpha(l_r + r)]k_r]}{(k_m + k_r)(1 - \alpha)}.$$

Let denote $b_c = y(p)$.

$$\frac{\partial b_c}{\partial p} = \frac{\partial y(p)}{\partial p} = \frac{k_r}{(k_m + k_r)(1 - \alpha)}.$$

$$\rightarrow \frac{\partial y(p)}{\partial p} > 0 \rightarrow y(p) \text{ is an increasing function of } p.$$

When p increases, b_c will be increased.

2. $\boldsymbol{w_c}$ increases.

According to discussions in Case 1 for w_c, it is also resulted that when b_C increases, w_c will be increased.

In addition, when p increases, b_c decreases.

Therefore, p increases, w_c also decreases.

3. $\boldsymbol{Q^*}$ decreases.

According to discussions in assumptions above, Q^* is a decreasing function of p.

Therefore, p increases, Q^* also decreases.

The results in the expected profits and the utilities of profits in the SC will be shown in the numerical examples part.