OPTIMAL PROMOTION PRICE AND PERIOD: BI-LEVEL LINEAR PROGRAMMING APPROACH

BY

NATTANA TANGSANGOB

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF MASTER OF
ENGINEERING (LOGISTICS AND SUPPLY CHAIN SYSTEMS
ENGINEERING)
SIRINDHORN INTERNATIONAL INSTITUTE OF TECHNOLOGY
THAMMASAT UNIVERSITY
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A Thesis Presented

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Abstract

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Optimizing promotion price is a complicated issue facing by retailer in every competitive market. This research empirically analyzes the retailer's optimal decision on price and period. A mathematical programming model which is based on an integration of retailer's pricing decision and customer's response is proposed. The objective of this research is to utilize the model to understand how optimal promotion decision should be made to maximize retailer's profit while facing strategic customers who make a purchasing decision to minimize their purchasing and holding costs. To accomplish this study, we formulate a nonlinear bi-level programming model with discrete and continuous variables to influence the customers' purchasing decision in such a way that retailer plan. Then transform into an equivalent single model by applying Duality Theory and linearize the model which can be solve by IBM ILOG CPLEX. Further, we investigate four key parameters: (1) Competitor price; (2) Wholesale price; (3) Holding cost of customer; and (4) Demand which affect the optimal promotion price and period. The main contribution of this research is we prove that optimal solution for solving the customer model by linear programming model and integer programming model is equal. Further, the model can provide the retailer's optimal promotion discount strategy and inventory policy that is applicable for managers in industry and researchers in academic area.

Keywords: promotion period, retail price, optimization, bi-level linear programming

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Chapter 1
Introduction

1.1 Background

Retailer has developed a variety of sale promotion methods to attract customers' demand and increase the market share more than a decade. Actually, the promotion tools and promotion formats are changing all the time depending on trends in economy. A very popular practice is temporary discount promotion, the retailer offers price reduction for a short period of time and then increase it as normal price. To develop the price discount promotion, retailer need to consider two main decision and two related question in mind: promotion depth and promotion frequency, order quantity and time (Tellis & Zufryden, 1995). Consumers are expected to response to retailer promotion in some ways such as purchase acceleration and stock-piling. In order to provide an understanding about how promotion works, (Teunter, 2002) have identified and summarized the concept and theory from prior literature of sales promotion response and reaction mechanism. The possible effects of sales promotion have been decomposed into five mechanisms and discussed in detail which are brand switching, store switching, repeat purchasing, purchasing timing and category expansion. (Grewal et al., 2011) outlined the recent innovations in price and promotion finding, they highlighted research issue and provide benefit suggestions for development and improvement the future research through three main area; (1) targeting promotion, (2) price and promotion model, and (3) promotion design.

Whether in the past or in recent year, promotion optimization seems to be a major issue for retailers. In this thesis, we examines the decision of a retailer and customers who want to maximize their welfare in response to other actions. The problem is considered as a two-level decision problem. The retailer will select the promotion plan based on his/her objective function which is to maximize his/her profit by considering consumer reaction. The multiple strategic customers will make a decision to buy product in such a way that they can minimize their purchasing and holding costs in response. Thus, the retailer promotion price and period is determined as a critical decision of our mathematical model.

A bi-level mathematical programming approach is developed to address the retailer optimization problem. In addition, the effect of other factors such as competitor price, wholesaler price, customers' holding cost and demand that pass through retailer's promotion selection is investigated.

1.2 Problem Statement

This paper discusses the problem of retailer decision on finding the optimal promotion price and period. When making decision about promotion, retailer must rely on their expectations of consumers' reactions (Kalwani & Yim, 1992).In this paper, we focus on the relationship of a retailer and multiple strategic customers. The strategic customer is the customer who will optimize his/her own purchase behavior in response to the pricing strategies of the firm (Talluri & Ryzin, 2004). Therefore, we aim to study and develop the mathematical programming model to answer the following questions; when the retailer should start the promotion?, how long it should take the promotion?, how much of price reduction for the promotion?, and how can we make the strategic customers react to retailer's announcement in making decision to purchase a product in such a way that retailer plan?.

1.3 Objective of Thesis

- To improve the efficiency of the supply chain by developing mathematical programming that optimizes a two-level decision problem of a retailer and multiple customers by finding optimal promotion price and period to maximize retailer's profit when facing strategic customers who want to minimize their own purchasing cost and holding cost
- To review the research papers that related to optimal promotion price and period and a bi-level linear mathematical programming model
- To use the developed model to understand how promotion decision both price and period should be made, and apply the promotion decision to the real world situation in an effective and efficient way

1.4 Significant Outcome of Thesis

The proposed optimal promotion price and period by using bi-level optimization approach is expected to provide the contributions to both academic and practitioner or industry. For contribution to academic community, especially in the fields of operations research in the supply chain, this proposed model transform a nonlinear bi-level optimization problem of a retailer and multiple customers into a linear single level optimization problem solvable using available commercial software such as IBM ILOG CPLEX. We believe that this model is a quite new and meaningful approach for practitioner and industry, retailer can adopt the model to solve optimal promotion price and period simultaneously. The results of our model provide insights about optimal promotion discount pattern.

1.5 Overview of Thesis

This thesis consists of five chapters which are organized as follows:

Chapter 1 is the introduction of the thesis which provides the background of optimal promotion price and period problem, the problem statement, the objective of the thesis, significant outcome of thesis and overview of this thesis.

Chapter 2 is the literature review which presents the related research literature in pricing and promotion area. A bi-level linear programming approach also is reviewed.

Chapter 3 is the study of multi-period pricing strategies in response to competitor discount. The retailer and customer model are developed. The customer numerical experiment is implemented to explain the retailer's price decision.

Chapter 4 presents the formulation of a bi-level mathematical programming model that propose to solve optimal promotion price and period problem. The numerical experiment and the results are also analyzed and discussed in detail.

Chapter 5 provides the conclusion of this thesis and the future research study.

Chapter 2
Literature Review

This chapter introduces the review of the research literature in 2 main bases area. The first research area involves pricing and promotion literature. The second research area presents a bi-level linear programming approach.

2.1 Pricing and Promotion

Pricing and promotion have been a key instrument for the retailer to increase his/her sale volume and profit for a long time, it is not surprised that they have been one of the most popular topic discussed by marketing area. It is useful to provide some main highlighted definition of sales promotion before emphasis in specific literature. (R.C. Blattberg & Neslin, 1990) stated that "sales promotion is action-oriented marketing event whose purpose is to have direct impact on the firm's customers." There are three types of sales promotion; (1) "consumer promotions" are promotions offer directly to customer by manufacturer, (2) "retailer promotions" are the promotions address to final customer by the retailer, and (3) "trade promotions" are promotions that manufacture offers to the retailer. (Kotler, 1988) provide another definition by stating "sales promotion is a diverse collection of incentive tools, mostly short term, designed to stimulate quicker and/or greater purchase of a particular product by consumers or the trade." According to (Quelch, 1989) defined sales promotion as "temporary incentives targeted at the trade (called trade promotions) or at end consumers (consumer promotions). While sales promotions generally aim to change purchase behavior, they very in whether they attempt to persuade trade customers or end consumers to buy a product for the first time, to buy more, to buy earlier or to buy more often."

From above definition, it can be noticed that sales promotion have a several kind of techniques and features to incentive the different target segment. There are a number of sales promotion techniques such as quantity deals, price deals, coupons, discounts, sampling, and premium gifts that users can offer to reach target market. However, the majority of this thesis has focused on price discount technique.

Price-discounted promotion is the issue that have been address and widely studied by both marketing and economics. There are two decision aspects which are promotion depth and promotion frequency that many researchers try to find the solution to answer and understand it. For example, (Achabal, McIntyre, & Smith, 1990) formulated a closed-form, analytical response and optimization model to analyze the profitability of periodic promotions. The result of the model pointed out the simple policy guideline for managers to choose the most profit promotional price and frequency. (Rao, 1991) proposed a multistage game modeling framework for making decision on promotion depth and frequency, focusing on the competition between a private label and national brand. (Tellis & Zufryden, 1995)developed a dynamic planning model that provide an optimal discount strategy for multiple brands over multiple time periods. The model also shows how the optimum promotion depth and timing discount response to the variety of key demand and key supply characteristics. The experiment study by (Alba, Mela, Shimp, & Urbany, 1999) investigated the effected and conditioned of discount frequency and depth on consumer's price-estimation judgments. (Kurata & Liu, 2007) developed a Markov switching AR(1) time series model to examines and analyzed how a retailer should make the decision on the depth and frequency of price discount promotion and capture the demand response to the promotion. The study also considered the information sharing under supply chain framework and analyzed the price format selection.

There has been extensive literature on customer response to price discount promotion, deal proneness seems to be one of the most important consideration topic that researchers studied to understand consumers. For instance,(Webster, 1965), (Massy & Frank, 1965), (R.C. Blattberg & Neslin, 1990), (R. Blattberg, Buesing, Peacock, & Sen, 1978) and (Lichtenstein, Burton, & Netemeyer, 1997). The pioneer, (Webster, 1965) who conducted the first deal proneness studies, defined deal proneness as a function of both the consumer's buying behavior the frequency with which a given brand is sold on deal.(Massy & Frank, 1965), studied the changing in price and dealing activities that effect the market segments of family purchasing, package size and distribution channels. (Lichtenstein et al., 1997) developed model to investigate the relationship of deal proneness between segments of consumers and eight sales promotion types.

Generally, the drivers of promotion response can be investigated in many different angles.(Ailawadi & Neslin, 1998) studied the effect of promotion on consumption and developed flexible usage rate function to capture the real consumption phenomenon. The long-term impact of promotion into brand switching and consumption have demonstrated by using a Monte Carlo simulation. Their work provided the important finding that can help managers understand the behavioral of flexible consumption. (Pierre Chandon & Gilles, 2000) suggested that monetary saving is not only reason that customers response to a sales promotion. There are many several benefits of a sale promotion that influenced purchasing behavior of consumers, the utilitarian products seem to be impacted by monetary promotions than hedonic products. (Kogan & Herbon, 2008) observed the changing in customer buying behavior and customer price sensitivity on a supply chain under limited-time promotion. The paper indicated that when the customer price sensitive increases, the wholesale equilibrium price decreases, product order increases and product price drops.

Several variables have been considered and analyzed as a key that affect the optimal promotion price. There has been substantial literature on coordinated promotion and holding inventory cost. (R. Blattberg et al., 1978) considered transaction costs, holding costs and stock out costs as variables to identify households affecting deal proneness. (Robert C. Blattberg, Eppen, & Lieberman, 1981) developed the consumer and retailer model to study how deals work and how the retailer transfers its inventory carrying cost to consumer in food industry. Assuming that the retailer and consumer would like to minimize their own cost, the derived model shows that the dealing occurs because the retailer who has higher inventory holding cost than consumer would like to reduce his/her holding cost by shifting the inventory to customer. The retailer is motivated to offer price reduction to consumer and consumer is willing to hold the inventory in trade off so the consumer and retailer decisions are interrelated. (Jeuland & Narasimhan, 1985) proposed the temporary price cuts model to prove that promotion will be effective if the correlation between demand rates and customer inventory costs are positive.

Price format is one of the competitive marketing tools that retailers need to address to their promotion strategy. The popular choices of promotion strategies have been selected by retailer are EDLP (Everyday Low Price) and Hi-Lo (High-Low pricing strategy). (Hoch, Drèze, & & Purk, 1994) defined EDLP as "the retailer charges a constant, lower everyday price with no temporary price discounts" whereas "Hi-lo strategy, the retailer charges to high prices on an everyday basis, but then runs frequent promotions in which prices are temporarily lowered sometimes below the EDLP level."

A number of researchers examine the benefit of EDLP and Hi-Lo strategy and provided the better understanding of which price format strategy that the retailers should select for their business. (Hoch et al., 1994) found that price reduction 10% in EDLP can help retailer increase 3% in sales volume but decreasing 18% in profit. On the other hand, price went down 10% in Hi-Lo led to a 3% sale decrease but raising 15% in profit. (Narasimhan, 1988) derived Hi-Lo policy to investigated the brand switchers who are brand loyalty in competitive market framework. The frequency and depth of discount also have been observed. (Lattin, 1991) observed that the business which driven by EDLP strategy like Wal-Mart could lower its advertising cost. (Bailey, 2008) implemented EDLP policy and studied how store loyalty factor and sale proneness influenced consumer buying behavior.

2.2 Bi-Level Linear Programming Approach

The bi-level programming problem is a hierarchical optimization problem involving two-level decision making. The first decision maker or upper level is the leader who dominant the second decision maker or lower level. The second maker is also considered as follower who executes the choice or strategy after seeing action of the leader.

The algorithms and formulation of bi-level programming problem was originated since mid-1970s. The first researchers who introduced and developed the globally optimal solution for bi-level and multilevel programming were (Candler & Townsley, 1982). The study was motivated by the Stackelberg game theory when the subset of leader's variables associated with the follower's optimal basic. Then (Bialas & Karwan, 1984) proposed "K th best" algorithm for an extreme point solution of a linear program. The "Kuhn-Tucker" approach is one of the most popular method for solving the linear bi-level programming problem. The basic idea of "Kuhn-Tucker" is dealing with complimentary slackness by a branch and bound approach. The reader is refer to (Fortuny-Amat & McCarl, 1981) and (Jonathan F. Bard & Falk, 1982)

(White & Anandalingam, 1993) presented the penalty approach to incur the optimal solution by applying with a duality gap. The equivalent single mathematical program seems to be the direct way to solve bi-level problem but it is not easy to do that. Though,(J.F. Bard, 1998) has proposed the interesting technique for transforming the model by employing Karush-Kuhn-Tucker condition.

The algorithms of bi-level programming problem has been developed and improved, it was caught attention by many researchers for a long time. (Vicente & Calamai, 1994) classified the solution and algorithms of bi-level programming into five different classes; Extreme point algorithms, Branch and bound algorithms, Complementarity pivot algorithms, Descent methods and Penalty function methods. (Dempe, 2003) has intensively reviewed theoretical literature and related work of bi-level programming problem.

However, we have no attempt to comprehensive review the algorithms, we focus on the application of bi-level optimization that can apply in real world situation problem and point some interesting related papers to the reader. There are a number of problem that involve decision making process with a hierarchical structure, for example transportation network design problem and management planning. (Candler, Fortuny-Amat, & McCarl, 1981) solved agricultural policy with two separate decision problems by extended the procedure presented in previous work of (Fortuny-Amat & McCarl, 1981). They applied the multilevel programming to an irrigation canal command problem and optimal pricing policy for fertilizer dealer.(J. Bard, Plummer, & Sourie, 1998) has developed grid search algorithm and nonlinear programming formulation of bi-level programing problem along with finding the solution to minimize tax credits of biofuel production for agricultural sector who want to maximize its own profit.

(Brotcorne, Labbé, Marcotte, & Savard, 2000) proposed bi-level programming formulation to examine a freight tariff setting problem between a group of carriers and a shipper. The objective of carries is to maximize their own profit from tariffs. On the other hand, the minimization of transportation is a shipper's objective function. In the study of (Ryu, Dua, & Pistikopoulos, 2004), enterprise-wide supply chain planning problems of distribution network planning at upper level corresponding with production planning at lower level has been illustrated and solved by model transformation of a bi-level problem into a single parametric problem.

Recently, (Sun, Gao, & Wu, 2008) described the logistics distribution centers location problem by using bi-level programming model. This study presented the planner and customer relationship, the leader decision maker determines the optimal location in order to minimize his/her cost whereas the follower decision maker provide an equilibrium demand distribution in response to minimize his/her cost.

(Cao & Chen, 2006) examined and solved a capacitated plant selection problem in decentralized manufacturing environments by applying a two-level decision framework. They investigated the plant selection problem by derived an equivalent single level model. A two-level nonlinear programing model was first developed, then they improved the model by transform and linearized it. In addition, the research methodology of this thesis is closed to the procedure by (Cao & Chen, 2006) but it is different in area of interest and application.

We can noticed from above literature reviews that bi-level programming problem has been growing applied to the more general application in a broader range of fields. Obviously, the retailer optimal promotion price and period can also be viewed as leader-follower framework where the retailer make a decision on promotion plan, the customers independently selected the store to purchase product. Hence, it is suitable to implement retailer promotion optimization problem with a Bi-level mathematical programming approach.

Chapter 3
Multi-period Pricing Strategies in Response to Competitor Discount: Strategic Customer Numerical Experiments

This chapter studies the multi-period pricing strategies in response to competitor discount. The problem description is simply introduced in Section 3.1. The background of mathematical model formulation is explained in Section 3.2. In Section 3.3, we present preliminary numerical experiment and result. Discussion of the model is presented in Section 3.4.

3.1 Problem Description

This study considers a two-stage supply chain composed of a single retailer who sells a single product to multiple strategic customers. However, there are a competitor who offers the same product to customers by providing different services. Customers can buy the product either from the retailer or from the competitor, depending on price of the product. Two model are developed in this study, one is a retailer model and another one is a customer model. The retailer tries to find a policy of promotion price and period that perform well in maximizing retailer's profit. And the customers also tries to minimize their purchasing and holding costs.

3.2 Mathematical Model

Before the mathematical programming model is presented, we first give the notations to be used in the model as follows.

3.2.1 Notations

Parameters:

T = set of time period

D_{jk} = demand of customer k during period j

h_{ijk} = holding cost of customer k to carry 1 unit of production from period i to period j

hr_i = holding cost at retailer in period i

Sc = fixed order cost at retailer

M = very large number

Pl = list price

pc_i = competitor price in period i

w_i = wholesale price in period i

p_i = retail price in period i

Decision Variables:

z_{ijk} = 1 if customer k purchases for retailer in period i and use in period j; 0 otherwise

γ_{ijk} = 1 if customer k purchases for competitor in period i and use in period j; 0 otherwise

x_i = order quantity that a retailer places in period i

I_i = inventory at the end of period i

S_i = 1 if there is ordering from retailer to purchase in period i; 0 otherwise

3.2.2 Retailer model

$$\textit{Maximize Profit} = \sum_k \sum_i \sum_{j \geq i} p_i D_{jk} z_{ijk} - \sum_i S_i Sc - \sum_i hr_i I_i - \sum w_i x_i \tag{3.1}$$

Subject to:

$$x_i + I_{i-1} - \sum_k \sum_j z_{ijk} D_{jk} = I_i \qquad \forall i = 1,...,T \tag{3.2}$$

$$x_i \leq S_i M \qquad \forall i \tag{3.3}$$

$$S_i \in \{0,1\} \qquad \forall i \tag{3.4}$$

$$x_i \geq 0 \qquad \forall i \tag{3.5}$$

$$I_i \geq 0 \qquad \forall i \tag{3.6}$$

From (3.1), the objective function of the model is to maximize the profit of retailer. Retailer's profit comes from the difference between total revenue and following cost: fixed setup cost (i.e. administration cost, transportation cost), inventory cost at retailer and ordering cost. The constraint (3.2) sets up the inventory at retailer, assuming that the beginning inventory is zero. Constraint (3.3) states that if the retailer order the product in period *i*, the binary variable S_i cannot be zero. Constraint (3.4) determines the binary variable. Constraint (3.5) and (3.6) represents the non-negativity variables.

3.2.3 Customer model

$$\textit{Minimize Cost} = \sum_{i}\sum_{j\geq i}\left(p_i + h_{ijk}\right)D_{jk}z_{ijk} + \sum_{i}\sum_{j\geq i}\left(pc_i + h_{ijk}\right)D_{jk}\gamma_{ijk} \qquad (3.7)$$

Subject to:

$$\sum_{i\leq j}\left(z_{ijk} + \gamma_{ijk}\right) \geq 1 \qquad \forall j \qquad (3.8)$$

$$z_{ijk} \in \{0,1\} \qquad \forall j, \forall i \leq j, \forall k \qquad (3.9)$$

$$\gamma_{ijk} \in \{0,1\} \qquad \forall j, \forall i \leq j, \forall k \qquad (3.10)$$

The objective function (3.7) is to minimize total cost of customers which consist of purchasing cost and holding cost at our retailer including holding cost of customer and purchasing cost at competitor's retailer. Constraint (3.8) shows the total demand that customer k purchases in period i and use in period j. Lastly, Constraint (3.9) and (3.10) determines the binary variable.

3.3 Numerical Experiments and Results

In this section, we use the mathematical programming model from previous section to generate the numerical results. We assume that the retail prices are given but many pricing strategies are experimented. We consider the operating in 8 periods and we have 20 customers, we use the parameter values and demand in following table.

Table 3.1: Parameter Values and Demand

	Values
Demand of customer	{54, 57, 54, 57, 54 , 51, 58, 56}
Holding cost at retailer	5
Fixed setup cost at retailer	1000
List price	100
Wholesale price	50

The percentage of holding cost per unit per time that suggested in textbooks according to (Waters, December 2012) ranges between 19% and 35% depending on the industry and field. Therefore, this paper we assume the holding cost of customer to carry the product per unit per time by using Normal Distribution. The holding cost of customer can see in Table 3.2.

Table 3.2: Holding cost of customer

Holding cost per unit per period	Number of customer (Frequency)
0.2	3
0.3	2
0.4	7
0.5	7
0.6	1

The effect of price discount is analyzed. We do the experiments by setting competitor price, we use pc_i = {100, 100, 100, 90, 90, 100, 100, 100} while others are constant as listed in Table 1 and 2. Next, we vary the retail price to see which one is the best pricing discount pattern. We use 5 patterns of discount price as shown in Table 3.3.

Table 3.3: The discount price scenario

	Scenario	Time period							
		1	2	3	4	5	6	7	8
1	Same discount	100	100	100	90	90	100	100	100
2	Long and shallow discount	100	100	90	90	90	90	100	100
3	Short and deep discount	100	100	100	85	85	100	100	100
4	No discount	100	100	100	100	100	100	100	100
5	One discount	100	100	100	89	100	100	100	100

Table 3.4: Decision of the model

	Time period							
	1	2	3	4	5	6	7	8
Scenario 1								
Retailer's sales	54	57	54	57	219	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	57	0	57	0	0	0	0	0
Order quantity	111	0	111	0	219	0	0	0
Retailer's profit	15720							
Customer's cost	41471							
Scenario 2								
Retailer's sales	54	57	54	57	54	165	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	54	0	0	0	0
Order quantity	165	0	0	111	0	165	0	0
Retailer's profit	14655							
Customer's cost	40866							
Scenario 3								
Retailer's sales	54	57	54	57	219	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	57	0	57	0	0	0	0	0
Order quantity	111	0	111	0	219	0	0	0
Retailer's profit	14340							
Customer's cost	40091							
Scenario 4								
Retailer's sales	54	57	54	0	0	0	0	0
Competitor's sales	0	0	0	57	219	0	0	0
Inventory	111	54	0	0	0	0	0	0
Order quantity	165	0	0	0	0	0	0	0
Retailer's profit	6425							
Customer's cost	41471							
Scenario 5								
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0
Retailer's profit	16310							
Customer's cost	41318							

After we run the model separately case by case the result is shown in Table 3.4. We can see from the above table that the strategic customers will change their purchasing behavior response to the retailer's pricing strategy by purchasing the product ahead on promotion discount period. However, the customers prefer to buy the product at the end of promotion period because the customers not only think about a price reduction but also consider their holding cost. The scenario when a retailer promotes the discount price with a slightly discount to cut competitor price seems to be the best decision to attract the strategic customers to buy product at the store according to the sales during discount period increase a lot compare to other periods.

In Figure 3.1 the profit and cost is resulted from the decision of the model in previous table, the retailer tries to increase the profit and compete the competitor by offering price discount. In addition, offering the promotion at the right price and period also help retailer to reduce the set up cost and inventory cost. We can see that the retailer will get the highest profit in Scenario 5 when it offers the discount promotion only one time in period 4. The lowest profit is scenario 4 when the retailer makes decision to offer no discount in any period, this decision causes the customers buy product at competitor during the discount promotion in large lots and makes retailer lost profit. However the total cost of customers are lowest in scenario short and deep discount.

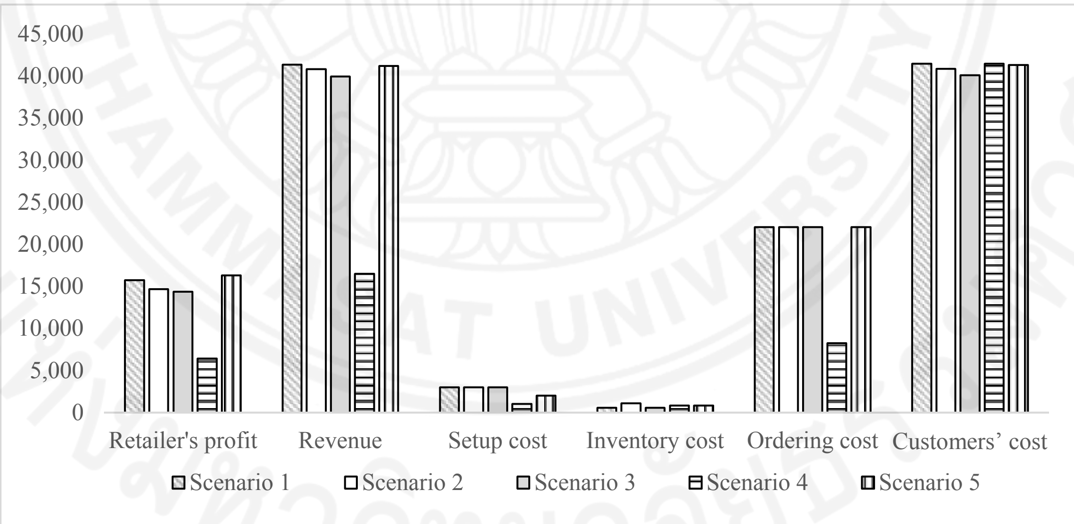

Figure 3.1: Numerical result

Next, we repeat formulating the exact experiments with competitor giving more discount to determine whether the model's response is consistent with the first experiment or not, we set pc_i = {100, 70, 70, 100, 100, 100, 100, 100 and then we adjust the 5 scenarios with the same patterns in Table 3.3 but different number relative to new competitor price in Table 3.5.

Table 3.5: The discount price scenario (2)

	Scenario	Time period							
		1	2	3	4	5	6	7	8
1	Same discount	100	70	70	100	100	100	100	100
2	Long and shallow discount	65	65	65	65	100	100	100	100
3	Short and deep discount	100	60	60	100	100	100	100	100
4	No discount	100	100	100	100	100	100	100	100
5	One discount	100	69	100	100	100	100	100	100

As we can see from the decision of the model in Table 3.6, the results suggest that the best scenario to maximize retailer's profit is scenario 5. The second numerical example supports the first experiment, the model still make decision in the same direction even changing the discount price and period. The strategic customer responses to discount promotion by deciding to buy products from the retailer that gives them the highest utility and satisfy their own demand.

Table 3.6: Decision of the model (2)

	Time period							
	1	2	3	4	5	6	7	8
Scenario 1								
Retailer's sales	54	57	330	0	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	57	0	0	0	0	0	0	0
Order quantity	111	0	330	0	0	0	0	0
Retailer's profit	8155							
Customer's cost	32814							
Scenario 2								
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0
Retailer's profit	3790							
Customer's cost	28881							
Scenario 3								
Retailer's sales	54	57	330	0	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	57	0	0	0	0	0	0	0
Order quantity	111	0	330	0	0	0	0	0
Retailer's profit	4285							
Customer's cost	28944							

Table 3.7: Decision of the model (2) continue.

	Time period							
	1	2	3	4	5	6	7	8
Scenario 4								
Retailer's sales	54	0	0	0	0	0	0	0
Competitor's sales	0	57	330	0	0	0	0	0
Inventory	0	0	0	0	0	0	0	0
Order quantity	54	0	0	0	0	0	0	0
Retailer's profit	1700							
Customer's cost	32814							
Scenario 5								
Retailer's sales	54	387	0	0	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	0	0	0	0	0	0	0	0
Order quantity	54	387	0	0	0	0	0	0
Retailer's profit	8222							
Customer's cost	32725							

Figure 3.2 illustrates profit and breakdown cost element of a retailer and customers respect to the decision of the model in Table 3.5. Actually, the revenue in scenario 1 is highest one but the decision of the model for ordering product in scenario 5 makes inventory equal to zero and it makes the retailer's total profit equal to 8222 that higher than other scenarios. So the retailer tends to offer one discount with slightly cut the competitor price to o maximize their own profit. We can see that offering the promotion in earlier period also helps the retailer to lower the inventory cost.

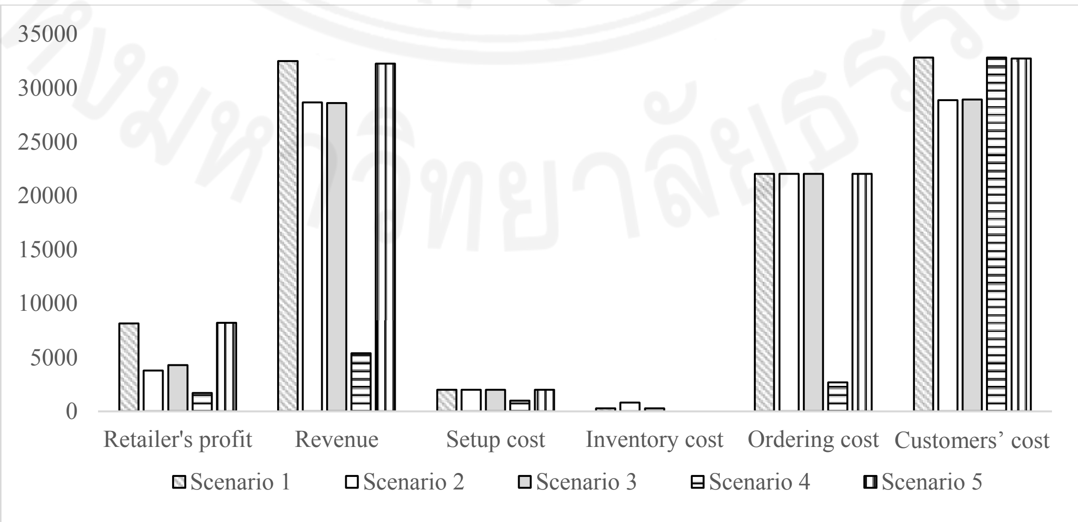

Figure 3.2: Numerical result (2)

3.4 Discussion of the model

In this study, we consider a two-stage supply chain composed of a single retailer who sells a single product to multiple strategic customers. We developed the mathematical model to find the multiple-period pricing strategy to maximize retailer's profit when facing strategic customers who want to minimize their purchasing and holding costs. From the experiment, the preliminary results indicate that the retailer will maximize its own profit when it offers promotion at the same period of competitor but it will provide only one period to customer. This promotion decision makes the strategic customers purchase the products in large quantity during discount period at the retailer. In addition, the retailer trends to order the product large lots to cover the demand of customers in future periods and to reduce the setup cost. Besides the ordering decision also considers the holding cost at retailer.

Chapter 4
Optimal Promotion Price and Period: Bi-Level Linear Programming Approach

In this chapter, we develop the model to extend the limitation of the previous chapter. A bi-level mathematical programming model is formulated to solve and find the solution of retailer's optimal promotion price and period strategy. In Section 4.1, we describe the basic situation of the problem. Then the mathematical programming model is presented in Section 4.2. Following this, the numerical experiments are derived and analyzed for each scenario. Lastly, the results and finding will be discussed in detail.

4.1 Problem Description

In this study, we have developed the scenario of a supply chain composed of a manufacturer, a retailer, a competitor and multiple strategic customers. However, this study mainly focus on the relationship of a retailer and multiple strategic customers. Each of them tries to optimize his/her own welfare in response to other actions. The retailer assumes the role of leader who provides the retail promotion price and period to the market. In response, the multiple strategic customers make a purchasing decision to minimize their purchasing and holding costs, this places customer as follower. To solve such problem, we have developed a mathematical programming model that makes customers react to retailer's announcement in making purchasing decision in such a way that retailer plan. According to improve the efficient of supply chain, we also study the effect of wholesaler and competitor price discount pass through retailer price.

The methodology for solving a retailer optimal promotion price and period is organized as follows. First, we develop the retailer and customer model. Second we formulate a bi-level mathematical programming model that composed of retailer's decision at the upper level and customer's decision at the lower level. Then we transform the model into an equivalent single level model and linearize the objective function of model. Finally, we solve the problem by using available optimization software (CPLEX).

4.2 Mathematical Model

The assumption that we used throughout this study are listed as following; the initial inventory is zero, demand of each period is independent and known, the inventory holding cost is known, and the competitor price is known.

Parameters:

T = set of time period

D_{jk} = demand of customer k during period j

h_{ijk} = holding cost of customer k to carry 1 unit of production from period i to period j

hr_i = holding cost at retailer in period i

Sc = fixed order cost at retailer

M = very large number

Pl = list price

pc_i = competitor price in period i

w_i = wholesale price in period i

Decision Variables:

z_{ijk} = 1 if customer k purchases for retailer in period i and use in period j; 0 otherwise

γ_{ijk} = 1 if customer k purchases for competitor in period i and use in period j; 0 otherwise

Pm = promotion price

p_i = retail price in period i

x_i = order quantity retailer place in period i

I_i = inventory at the end of period i

S_i = 1 if there is ordering from retailer to purchase in period i; 0 otherwise

Pr_{be} = 1 if promotion starts from period b to period e; 0 otherwise

y_i = 1 if period i outside promotion period b and e; 0 if period i is inside period b and e

4.2.1 Retailer Model

The retailer's objective is to maximize his/her profit. The profit of retailer comes from the difference between revenue and total cost that compose of fixed set up cost, inventory cost at retailer and ordering cost.

$$\textit{Maximize Profit} = \sum_k \sum_i \sum_{j \geq i} p_i D_{jk} z_{ijk} - \sum_i S_i Sc - \sum_i hr_i I_i - \sum w_i x_i \qquad (4.1)$$

Subject to:

$$x_i + I_{i-1} - \sum_k \sum_j z_{ijk} D_{jk} = I_i \qquad \forall i = 1,...,T \qquad (4.2)$$

$$x_i \leq S_i M \qquad \forall i \qquad (4.3)$$

$$p_i \leq Pl \qquad \forall i \qquad (4.4)$$

$$p_i \geq Pl - (1 - y_i) M \qquad \forall i \qquad (4.5)$$

$$p_i \leq Pm + y_i M \qquad \forall i \qquad (4.6)$$

$$p_i \geq Pm \qquad \forall i \qquad (4.7)$$

$$\sum_b \sum_{e \geq b} Pr_{be} \leq 1 \qquad (4.8)$$

When no promotion period is selected ($\sum_b \sum_{e \geq b} Pr_{be} = 0$), force $y_i = 1$ and $p_i = Pl$ for all periods

$$1 - M \sum_b \sum_{e \geq b} Pr_{be} \leq y_i \qquad \forall i = 1,...,T \qquad (4.9)$$

$$Pr_{be} \leq y_i \qquad \forall i = e+1 ... T \qquad (4.10)$$

$$Pr_{be} \leq y_i \qquad \forall i = 1,...,b-1 \qquad (4.11)$$

$$1 - Pr_{be} \geq y_i \qquad \forall i = b,...,e \qquad (4.12)$$

$$S_i \in \{0,1\} \qquad \forall i \qquad (4.13)$$

$$Pr_{be} \in \{0,1\} \qquad \forall i \qquad (4.14)$$

$$y_i \in \{0,1\} \qquad \forall i \qquad (4.15)$$

Constrains (4.2) determines the set up inventory at a retailer. Constraints (4.3) the binary variable (S_i) cannot be zero when the retailer makes an order in period i. Constraint (4.4) and (4.5) shows that when $y_i = 1$, p_i will be equal Pl. Constraint (4.6) and (4.7) will make p_i be equal to Pm when the retailer offers promotion to customers ($y_i = 0$). Constrain (4.8) determines that there is promotion between period b and e. Constrain (4.9) states that when no promotion is selected, the binary variable (y_i) cannot be zero. Constraint (4.10), (4.11) and (4.12) determines that $y_i = 1$ if period i outside promotion period b and e and $y_i = 0$ if period i is inside promotion period b and e. Constrain (4.13), (4.14) and (4.15) represent the binary variable. Figure 4.1 shows the promotion structure of the retailer model.

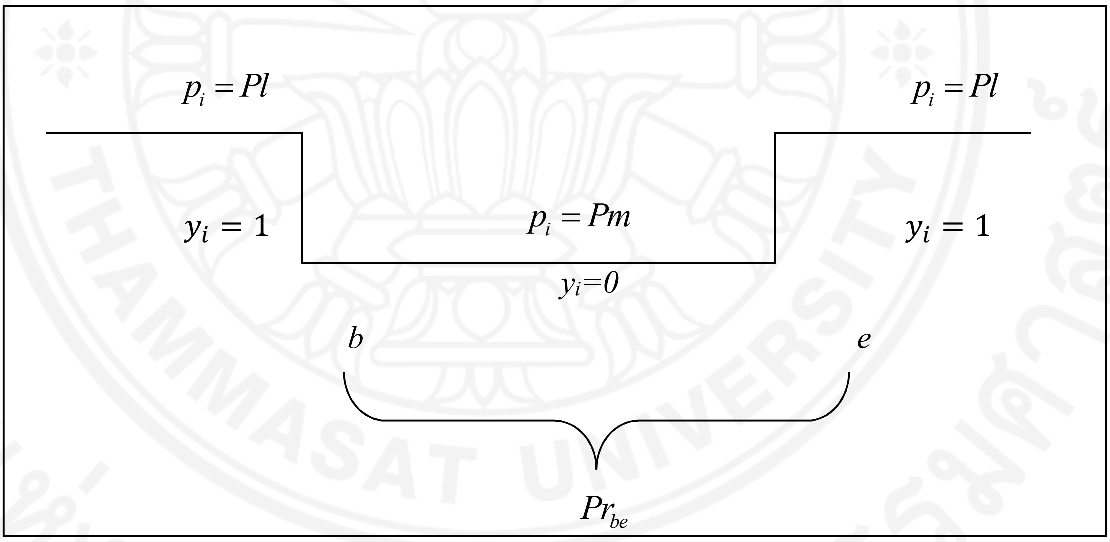

Figure 4.1: The promotion structure of the retailer model

4.2.2 Customer Model

It is assumed that we are dealing with the strategic customers. The customers' objective function is to minimize their total cost. The customer cost is composed of purchasing and inventory holding costs.

$$\textit{Minimize Cost} = \sum_{i}\sum_{j\geq i}\left(p_i + h_{ijk}\right)D_{jk}z_{ijk} + \sum_{i}\sum_{j\geq i}\left(pc_i + h_{ijk}\right)D_{jk}\gamma_{ijk} \tag{4.16}$$

Subject to:

$$\sum_{i\leq j}\left(z_{ijk} + \gamma_{ijk}\right) \geq 1 \qquad \forall j \tag{4.17}$$

$$z_{ijk} \in \{0,1\} \qquad \forall j, \forall i \leq j, \forall k \tag{4.18}$$

$$\gamma_{ijk} \in \{0,1\} \qquad \forall j, \forall i \leq j, \forall k \tag{4.19}$$

4.2.3 Bi-level Mathematical Programming Model

The retailer at the upper level selects the promotion price and period by anticipating the customers' response, the retailer considers to maximize retailer's profit when facing strategic customers. The multiple strategic customers' purchasing decision at lower level is making independently to minimize their own purchasing and holding cost. This optimal promotion price and period problem can be viewed as a bi-level mathematical programming problem so we put the customer model as the constraints in the retailer model.

Since the customer model is an integer programming model (IP). We need to convert the model to be a linear programming model (LM) first before applying bi-level linear programming model. We can relax the variable z_{ijk} and γ_{ijk} from binary as below.

Customer Model: Integer programming model (IP)

$$\textit{Minimize Cost} = \sum_{i}\sum_{j\geq i}\left(p_i + h_{ijk}\right)D_{jk}z_{ijk} + \sum_{i}\sum_{j\geq i}\left(pc_i + h_{ijk}\right)D_{jk}\gamma_{ijk} \tag{4.16}$$

Subject to:

$$\sum_{i\leq j}\left(z_{ijk} + \gamma_{ijk}\right) \geq 1 \qquad \forall j \tag{4.17}$$

$$z_{ijk} \in \{0,1\} \qquad \forall j, \forall i \leq j, \forall k \tag{4.18}$$

$$\gamma_{ijk} \in \{0,1\} \qquad \forall j, \forall i \leq j, \forall k \tag{4.19}$$

Customer Model: Linear programming model (LM)

Minimize Cost = $\sum_{i}\sum_{j\geq i}\left(p_i + h_{ijk}\right)D_{jk}z_{ijk} + \sum_{i}\sum_{j\geq i}\left(pc_i + h_{ijk}\right)D_{jk}\gamma_{ijk}$ *(4.16)*

Subject to:

$$\sum_{i\leq j}\left(z_{ijk} + \gamma_{ijk}\right) \geq 1 \qquad \forall j \tag{4.17}$$

$$z_{ijk} \geq 0 \qquad \forall j, \forall i \leq j, \forall k \tag{4.18'}$$

$$\gamma_{ijk} \geq 0 \qquad \forall j, \forall i \leq j, \forall k \tag{4.19'}$$

Proposition 1 *will show that the result is achieved through a linear programming model, it is also the optimal solution to the customer model that solve by an integer programming model.*

Proof. See in Appendix

The model for solving retailer optimal promotion price and period problem by using a bi-level programming approach is formulated follow the general form that presented in (J.F. Bard, 1998).

Maximize Profit = $\sum_{k}\sum_{i}\sum_{j\geq i} p_i D_{jk} z_{ijk} - \sum_{i} S_i Sc - \sum_{i} hr_i I_i - \sum w_i x_i$ *(4.1)*

Subject to:

$$x_i + I_{i-1} - \sum_{k}\sum_{j} z_{ijk} D_{jk} = I_i \qquad \forall i = 1,\ldots,T \tag{4.2}$$

$$x_i \leq S_i M \qquad \forall i \tag{4.3}$$

$$p_i \leq Pl \qquad \forall i \tag{4.4}$$

$$p_i \geq Pl - (1 - y_i)M \qquad \forall i \tag{4.5}$$

$$p_i \leq Pm + y_i M \qquad \forall i \tag{4.6}$$

$$p_i \geq Pm \qquad \forall i \tag{4.7}$$

$$\sum_{b}\sum_{e\geq b} Pr_{be} \leq 1 \tag{4.8}$$

When no promotion period is selected ($\sum_{b}\sum_{e\geq b} Pr_{be}=0$*), force* $y_i=1$ *and* $p_i=Pl$ *for all periods*

$$1-M\sum_{b}\sum_{e\geq b} Pr_{be}\leq y_i \qquad \forall i=1,...,T \qquad (4.9)$$

$$Pr_{be}\leq y_i \qquad \forall i=e+1...T \qquad (4.10)$$

$$Pr_{be}\leq y_i \qquad \forall i=1,...,b-1 \qquad (4.11)$$

$$1-Pr_{be}\geq y_i \qquad \forall i=b,...,e \qquad (4.12)$$

$$S_i\in\{0,1\} \qquad \forall i \qquad (4.13)$$

$$Pr_{be}\in\{0,1\} \qquad \forall i \qquad (4.14)$$

$$y_i\in\{0,1\} \qquad \forall i \qquad (4.15)$$

$$\text{Minimize Cost} = \sum_{i}\sum_{j\geq i}\left(p_i+h_{ijk}\right)D_{jk}z_{ijk}+\sum_{i}\sum_{j\geq i}\left(pc_i+h_{ijk}\right)D_{jk}\gamma_{ijk} \qquad (4.16)$$

Subject to:

$$\sum_{i\leq j}\left(z_{ijk}+\gamma_{ijk}\right)\geq 1 \qquad \forall j \qquad (4.17)$$

$$z_{ijk}\geq 0 \qquad \forall j,\forall i\leq j,\forall k \qquad (4.18')$$

$$\gamma_{ijk}\geq 0 \qquad \forall j,\forall i\leq j,\forall k \qquad (4.19')$$

4.2.4 Transform into an equivalent single level model

As we describes in above section that retailer optimal promotion price and period are two level cooperation between the purchasing decision making at the lower level and selected promotion price and period at the upper level. In this section, we first presents model transformation then the model is linearized in next section. The primal-dual theory and complimentary slackness is implemented into the customer model at the lower level.

4.2.4.1 Primal-Dual problem

Every linear programming problem, referred to as primal problem has associated with it related linear programming problem called its dual. The standard form for primal-dual problem has shown in Figure 4.2:

(Primal Problem)

Maximize $z = \sum_{j=1}^{n} c_j x_j$

Subject to:

$\sum_{j=1}^{n} a_{ij} x_j \le b_i$ for i = 1, 2,…,m

$x_j \ge 0$ for j = 1, 2, …, n

(Dual Problem)

Minimize $w = \sum_{i=1}^{m} b_i y_i$

Subject to:

$\sum_{i=1}^{m} a_{ij} y_i \ge c_j$ for j = 1, 2, …, n

$y_i \ge 0$ for i = 1, 2,…,m

Figure 4.2: Standard form for primal-dual problem

Now, using the rules in above figure we can transform our customer model (primal problem) to dual problem as below.

Primal model:

$$\textit{Minimize Cost} = \sum_{i}\sum_{j \ge i}\left(p_i + h_{ijk}\right) D_{jk} z_{ijk} + \sum_{i}\sum_{j \ge i}\left(pc_i + h_{ijk}\right) D_{jk} \gamma_{ijk} \qquad (4.16)$$

Subject to:

$$\sum_{i \le j}\left(z_{ijk} + \gamma_{ijk}\right) \ge 1 \qquad \forall j \qquad (4.17)$$

$$z_{ijk} \ge 0 \qquad \forall j, \forall i \le j, \forall k \qquad (4.18')$$

$$\gamma_{ijk} \ge 0 \qquad \forall j, \forall i \le j, \forall k \qquad (4.19')$$

Dual model:

$$\textit{Maximize} = \sum_{j} \lambda_{jk} \qquad (4.20)$$

Subject to:

$$\lambda_{jk} \le (p_i + h_{ijk}) D_{jk} \qquad \forall j, \forall i \le j, \forall k \qquad (4.21)$$

$$\lambda_{jk} \le (pc_i + h_{ijk}) D_{jk} \qquad \forall j, \forall i \le j, \forall k \qquad (4.22)$$

$$\lambda_{jk} \ge 0 \qquad \forall j, k \qquad (4.23)$$

The objective function (4.20) is the maximization problem. The constraint (4.21) and (4.22) are generated from primal objective function (4.16). Constraint (4.23) is dual variable of the constraint (4.17).

4.2.4.2 Complementary Slackness Condition

The concept of complementary slackness refers to a relationship between the slackness in a primal constraints and the slackness of the associated dual variable. Use the standard form of primal-dual problem. Assuming problem (P) has a feasible solution x and problem (D) has a feasible solution y. Then x and y are optimal if and only if the following conditions are satisfied:

$(b_i - \sum_j a_{ij} x_j) y_i = 0, \forall_i$ where $b_i - \sum_j a_{ij} x_j$ is the slackness of primal.

$(\sum_i a_{ij} y_i - c_j) x_j = 0, \forall_j$ where $\sum_i a_{ij} y_i - c_j$ is the slackness of dual.

It can also say that primal-dual problem have optimal solution if:

1) Primal solution is feasible

2) Dual solution is feasible

3) Complementary slackness condition are satisfy

(Slack of Primal) x (Dual Variable) = 0 and

(Slack of Dual) x (Primal Variable) = 0

Complimentary Slackness:

$$Sp_{jk} = \sum_{i \le j} \left(z_{ijk} + \gamma_{ijk} \right) - 1 \qquad \forall j, k \tag{4.24}$$

$$Sd_{ijk} = (p_i + h_{ijk}) D_{jk} - \lambda_{jk} \qquad \forall j, \forall i \le j, \forall k \tag{4.25}$$

$$Sdc_{ijk} = (pc_i + h_{ijk}) D_{jk} - \lambda_{jk} \qquad \forall j, \forall i \le j, \forall k \tag{4.26}$$

$$(1 - fp_{jk}) \varepsilon \le Sp_{jk} \le (1 - fp_{jk}) M \qquad \forall j, k \tag{4.27}$$

$$fp_{jk} \varepsilon \le \lambda_{jk} \le fp_{jk} M \qquad \forall j, k \tag{4.28}$$

$$(1 - fd_{ijk}) \varepsilon \le Sd_{ijk} \le (1 - fd_{ijk}) M \qquad \forall j, \forall i \le j, \forall k \tag{4.29}$$

$$fd_{ijk} \varepsilon \le z_{ijk} \le fd_{ijk} M \qquad \forall j, \forall i \le j, \forall k \tag{4.30}$$

$$(1-fdc_{ijk})\varepsilon \le Sdc_{ijk} \le (1-fdc_{ijk})M \quad \forall j, \forall i \le j, \forall k \tag{4.31}$$

$$fdc_{ijk}\varepsilon \le \gamma_{ijk} \le fdc_{ijk}M \quad \forall j, \forall i \le j, \forall k \tag{4.32}$$

$$fp_{jk} \in \{0,1\} \quad \forall j,k \tag{4.33}$$

$$fd_{ijk} \in \{0,1\} \quad \forall j, \forall i \le j, \forall k \tag{4.34}$$

$$fdc_{ijk} \in \{0,1\} \quad \forall j, \forall i \le j, \forall k \tag{4.35}$$

The constraint (4.24), (4.25) and (4.26) are setup slack of primal and slack of dual. Where in (4.27)-(4.32) M is a large positive number and ε is a small positive number. Constraint (4.27) and (4.28) shows that when slack of primal is positive, it forces $fp_{jk} = 0$ then the associated variable (λ_{jk}) is zero. It also says that if slack of primal is binding, fp_{jk} becomes 1 and make variable is positive. The same concept is applied through constraint (4.29)-(4.32). Constraint (4.33)-(4.35) determine the binary variable.

Now we can transform nonlinear bi-level optimization problem into the single level problem as following model.

Single level optimization problem:

Maximize Profit = $$\sum_k \sum_i \sum_{j \ge i} p_i D_{jk} z_{ijk} - \sum_i S_i Sc - \sum_i hr_i I_i - \sum w_i x_i \tag{4.1}$$

Subject to:

Retailer

$$x_i + I_{i-1} - \sum_k \sum_j z_{ijk} D_{jk} = I_i \quad \forall i = 1,...,T \tag{4.2}$$

$$x_i \le S_i M \quad \forall i \tag{4.3}$$

$$p_i \le Pl \quad \forall i \tag{4.4}$$

$$p_i \ge Pl - (1-y_i)M \quad \forall i \tag{4.5}$$

$$p_i \le Pm + y_i M \quad \forall i \tag{4.6}$$

$$p_i \ge Pm \quad \forall i \tag{4.7}$$

$$\sum_b \sum_{e \ge b} Pr_{be} \le 1 \tag{4.8}$$

When no promotion period is selected ($\sum_{b}\sum_{e\geq b} Pr_{be} = 0$*), force* $y_i = 1$ *and* $p_i = Pl$ *for all periods*

$$1 - M\sum_{b}\sum_{e\geq b} Pr_{be} \leq y_i \qquad \forall i = 1,...,T \qquad (4.9)$$

$$Pr_{be} \leq y_i \qquad \forall i = e+1...T \qquad (4.10)$$

$$Pr_{be} \leq y_i \qquad \forall i = 1,...,b-1 \qquad (4.11)$$

$$1 - Pr_{be} \geq y_i \qquad \forall i = b,...,e \qquad (4.12)$$

$$S_i \in \{0,1\} \qquad \forall i \qquad (4.13)$$

$$Pr_{be} \in \{0,1\} \qquad \forall i \qquad (4.14)$$

$$y_i \in \{0,1\} \qquad \forall i \qquad (4.15)$$

Customer's optimal decision:

Primal of Customer k

$$\sum_{i\leq j}\left(z_{ijk} + \gamma_{ijk}\right) \geq 1 \qquad \forall j \qquad (4.17)$$

$$z_{ijk} \geq 0 \qquad \forall j, \forall i \leq j, \forall k \qquad (4.18')$$

$$\gamma_{ijk} \geq 0 \qquad \forall j, \forall i \leq j, \forall k \qquad (4.19')$$

Dual of Customer k

$$\lambda_{jk} \leq (p_i + h_{ijk})D_{jk} \qquad \forall j, \forall i \leq j, \forall k \qquad (4.21)$$

$$\lambda_{jk} \leq (pc_i + h_{ijk})D_{jk} \qquad \forall j, \forall i \leq j, \forall k \qquad (4.22)$$

$$\lambda_{jk} \geq 0 \qquad \forall j, k \qquad (4.23)$$

Complimentary Slackness

$$Sp_{jk} = \sum_{i\leq j}\left(z_{ijk} + \gamma_{ijk}\right) - 1 \qquad \forall j, k \qquad (4.24)$$

$$Sd_{ijk} = (p_i + h_{ijk})D_{jk} - \lambda_{jk} \qquad \forall j, \forall i \leq j, \forall k \qquad (4.25)$$

$$Sdc_{ijk} = (pc_i + h_{ijk})D_{jk} - \lambda_{jk} \qquad \forall j, \forall i \leq j, \forall k \qquad (4.26)$$

$$(1 - fp_{jk})\varepsilon \leq Sp_{jk} \leq (1 - fp_{jk})M \qquad \forall j, k \qquad (4.27)$$

$$fp_{jk}\varepsilon \leq \lambda_{jk} \leq fp_{jk}M \qquad \forall j, k \qquad (4.28)$$

$$(1-fd_{ijk})\varepsilon \le Sd_{ijk} \le (1-fd_{ijk})M \qquad \forall j, \forall i \le j, \forall k \tag{4.29}$$

$$fd_{ijk}\varepsilon \le z_{ijk} \le fd_{ijk}M \qquad \forall j, \forall i \le j, \forall k \tag{4.30}$$

$$(1-fdc_{ijk})\varepsilon \le Sdc_{ijk} \le (1-fdc_{ijk})M \qquad \forall j, \forall i \le j, \forall k \tag{4.31}$$

$$fdc_{ijk}\varepsilon \le \gamma_{ijk} \le fdc_{ijk}M \qquad \forall j, \forall i \le j, \forall k \tag{4.32}$$

$$fp_{jk} \in \{0,1\} \qquad \forall j, k \tag{4.33}$$

$$fd_{ijk} \in \{0,1\} \qquad \forall j, \forall i \le j, \forall k \tag{4.34}$$

$$fdc_{ijk} \in \{0,1\} \qquad \forall j, \forall i \le j, \forall k \tag{4.35}$$

We can notice that the converting a bi-level optimization problem to a single level optimization problem of our study is similar to the model transformation structure that presented by (J.F. Bard, 1998), we can see it in below figures.

$$\begin{aligned}
&\min_{x \in X} F(x,y) = c_1x + d_1y \\
&\text{s.t.} \\
&\qquad A_1x + B_1y \le b_1 \\
&\qquad \min_{y \in Y} f(x,y) = c_2x + d_2y \\
&\qquad \text{s.t.} \\
&\qquad\qquad A_2x + B_2y \le b_2
\end{aligned}$$

Figure 4.3: General form of bi-level programming model (J.F. Bard, 1998)

$$\begin{array}{lcl}
\displaystyle\min_{\mathbf{x} \in X} & \mathbf{c_1x} + \mathbf{d_1y} & \\
\text{s.t.} & & \\
 & A_1\mathbf{x} + \mathbf{B_1y} & \le \mathbf{b_1} \\
 & \mathbf{u}B_2 - \mathbf{v} & = -\mathbf{d_2} \\
 & \mathbf{u}(\mathbf{b_2} - A_2\mathbf{x} - B_2\mathbf{y}) + \mathbf{vy} & = 0 \\
 & A_2\mathbf{x} + B_2\mathbf{y} & \le \mathbf{b_2} \\
 & \mathbf{x} \ge 0, \ \ \mathbf{y} \ge 0, \ \ \mathbf{u} \ge 0, \ \ \mathbf{v} \ge 0 &
\end{array}$$

Figure 4.4: An equivalent single level model (J.F. Bard, 1998)

4.2.5 Linearization Model

Recall the objective function of retailer optimization model,

$$\textit{Maximize Profit} = \sum_{k}\sum_{i}\sum_{j\geq i} p_i D_{jk} z_{ijk} - \sum_{i} S_i Sc - \sum_{i} hr_i I_i - \sum w_i x_i \qquad (4.1)$$

We can see that the objective function (4.1) is nonlinear because of the term $\sum_{k}\sum_{i}\sum_{j\geq i} p_i D_{jk} z_{ijk}$ (both p_i and $z_i$are decision variables). We need to linearize the objective function so it can be easily solved by available commercial software.

As presented in Section 4.2.4, the customer model is forced to be at optimal points due to the primal feasibility, dual feasibility and complementary slackness constraints, the primal objective and dual objective are equal.

$$\sum_{i}\sum_{j\geq i}\left(p_i + h_{ijk}\right) D_{jk} z_{ijk} + \sum_{i}\sum_{j\geq i}\left(pc_i + h_{ijk}\right) D_{jk} \gamma_{ijk} = \sum_{j} \lambda_{jk}$$

$$\sum_{i}\sum_{j\geq i} p_i D_{jk} z_{ijk} + \sum_{i}\sum_{j\geq i} h_{ijk} D_{jk} z_{ijk} + \sum_{i}\sum_{j\geq i}\left(pc_i + h_{ijk}\right) D_{jk} \gamma_{ijk} = \sum_{j} \lambda_{jk}$$

$$\sum_{i}\sum_{j\geq i} p_i D_{jk} z_{ijk} = \sum_{j} \lambda_{jk} - \sum_{i}\sum_{j\geq i} h_{ijk} D_{jk} z_{ijk} - \sum_{i}\sum_{j\geq i}\left(pc_i + h_{ijk}\right) D_{jk} \gamma_{ijk}$$

Now we can linearize the objective function (4.1) by replacing nonlinear term by linear term as following.

$$\textit{Maximize Profit} = \sum_{k}\sum_{i}\sum_{j\geq i} p_i D_{jk} z_{ijk} - \sum_{i} S_i Sc - \sum_{i} hr_i I_i - \sum w_i x_i \qquad (4.1)$$

$$= \sum_{k}\left(\sum_{j} \lambda_{jk} - \sum_{i}\sum_{j\geq i} h_{ijk} D_{jk} z_{ijk} - \sum_{i}\sum_{j\geq i}\left(pc_i + h_{ijk}\right) D_{jk} \gamma_{ijk}\right) - \sum_{i} S_i Sc - \sum_{i} hr_i I_i - \sum w_i x_i$$

$$(4.1')$$

4.3 Numerical Experiments and Results

In this section, the numerical experiments are interpret to determine and discuss the implication of the retailer optimization model that has been developed in the previous part. In the experiments, the parameters of the model are varied to demonstrate the two-level decision making process of a retailer and strategic customers for retailer's pricing strategy. Related cost such as setup cost, inventory cost and ordering cost are considered.

To test the implication of the model, the input data in following table will be used. We assume the demand and holding cost of customers by utilize Normal Distribution, setting mean at 20 and standard deviation of 5. In addition, we considers 8 periods (weeks) and 20 customers in this experiment.

Table 4.1: Input data of base case

	Values
Demand of customer	{54, 57, 54, 57, 54 , 51, 58, 56}
Holding cost at retailer	5
Holding cost at customers	{0.52, 0.53, 0.46, 0.52, 0.44, 0.35, 0.40, 0.37, 0.22, 0.44, 0.34, 0.46, 0.46, 0.37, 0.52, 0.37, 0.20, 0.56, 0.37, 0.25}
Fixed setup cost at retailer	1000
List price	100
Wholesale price	{50, 50, 50, 50, 50, 50, 50, 50}
Competitor price	{100,100,100,80,80,100,100,100}

Table 4.2: The optimal decision of base case

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Retail price	100	100	100	79.44	100	100	100	100
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0
Retailer's profit	13549.67							
Revenue	38424.67							
Setup cost	2000							
Inventory cost	825							
Ordering cost	22050							
Customer's cost	38640.99							

Table 4.2 presents the optimal decision of retailer and associated results of base case scenario which will be used to capture the relationship of each parameter afterward. In the optimal decision of base case scenario, we can see that the retailer decides to offer promotion at the first period of competitor promotion and slight cut the price to compete customer demand from competitor.

4.3.1 Competitor price analysis

In this section, we analyze the impact of competitor price that affects the decision of retailer price. The main parameters of these experiments are the same as base case input data that presented in Table 4.1 but the competitor price is modified for several scenarios. For instance, short and deep discount, long and shallow discount and everyday low price discount (EDLP)

Table 4.3- 4.9 present the significant contribution of results. In case Pc1, when competitor announces the promotion one period with 10% discount, the retailer decides to apply the same price discount strategy. Case Pc2, the competitor utilize EDLP format by selling the product lower than normal list price 10% in every period, the retailer reacts to this promotion plan by cutting the promotion price less from 90 to 86.7759 since the first period of demand. It leads to forward buying from customers who have low inventory cost. However, the strategic customers who have high holding cost will purchase the product as their normal demand at the competitor which provide them with lower price. The competitor price with long and shallow discount in case Pc3 makes retailer optimize its profit by offer promotion discount at the beginning of competitor's promotion with 88.3116 discount price. In case Pc4, the competitor offers the different deal scenario, it has two frequency discounts away from each other in period 2 and period 8. Retailer response to this competitive discount by cutting the price since the first period ahead of competitor promotion at 87.3773. This decision helps retailer increase its own profit because it causes retailer order less often and it consequently reduces set up and inventory carrying cost. We can notice from example case Pc1 –Pc4 that the competitor offers the same amount of discount price at 90 but different promotion patterns. The optimization decision of the model suggest that the longer duration of competitor's discount, the higher discount promotion retailer should do.

Next, we set the discount equal to 30% of normal price list in case Pc5, the result is the same direction as earlier case. The result derives from case Pc6 suggest that the retailer will have no promotion in any period if it see that it is not worth for making discount because the competitor makes a larger discount until the retailer cannot complete it. Lastly, when the competitor offers stronger discount and the promotion period is very far away from each other in period 1 and period 8, the model decides to set the price equal to competitor's promotion price to influence customer demand as much as it can, then let it go the demand of customers that it cannot complete to competitor who provide the lower product price.

The results of competitor price comparison in Table 4.10 indicates that the optimal promotion price and period of retailer is sensitive to the competitor's decision in the same market. The retailer will adjust the price to complete strategic customers while maximize its own profit.

Further, we can observe the reorder pattern of retailer that is the retailer will order the product from wholesaler at the first period then reorder it at the promotion period. Retailer has to make sure that it places enough order quantity at the first period to cover the customer's demand until next reorder. Sometimes, retailer decides to combine order and promotion price to be at the same period to reduce set up cost like case Pc4.

In addition, we see that the retailer decides to let go only a few of customers' demand for example let go 5 unit of demand in case Pc2 and 1 unit in both Pc4 and Pc5. The question is why the model doesn't make the decision to complete all customers' demand in all period. So we do some sensitivity analysis in case Pc2. The results show that if retailer decreases its price from 86.7759 to 86, it will gain all customers' demand including 5 units that it used to lose to competitor but the total retailer's profit decrease from 15,034 to 14,876. Therefore, the reason that sometimes retailer decides to lose some amount of demand to competitor because it is not worth for doing discount promotion to attract the customers to make a purchase only a few units but the retailer has to trade with losing marginal profit

Table 4.3: Results of competitor price variation (Case P1)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	90	100	100	100	100
Retail price	100	100	100	90	100	100	100	100
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0
Retailer's profit	16,465							
Revenue	41,340							
Setup cost	2,000							
Inventory cost	825							
Ordering cost	22,050							
Customer's cost	41,556							

Table 4.4: Results of competitor price variation (Case P2)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	90	90	90	90	90	90	90	90
Retail price	86.7759	100	100	100	100	100	100	100
Retailer's sales	436	0	0	0	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	5
Inventory	0	0	0	0	0	0	0	0
Order quantity	436	0	0	0	0	0	0	0
Retailer's profit	15,034							
Revenue	37,834							
Setup cost	1,000							
Inventory cost	0							
Ordering cost	21,800							
Customer's cost	38,870							

Table 4.5: Results of competitor price variation (Case P3)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	90	90	90	90	100	100
Retail price	100	100	88.3116	100	100	100	100	100
Retailer's sales	54	57	330	0	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	57	0	0	0	0	0	0	0
Order quantity	111	0	330	0	0	0	0	0
Retailer's profit	15,908							
Revenue	40,243							
Setup cost	2,000							
Inventory cost	285							
Ordering cost	22,050							
Customer's cost	40,567							

Table 4.6: Results of competitor price variation (Case P4)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	90	100	100	100	90	100	100
Retail price	87.3773	100	100	100	100	100	100	100
Retailer's sales	440	0	0	0	0	0	0	0
Competitor's sales	0	0	0	0	0	1	0	0
Inventory	0	0	0	0	0	0	0	0
Order quantity	440	0	0	0	0	0	0	0
Retailer's profit	15,446							
Revenue	38,446							
Setup cost	1,000							
Inventory cost	0							
Ordering cost	22,000							
Customer's cost	15,446							

Table 4.7: Results of competitor price variation (Case P5)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	70	100	100	100	70	100	100
Retail price	100	67.9018	100	100	100	100	100	100
Retailer's sales	54	386	0	0	0	0	0	0
Competitor's sales	0	0	0	0	0	1	0	0
Inventory	0	0	0	0	0	0	0	0
Order quantity	54	386	0	0	0	0	0	0
Retailer's profit	7,610							
Revenue	31,610							
Setup cost	2,000							
Inventory cost	0							
Ordering cost	22,000							
Customer's cost	32,131							

Table 4.8: Results of competitor price variation (Case P6)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	50	100	100	100	90	100	100
Retail price	100	100	100	100	100	100	100	100
Retailer's sales	54	0	0	0	0	0	0	0
Competitor's sales	0	387	0	0	0	0	0	0
Inventory	0	0	0	0	0	0	0	0
Order quantity	54	0	0	0	0	0	0	0
Retailer's profit	1,700							
Revenue	5,400							
Setup cost	1,000							
Inventory cost	0							
Ordering cost	2,700							
Customer's cost	25,203							

Table 4.9: Results of competitor price variation (Case P7)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	60	100	100	100	100	100	100	60
Retail price	60	100	100	100	100	100	100	100
Retailer's sales	385	0	0	0	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	56
Inventory	0	0	0	0	0	0	0	0
Order quantity	385	0	0	0	0	0	0	0
Retailer's profit	2,850							
Revenue	23,100							
Setup cost	1,000							
Inventory cost	0							
Ordering cost	19,250							
Customer's cost	26,910							

Table 4.10: Comparison of competitor price variation

	Time period							
Base case	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0

	Time period							
Case P1	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	90	100	100	100	100
Retail price	100	100	100	90	100	100	100	100
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0

	Time period							
Case P2	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	90	90	90	90	90	90	90	90
Retail price	86.7759	100	100	100	100	100	100	100
Retailer's sales	436	0	0	0	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	5
Order quantity	436	0	0	0	0	0	0	0

	Time period							
Case P3	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	90	90	90	90	100	100
Retail price	100	100	88.3116	100	100	100	100	100
Retailer's sales	54	57	330	0	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Order quantity	111	0	330	0	0	0	0	0

	Time period							
Case P4	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	90	100	100	100	90	100	100
Retail price	87.3772	100	100	100	100	100	100	100
Retailer's sales	440	0	0	0	0	0	0	0
Competitor's sales	0	0	0	0	0	1	0	0
Order quantity	440	0	0	0	0	0	0	0

	Time period							
Case P5	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	70	100	100	100	70	100	100
Retail price	100	67.9018	100	100	100	100	100	100
Retailer's sales	54	386	0	0	0	0	0	0
Competitor's sales	0	0	0	0	0	1	0	0
Order quantity	54	386	0	0	0	0	0	0

	Time period							
Case P6	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	50	100	100	100	90	100	100
Retail price	100	100	100	100	100	100	100	100
Retailer's sales	54	0	0	0	0	0	0	0
Competitor's sales	0	387	0	0	0	0	0	0
Order quantity	54	0	0	0	0	0	0	0

	Time period							
Case P7	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	60	100	100	100	100	100	100	60
Retail price	60	100	100	100	100	100	100	100
Retailer's sales	385	0	0	0	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	56
Order quantity	385	0	0	0	0	0	0	0

4.3.2 Wholesale price analysis

According to (R.C. Blattberg & Neslin, 1990), trade promotions are promotions that manufacture offers to the retailer so wholesale price is standard approach of trade promotions. Even though some researchers see trade promotion as a valueless approach to business because of forward buying and bullwhip effect, (Kotler, 1988) argued that it still have some benefits and positive effects to manufacturer and retailer. For instance, wholesaler temporary price discount can persuade the retailer to carry the brand and store more amount of products than normal.

In this part, we decide to test our model using various different period of wholesale price $(\boldsymbol{w_i})$ to determine how the trade promotion from wholesaler pass through the retailer's promotional decision $(\boldsymbol{p_i})$ and how the trade promotion affect the entire supply chain

We reduce 20 percent of wholesale price that offer from manufacture along each case scenario instead of constant wholesale price. Noted that parameters using in this experiment is the same as input data of base case. The results of derived model are displayed in Table 4.11- 4.21, we set the wholesale price equal to 40 ahead of competitor's promotion in period 3, period 2, and period 1 in scenario case W1, W2 and W3 respectively. The model demonstrates that the retailer will offer the promotion follow the wholesale price structure but the earlier period, the lower price offer as follow; 78.8744, 78.3116 and 77.7488. This experiment leads to lower retailer's revenue as price decrease however it helps retailer save inventory cost too. In case W4 – case W6, we set the wholesale price after competitor's promotion. From case W4, the retailer response to trade promotion by extending its promotion to take advantage of wholesaler discount. To see the significant effect of trade promotion, we do more experiment with different wholesale price pattern in case W5 and W6, it shows that the optimal price and period is the same as base case.

In case W7 –W9, we implement the model and set the scenario to cover promotion of competitor so the frequency of wholesale price discount is two times before and after competitor's promotion. There is no change in retailer's decision, the results are the same as presented in case W1- W3. However, we test the wholesale price again by setting trade promotion the same period with competitor's discount time in case W10, the optimal retail price and total supply chain profit is the same as base case.

Table 4.22 shows the comparison of wholesale price variation, it indicates that wholesale price can affect the retailer's optimal promotion price and period. The wholesale price discount passes through the decision of retailer and it leads to the higher sales in that period. The supply chain will gain the highest total profit when the wholesale price is stable, no discount offers to retailer at any period (base case), including case W6, case W7 and case W10 which total supply chain profit is equal to 20,165 and the retail price is 79.4372. Nevertheless, the manufacturer's profit and retailer's profit is different. The retailer will gain more profit if the manufacturer provides the discount same period as competitor's promotion.

Further, the experiment indicate that the manufacturer can help control retailer promotion if it offer the proper promotion wholesale price discount so it helps improve the efficiency of total supply chain so we can say that the manufacturer has a choice. The retailer can also pass some trade promotion from manufacturer to customers to help them minimize their total cost.

Table 4.11: Results of wholesale price variation (Base Case)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Wholesale price	50	50	50	50	50	50	50	50
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0
Retailer's profit	13,550							
Revenue	38,425							
Setup cost	2,000							
Inventory cost	825							
Ordering cost	22,050							
Customer's cost	38,641							

Table 4.12: Results of wholesale price variation (Case W1)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Wholesale price	50	50	40	50	50	50	50	50
Retail price	100	100	78.8744	100	100	100	100	100
Retailer's sales	54	57	330	0	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	57	0	0	0	0	0	0	0
Order quantity	111	0	330	0	0	0	0	0
Retailer's profit	16,094							
Revenue	37,129							
Setup cost	2,000							
Inventory cost	285							
Ordering cost	18,750							
Customer's cost	37,453							

Table 4.13: Results of wholesale price variation (Case W2)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Wholesale price	50	40	50	50	50	50	50	50
Retail price	100	78.3116	100	100	100	100	100	100
Retailer's sales	54	387	0	0	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	0	0	0	0	0	0	0	0
Order quantity	54	387	0	0	0	0	0	0
Retailer's profit	15,527							
Revenue	35,707							
Setup cost	2,000							
Inventory cost	0							
Ordering cost	18,180							
Customer's cost	36,159							

Table 4.14: Results of wholesale price variation (Case W3)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Wholesale price	40	50	50	50	50	50	50	50
Retail price	77.7488	100	100	100	100	100	100	100
Retailer's sales	441	0	0	0	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	0	0	0	0	0	0	0	0
Order quantity	441	0	0	0	0	0	0	0
Retailer's profit	15,647							
Revenue	34,287							
Setup cost	1,000							
Inventory cost	0							
Ordering cost	17,640							
Customer's cost	15,647							

Table 4.15: Results of wholesale price variation (Case W4)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Wholesale price	50	50	50	50	50	40	50	50
Retail price	100	100	100	80	80	80	100	100
Retailer's sales	54	57	54	57	54	165	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	54	0	0	0	0
Order quantity	165	0	0	111	0	165	0	0
Retailer's profit	14,085							
Revenue	38,580							
Setup cost	3,000							
Inventory cost	1,095							
Ordering cost	20,400							
Customer's cost	14,085							

Table 4.16: Results of wholesale price variation (Case W5)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Wholesale price	50	50	50	50	50	50	40	50
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0
Retailer's profit	13,550							
Revenue	38,425							
Setup cost	2,000							
Inventory cost	825							
Ordering cost	22,050							
Customer's cost	38,641							

Table 4.17: Results of wholesale price variation (Case W6)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Wholesale price	50	50	50	50	50	50	50	40
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0
Retailer's profit	13,550							
Revenue	38,425							
Setup cost	2,000							
Inventory cost	825							
Ordering cost	22,050							
Customer's cost	38,641							

Table 4.18: Results of wholesale price variation (Case W7)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Wholesale price	50	50	40	50	50	40	50	50
Retail price	100	100	78.8744	100	100	100	100	100
Retailer's sales	54	57	330	0	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	57	0	0	0	0	0	0	0
Order quantity	111	0	330	0	0	0	0	0
Retailer's profit	16,094							
Revenue	37,129							
Setup cost	2,000							
Inventory cost	285							
Ordering cost	18,750							
Customer's cost	37,453							

Table 4.19: Results of wholesale price variation (Case W8)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Wholesale price	50	40	50	50	50	50	40	50
Retail price	100	78.3116	100	100	100	100	100	100
Retailer's sales	54	387	0	0	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	0	0	0	0	0	0	0	0
Order quantity	54	387	0	0	0	0	0	0
Retailer's profit	15,527							
Revenue	35,707							
Setup cost	2,000							
Inventory cost	0							
Ordering cost	18,180							
Customer's cost	36,159							

Table 4.20: Results of wholesale price variation (Case W9)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Wholesale price	40	50	50	50	50	50	50	40
Retail price	77.7488	100	100	100	100	100	100	100
Retailer's sales	441	0	0	0	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	0	0	0	0	0	0	0	0
Order quantity	441	0	0	0	0	0	0	0
Retailer's profit	15,647							
Revenue	34,287							
Setup cost	1,000							
Inventory cost	0							
Ordering cost	17,640							
Customer's cost	15,647							

Table 4.21: Results of wholesale price variation (Case W10)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Wholesale price	50	50	50	40	40	50	50	50
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0
Retailer's profit	16,310							
Revenue	38,425							
Setup cost	2,000							
Inventory cost	825							
Ordering cost	19,290							
Customer's cost	38,641							

Table 4.22: Comparison of wholesale price variation

	Time period								*Total Supply chain profit*	*Manufacturer's profit*	*Retailer's profit*
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**			
Base Case	50	50	50	50	50	50	50	50	20,165	6,615	13,550
Optimal P	100	100	100	79.4372	100	100	100	100			
Case W1	50	50	40	50	50	50	50	50	19,409	3,315	16,094
Optimal P	100	100	78.8744	100	100	100	100	100			
Case W2	50	40	50	50	50	50	50	50	18,272	2,745	15,527
Optimal P	100	78.3116	100	100	100	100	100	100			
Case W3	40	50	50	50	50	50	50	50	17,852	2,205	15,647
Optimal P	77.7488	100	100	100	100	100	100	100			
Case W4	50	50	50	50	50	40	50	50	19,050	4,965	14,085
Optimal P	100	100	100	80	80	80	100	100			
Case W5	50	50	50	50	50	50	40	50	20,165	6,615	13,550
Optimal P	100	100	100	79.4372	100	100	100	100			
Case W6	50	50	50	50	50	50	50	40	20,165	6,615	13,550
Optimal P	100	100	100	79.4372	100	100	100	100			
Case W7	50	50	40	50	50	40	50	50	19,409	3,315	16,094
Optimal P	100	100	78.8744	100	100	100	100	100			
Case W8	50	40	50	50	50	50	40	50	18,272	2,745	15,527
Optimal P	100	78.3116	100	100	100	100	100	100			
Case W9	40	50	50	50	50	50	50	40	17,852	2,205	15,647
Optimal P	77.7488	100	100	100	100	100	100	100			
Case W10	50	50	50	40	40	50	50	50	20,165	3,855	16,310
Optimal P	100	100	100	79.4372	100	100	100	100			

4.3.3 Inventory holding cost of customer analysis

In this section, we develop numerical examples to examine the decision of the model when the distribution of customer's holding cost is changed. As we propose in base case, the holding cost of customer use Normal Distribution by setting mean equal to 20 and standard deviation is 5.

According to the results of holding cost variation in Table 4.23-4.27, when we test the model by increasing the mean of holding cost 40% in case H1, the optimal retailer promotion price changes from 79.4372 to 79.3246 in the same period. The retailer profit is decreasing in the same direction from 13,550 to 13,519. In contrast, when we reduce the mean of holding cost 40% for case H2, the optimal decision of retailer and retailer profit increase. We notice that shifting mean of customer's holding cost does not affect the setup cost, inventory cost and ordering cost including the customer's purchasing decision.

While we modify the standard deviation of customer's holding cost by rising it to 10 in case H3, the optimal price is also increasing from 79.4372 to 79.4630 and the retailer profit is growing too. However, when we decrease standard deviation of customer's holding cost from 5 to 0 in case Hc4, the optimal retailer price is also reduced but at the same period.

It is practical that when the holding cost of customers decrease, the retailer can increase its price. Then it leads to increase retailer and total supply chain profit. However the changing in holding cost effect only promotion price discount not period discount. The optimal promotion is still the same period as base case. It is reasonable to say that the retailer will prefer the customers who have lower holding cost.

Table 4.23: Results of customer's holding variation (Base Case)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Wholesale price	50	50	50	50	50	50	50	50
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0
Retailer's profit	13,550							
Revenue	38,425							
Setup cost	2,000							
Inventory cost	825							
Ordering cost	22,050							
Customer's cost	38,641							

Base case: *mean of holding cost = 20 and S.D of holding cost =5*

Table 4.24: Results of customer's holding variation (Case H1)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Retail price	100	100	100	79.2121	100	100	100	100
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0
Retailer's profit	13,488							
Revenue	38,363							
Setup cost	2,000							
Inventory cost	825							
Ordering cost	22,050							
Customer's cost	38,665							

Case Hc1: *mean of holding cost =30 and S.D of holding cost = 5*

Table 4.25: Results of customer's holding variation (Case H2)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Retail price	100	100	100	79.6623	100	100	100	100
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0
Retailer's profit	13,612							
Revenue	38,487							
Setup cost	2,000							
Inventory cost	825							
Ordering cost	22,050							
Customer's cost	38,616							

Case Hc2: *mean of holding cost =10 and S.D of holding cost = 5*

Table 4.26: Results of customer's holding variation (Case H3)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Retail price	100	100	100	79.463	100	100	100	100
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0
Retailer's profit	13,557							
Revenue	38,432							
Setup cost	2,000							
Inventory cost	825							
Ordering cost	22,050							
Customer's cost	38,657							

Case Hc3: *mean of holding cost =20 and S.D of holding cost = 10*

Table 4.27: Results of customer's holding variation (Case H4)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Retail price	100	100	100	79.4254	100	100	100	100
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0
Retailer's profit	13,546							
Revenue	38,421							
Setup cost	2,000							
Inventory cost	825							
Ordering cost	22,050							
Customer's cost	38,651							

Case Hc4: *mean of holding cost =10 and S.D of holding cost = 0*

Base case: *mean of holding cost = 20 and S.D of holding cost =5*

	Time period								Retailer's profit
	1	2	3	4	5	6	7	8	
Retail price	100	100	100	79.4372	100	100	100	100	13,550
Retailer's sales	54	57	54	276	0	0	0	0	
Competitor's sales	0	0	0	0	0	0	0	0	
Order quantity	165	0	0	276	0	0	0	0	

Mean of holding cost =30 and S.D of holding cost = 5

	Time period								Retailer's profit
	1	2	3	4	5	6	7	8	
Retail price	100	100	100	79.2121	100	100	100	100	13,488
Retailer's sales	54	57	54	276	0	0	0	0	
Competitor's sales	0	0	0	0	0	0	0	0	
Order quantity	165	0	0	276	0	0	0	0	

Mean of holding cost =10 and S.D of holding cost = 5

	Time period								Retailer's profit
	1	2	3	4	5	6	7	8	
Retail price	100	100	100	79.6623	100	100	100	100	13,612
Retailer's sales	54	57	54	276	0	0	0	0	
Competitor's sales	0	0	0	0	0	0	0	0	
Order quantity	165	0	0	276	0	0	0	0	

Mean of holding cost =20 and S.D of holding cost = 10

	Time period								Retailer's profit
	1	2	3	4	5	6	7	8	
Retail price	100	100	100	79.463	100	100	100	100	13,556
Retailer's sales	54	57	54	276	0	0	0	0	
Competitor's sales	0	0	0	0	0	0	0	0	
Order quantity	165	0	0	276	0	0	0	0	

Mean of holding cost =20 and S.D of holding cost = 0

	Time period								Retailer's profit
	1	2	3	4	5	6	7	8	
Retail price	100	100	100	79.4254	100	100	100	100	13,546
Retailer's sales	54	57	54	276	0	0	0	0	
Competitor's sales	0	0	0	0	0	0	0	0	
Order quantity	165	0	0	276	0	0	0	0	

Figure 4.5: Comparison of customer's holding cost variation

4.3.4 Demand analysis

As the base case scenario, we generate the demand of each period for individual customer by utilizing the random number function and Normal Distribution in Microsoft excel, we set mean of demand equal to 20 and standard deviation is 5.

To implement demand analysis, we vary the demand distribution by increasing and decreasing mean of demand. We also consider changing the distribution function to understand whether the skewed of demand distribution will affect the retailer's decision or not. The results demonstrate in Case D1 and D2 that adjust the mean of demand $\pm$ 20%, the optimal price is still the same as base case. The retailer profit is changing follow the demand.

Then we raise the standard deviation of demand in Case D3 equal to 10, the optimal price is 79.4372. We formulate demand distribution in case D4 and D5 by applying probability distribution, we set alpha = 1.5 and beta = 5 in case D4 and set alpha = 5 and beta = 1.5 for case D5. Note that we control the total demand. Even the demand is follow a skewed distribution, the optimal promotion price and period does not change.

We can obviously confirm the results of demand variation that changing demand distribution does not have any impact to the retailer's optimal price and period.

Table 4.28: Results of customer's demand variation (Base Case)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Wholesale price	50	50	50	50	50	50	50	50
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	111	54	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0
Retailer's profit	13,550							
Revenue	38,425							
Setup cost	2,000							
Inventory cost	825							
Ordering cost	22,050							
Customer's cost	38,641							

Base case: *mean of demand = 20 and S.D. of demand = 5*

Table 4.29: Results of customer's demand variation (Case D1)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	64	67	63	323	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	130	63	0	0	0	0	0	0
Order quantity	194	0	0	323	0	0	0	0
Retailer's profit	16,243							
Revenue	45,058							
Setup cost	2,000							
Inventory cost	965							
Ordering cost	25,850							
Customer's cost	31,969							

Case D1: *mean of demand = 25 and S.D. of demand = 5*

Table 4.30: Results of customer's demand variation (Case D2)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	44	47	45	229	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	92	45	0	0	0	0	0	0
Order quantity	136	0	0	229	0	0	0	0
Retailer's profit	10,856							
Revenue	31,791							
Setup cost	2,000							
Inventory cost	685							
Ordering cost	18,250							
Customer's cost	38,620							

Case D2: *mean of demand = 15 and S.D. of demand = 5*

Table 4.31: Results of customer's demand variation (Case D3)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	56	53	55	277	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	108	55	0	0	0	0	0	0
Order quantity	164	0	0	277	0	0	0	0
Retailer's profit	13,539							
Revenue	38,404							
Setup cost	2,000							
Inventory cost	815							
Ordering cost	22,050							
Customer's cost	38,617							

Case D3: *mean of demand = 20 and S.D. of demand = 10*

Table 4.32: Results of customer's demand variation (Case D4)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	54	57	59	271	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	116	59	0	0	0	0	0	0
Order quantity	170	0	0	271	0	0	0	0
Retailer's profit	13,603							
Revenue	38,528							
Setup cost	2,000							
Inventory cost	875							
Ordering cost	22,050							
Customer's cost	38,730							

Case D4: *mean of demand = 20 and S.D. of demand = 10 and alpha =1.5 and beta =5*

Table 4.33: Results of customer's demand variation (Case D5)

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Competitor price	100	100	100	80	80	100	100	100
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	57	52	56	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Inventory	108	56	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0
Retailer's profit	13,555							
Revenue	38,425							
Setup cost	2,000							
Inventory cost	820							
Ordering cost	22,050							
Customer's cost	38,636							

Case D5: *mean of demand = 20 and S.D. of demand = 10 and alpha =5 and beta =1.5*

Base case: *mean of demand = 20 and S.D. of demand = 5*

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Period Demand	54	57	54	57	54	51	58	56
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	54	57	54	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0

Mean of demand = 25 and S.D. of demand = 5

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Period Demand	64	67	63	66	63	60	68	66
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	64	67	63	323	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Order quantity	194	0	0	323	0	0	0	0

Mean of demand = 15 and S.D. of demand = 5

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Period Demand	44	47	45	48	45	42	48	46
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	44	47	45	229	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Order quantity	136	0	0	229	0	0	0	0

Mean of demand = 20 and S.D. of demand = 10

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Period Demand	56	53	55	55	56	55	52	59
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	56	53	55	277	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Order quantity	164	0	0	277	0	0	0	0

Alpha =1.5 and Beta =5

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Period Demand	54	57	59	56	54	56	53	52
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	54	57	59	271	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Order quantity	170	0	0	271	0	0	0	0

Alpha =5 and Beta =1.5

	Time period							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Period Demand	57	52	56	55	55	54	56	56
Retail price	100	100	100	79.4372	100	100	100	100
Retailer's sales	57	52	56	276	0	0	0	0
Competitor's sales	0	0	0	0	0	0	0	0
Order quantity	165	0	0	276	0	0	0	0

Figure 4.6: Comparison of demand variation

4.4 Discussion of the model

The aim of the numerical experiment in this chapter is to determine how each parameter would impact the optimization decision of retailer's promotion. The propose model in Section 4.2 is derived. Our retailer optimization model can solve a two-level problem, the customer will make the purchasing decision on the discount promotion as the retailer plan. We implement four analysis as follows; (1) Competitor price analysis; (2) Wholesale price analysis; (3) Holding cost of customer analysis; and (4) Demand analysis

First, we found that when the competitor announces the promotion, the retailer seems to make the decision on pricing strategy in the same direction which is the retailer cut the price with stronger deep discount at the first period of competitor and do it only one time, no matter the long duration of competitor's promotion. And the strategic customer responses to this promotion by changing purchasing timing in ahead of time during the promotion occurs. However, the retailer should offer a larger discount to induce customers to purchase enough quantity during discount promotion. The reorder pattern of retailer is noticeable that it will make an order at first period and at the promotion period by considering appropriate amount of order quantity to lower setup cost.

Second, the experiment indicate that the discount promotion from wholesale price can pass through retailer's price and if the manufacturer provide the proper discount at right price and period it will pass some of trade promotion to the end customer. Consequently, it enhance the total supply chain profit. In addition, wholesale price have an effect to both retailer's promotion price and time. Manufacturer also has a choice to make a decision on wholesale price and control retailer's action.

Third, the results of model shows some interrelated relationship between retailer's decision and holding cost of customers. The retailer tries to capture the demand of customers who have low holding cost. At the same time, the customers are willing to purchase the product and take the advantage of temporary price discount if they see that the holding cost is acceptable. We can say that the customers who have low holding cost may take advantage of promotion price by forward buying.

Fourth, the analysis of demand distribution shows that a skewed of demand distribution does not affect the retailer's decision. The optimal promotion price and period is the same. However the retailer profit and associated cost is changed.

Chapter 5
Conclusions and Recommendations

5.1 Conclusion of Thesis

This research formulates the mathematical programming model to determine the optimal decision of retailer's price promotion. A bi-level linear programming approach is utilized and then transformed it into an equivalence single level to solve a two decision problem of the retailer who wants to maximize his/her profit and the strategic customers who also minimize their own purchasing and holding cost simultaneously.

A simple framework of supply chain is developed to understand how the model works. The numerical experiments show the retailer's optimal promotion price and period and provide some guideline of inventory policy. The results of the model also demonstrate the related cost and profit such as retailer's profit, retailer's revenue, setup cost, inventory cost at retailer, customer's cost, manufacture's profit and total supply chain profit.

Further, we presents the optimal promotion price and period vary with the 4 key parameters which are: (1) Competitor price; (2) Wholesale price; (3) Holding cost of customer; and (4) Demand. The key parameters are investigated to describe the insight pattern of promotion discount that is useful for researcher and managers as follows. The competitor price and wholesale price discount from manufacturer have a great impact in retailer's promotion both depth and timing. The retailer should offer the discount promotion at the beginning period of competitor's promotion with lower price. And it should take advantage of trade deal by offering the promotion the same period as manufacturer provide discount price. It assumed from the model that the retailer prefers the customers who have low holding cost. However, the optimal pricing of retailer does not change follow the demand distribution.

All numerical experiments in this research is derived from IBM ILOG CPLEX, a commercial optimization software package. It is friendly user for solving retailer's optimizing promotional problem.

The main contribution of this research is we prove that optimal solution for solving the customer model by linear programming and integer programming model is equal. Further, the model can provide the optimal promotion discount strategy that is applicable for managers in industry and researchers in academic area.

5.2 Recommendations for Further Study

The model which we has developed in this study has some limitations. There are only one retailer and one competitor in the model, it would be meaningful to study multi-retailers and multi-competitors model. The experiment in this thesis is the short-term observation, the model should be extended to longer period to determine the retailer optimal decision and related cost. In addition, there are many other different demand and holding cost forms we can consider including the real data set. There are several extensions of this study that could be considered for future researches to bring out the managerial insight from the model. For instance, the effect of promotion that impact the customer's decision process to follow the retailer's promotion decision. The relationship of inventory decision and optimal pricing promotion. It would be more interesting if the future model explore the retailer price formats between "Everyday Low Price" and "Hi-Lo" pricing strategy.

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Appendix A
Proposition 1

Proposition 1 *will show that the result is achieved through a linear programming model, it is also the optimal solution to the customer model that solve by an integer programming model.*

To prove this theorem it is sufficient to show that the variables z_{ijk} and γ_{ijk} can only take integer values at the optimal solution of **LM**. If this is the case, then only the values 0 and 1 may arise, because of the constraints and the type of problem which the model **IP** defines.

(Hoffman & Kruskal, 2010) have shown that a linear program in the form like $\{\min cx \mid Ax > b, x \geq 0\}$ or $\{\max cx \mid Ax < b\}$ always has an integer optimal solution for any arbitrary integer vector b, if the matrix A is a totally unimodular matrix. (Luenberger, 1973) has shown that in the system of equations Ax = b, assume that A is totally unimodular and that all elements of A and b are integers so all basic solutions have integer components.

A matrix A is said to be totally unimodular if and only if every subdeterminant of A has value +1, -1, or 0.

Proof

For example, planning period is 4. We get the form of matrix A, let A be an m

	z_{11} i=1	z_{12} i=1	z_{13} i=1	z_{14} i=1	z_{22} i=2	z_{23} i=2	z_{24} i=2	z_{33} i=3	z_{34} i=3	z_{44} i=4	γ_{11} i=1	γ_{12} i=1	γ_{13} i=1	γ_{14} i=1	γ_{22} i=2	γ_{23} i=2	γ_{24} i=2	γ_{33} i=3	γ_{34} i=3	γ_{44} i=4		
j=1	1										1										≥	1
j=2		1			1							1			1						≥	1
j=3			1			1		1					1			1		1			≥	1
j=4				1			1		1	1				1			1		1	1	≥	1

by n matrix as follow:

Group A1 Group A2

|Sum of a_{mn} in A1-Sum of a_{mn} in A2|=0≤ 1 for all m so primal model has total unimodularity property.

Appendix B
CPEX Source Code

1. Retailer optimization model

The source code of the retailer model is divided into two sub-files which are model file and data file. The source code is as follows.

1.1 Retailer model file

```
/*********************************************
 * OPL 12.3 Model
 * Author: Windows7
 *********************************************/

//Parameters
int M = 10000;
float e = 0.0001;
int T = 8;
range index_i0 = 0..T;
//range index_i_1 = 1..(T-1);
range index_i = 1..T;
range index_j = 1..T;
{int}index_k = {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20};

float D [index_j][index_k]=...;

float h [index_i][index_j][index_k];

tuple holding_cost{
	int index_i;
	int index_j;
	int index_k;
	float value;
}
{holding_cost}hc=...;

execute{

	for (var h1 in hc){
		h[h1.index_i][h1.index_j][h1.index_k]=h1.value;
	}
}

float hr [index_i]=...;
```

```
float Sc =...;
float Pl =...;
float Pc [index_i]=...;
float w [index_i] =...;
float customer_cost_primal[index_k] ;
float customer_cost_dual[index_k] ;


//decision variables
dvar boolean z[index_i][index_j][index_k];
dvar boolean gamma[index_i][index_j][index_k];
dvar float+ Pm;
dvar float+ P [index_i];
dvar float+ x[index_i];
dvar float+ I[index_i0];
dvar boolean S[index_i];
dvar boolean Pr[index_i][index_i];
dvar boolean y[index_i];
dvar float+ lamda[index_j][index_k];
dvar float total_profit;

//variable of complimentary slackness
dvar float+ Sp [index_j][index_k];
dvar float+ Sd [index_i][index_j][index_k];
dvar float+ Sdc [index_i][index_j][index_k];
dvar boolean fp [index_j][index_k];
dvar boolean fd [index_i][index_j][index_k];
dvar boolean fdc [index_i][index_j][index_k];
dvar float k1;
dvar float k2;
dvar float k3;
dvar float k4;
dvar float sales[index_i];
dvar float customer_sales[index_i][index_k];
dvar float comp_sales[index_i];
dvar float total_demand;
dvar float period_demand[index_i];


//objective function
maximize total_profit;

subject to {
  total_profit ==  sum (k in index_k)(sum (j in index_j)lamda[j][k]
  -sum (i in index_i,j in i..T)(h[i][j][k]*D[j][k]*z[i][j][k])
  -sum (i in index_i,j in i..T)((Pc[i]+h[i][j][k])*D[j][k]*gamma[i][j][k]))
  -sum (i in index_i)(S[i]*Sc)
```

```
 - sum (i in index_i)(hr[i]*I[i])
 - sum (i in index_i)(w[i]*x[i]);
  forall (i in index_i)
          sales[i]==sum (k in index_k, j in i..T)(D[j][k]*z[i][j][k]);
  forall (i in index_i, k in index_k)
          customer_sales[i][k]==sum ( j in i..T)(D[j][k]*z[i][j][k]);
  forall (i in index_i)
          comp_sales[i]==sum (k in index_k, j in i..T)(D[j][k]*gamma[i][j][k]);
          total_demand==sum (k in index_k, j in index_j)(D[j][k]);
  forall (i in index_i)
          period_demand[i]==sum (k in index_k)(D[i][k]);

//retailer
k1== sum (k in index_k)(sum (j in index_j)lamda[j][k]
 -sum (i in index_i,j in i..T)(h[i][j][k]*D[j][k]*z[i][j][k])
 -sum (i in index_i,j in i..T)((Pc[i]+h[i][j][k])*D[j][k]*gamma[i][j][k]));
k2==sum (i in index_i)(S[i]*Sc);
k3==sum (i in index_i)(hr[i]*I[i]);
k4==sum (i in index_i)(w[i]*x[i]);
I[0]==0;
 forall (i in index_i)
   x[i]+I[i-1]-sum (k in index_k,j in index_j)(z[i][j][k]*D[j][k])== I[i];
 forall (i in index_i)
   x[i] <= S[i]*M;

 forall (i in index_i)
   P[i] <= Pl;
 forall (i in index_i)
   P[i] >= Pl-(1-y[i])*M;

 forall (i in index_i)
   P[i] <= Pm+y[i]*M;
 forall (i in index_i)
   P[i] >= Pm;

 sum (b in index_i, e in b..T)(Pr[b][e]) <= 1;

 forall (b in index_i, e in b..T, i in (e+1..T))
          Pr[b][e] <= y[i];

 forall (b in index_i, e in b..T, i in (1..b-1))
          Pr[b][e] <= y[i];

 forall (b in index_i, e in b..T, i in (b..e))
          1- Pr[b][e] >= y[i];
```

```
 //when no promotion period
 forall (i in index_i)
 1-M*sum (b in index_i, e in b..T)(Pr[b][e]) <= y[i];


//Primal of Customer k
         forall ( k in index_k, j in index_j)
           sum (i in 1..j) (z[i][j][k]+gamma[i][j][k]) >= 1;

//Dual of Customer k
   forall (j in index_j,i in 1..j, k in index_k)
     lamda[j][k] <= (P[i]+h[i][j][k])*D[j][k];
   forall (j in index_j,i in 1..j,k in index_k)
     lamda[j][k] <= (Pc[i]+h[i][j][k])*D[j][k];

//Complimentary Slackness
forall (k in index_k, j in index_j)
  Sp[j][k] == sum (i in 1..j)(z[i][j][k]+gamma[i][j][k])-1;

forall (j in index_j, i in 1..j, k in index_k)
  Sd[i][j][k] == (P[i]+ h[i][j][k]) * D[j][k] - lamda[j][k];

forall (j in index_j, i in 1..j, k in index_k)
  Sdc[i][j][k] == (Pc[i]+ h[i][j][k]) * D[j][k] - lamda[j][k];

forall (j in index_j,k in index_k)
  (1-fp[j][k])*e <= Sp[j][k];
forall (j in index_j, k in index_k)
  Sp[j][k] <= (1-fp[j][k])*M;

forall (j in index_j, k in index_k)
  fp[j][k]*e <= lamda[j][k];
forall (j in index_j, k in index_k)
  lamda[j][k] <= fp[j][k]*M;

forall (j in index_j, i in 1..j, k in index_k)
  (1-fd[i][j][k])*e <= Sd[i][j][k];
forall (j in index_j, i in 1..j, k in index_k)
  Sd[i][j][k] <= (1-fd[i][j][k])*M;

forall (j in index_j, i in 1..j, k in index_k)
   fd[i][j][k]*e <= z[i][j][k];
forall (j in index_j, i in 1..j, k in index_k)
   z[i][j][k] <= fd[i][j][k]*M;

 forall (j in index_j, i in 1..j, k in index_k)
   (1-fdc[i][j][k])*e <= gamma[i][j][k];
```

```
 forall (j in index_j, i in 1..j, k in index_k)
   gamma[i][j][k] <= fdc[i][j][k]*M;


}

execute{


for (var k in index_k) {

        customer_cost_dual[k] =0;
        customer_cost_primal[k]=0;
        for (var j in index_j) {
                customer_cost_dual[k]=customer_cost_dual[k]+lamda[j][k];
                for(var i in index_i  )
                        if (j>=i) {
                        customer_cost_primal[k]=customer_cost_primal[k]
+((P[i]+h[i][j][k])*D[j][k]*z[i][j][k]+((Pc[i]+h[i][j][k])*D[j][k]*gamma[i][j][k]));

                        }
        }
}
}
```

1.2 Retailer data file

```
/*********************************************
 * OPL 12.3 Data
 * Author: Windows7
*********************************************/

SheetConnection sheet("Input.xlsx");
// Change D to see optimal of retailer price, different patterns
D from SheetRead(sheet, "Demand1!B2:U9");
//D from SheetRead(sheet, "Demand2!B2:U9");
//D from SheetRead(sheet, "Demand3!B2:U9");
//D from SheetRead(sheet, "Demand4!B2:U9");
//D from SheetRead(sheet, "Demand5!B2:U9");
//D from SheetRead(sheet, "Demand6!B2:U9");
//D from SheetRead(sheet, "Demand7!B2:U9");
//D from SheetRead(sheet, "Demand8!B2:U9");

// Change hc to see optimal of retailer price, different patterns
hc from SheetRead(sheet,"Holding1!A2:D1281");
//hc from SheetRead(sheet,"Holding2!A2:D1281");
```

```
//hc from SheetRead(sheet,"Holding3!A2:D1281");
//hc from SheetRead(sheet,"Holding4!A2:D1281");
//hc from SheetRead(sheet,"Holding5!A2:D1281");
//hc from SheetRead(sheet,"Holding6!A2:D1281");


hr from SheetRead(sheet, "Demand1!B12:i12");
Sc = 1000;
Pl = 100;

// Change Pc to see optimal of retailer price, different patterns
Pc from SheetRead(sheet, "Sheet1!B15:i15");
//Pc = [100    100    100    90     100    100    100    100];
//Pc = [90 90 90 90 90 90 90 90];
//Pc = [100    100    90     90     90     90     100    100];
//Pc = [100    100    100    80     80     100    100    100];
//Pc = [100    70     100    100    100    70     100    100];
//Pc = [100    90     100    100    100    50     100    100];
//Pc = [100    50     100    100    100    90     100    100];

// Change W to see optimal of retailer price, different patterns
w from SheetRead(sheet, "Sheet1!L15:S15");
//w = [50      50     50     50     50     50     50     50];
//w = [50      40     40     50     50     50     50     50];
//w = [50      40     40     50     50     40     40     50];
//w = [50      35     50     50     50     50     50     50];
//w = [50      50     45     45     45     45     50     50];
//w = [50      50     50     50     50     40     40     50];
//w = [40      50     50     50     50     50     50     50];
//w = [50      50     50     50     50     50     50     40];

//Postprocessing file
//Change file name here
SheetConnection fileout("8631.xlsx");
w to SheetWrite(fileout, "Result!d2:k2");
Pc to SheetWrite(fileout, "Result!d3:k3");
P to SheetWrite(fileout, "Result!d4:k4");
comp_sales to SheetWrite(fileout, "Result!d5:k5");
I to SheetWrite(fileout, "Result!c6:k6");
period_demand to SheetWrite(fileout, "Result!d7:k7");
S to SheetWrite(fileout, "Result!d8:k8");
sales to SheetWrite(fileout, "Result!d9:k9");
x to SheetWrite(fileout, "Result!d10:k10");
y to SheetWrite(fileout, "Result!d11:k11");
total_demand to SheetWrite(fileout, "Result!c12");
total_profit to SheetWrite(fileout, "Result!c13");
k1 to SheetWrite(fileout, "Result!c14");
```

```
k2 to SheetWrite(fileout, "Result!c15");
k3 to SheetWrite(fileout, "Result!c16");
k4 to SheetWrite(fileout, "Result!c17");
```

2. Customer optimization model

The source code of the customer model to find the customer cost is as follows.

2.1 Customer model file

```
/*********************************************
 * OPL 12.3 Model
 * Author: Windows7
 *********************************************/
//Parameters
int T = 8;
range index_i = 1..T;
range index_j = 1..T;
{int}index_k = {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20};

float D [index_j][index_k]=...;
float P [index_i]=...;
float h [index_i][index_j][index_k];

tuple holding_cost{
	int index_i;
	int index_j;
	int index_k;
	float value;
}
{holding_cost}hc=...;
execute{

	for (var h1 in hc){
		h[h1.index_i][h1.index_j][h1.index_k]=h1.value;
	}
}

float Pc[index_i]=...;
//decision variables
dvar boolean z[index_i][index_j][index_k];
dvar boolean gamma[index_i][index_j][index_k];
dvar float customer_cost;

//objective function
minimize customer_cost;
subject to {
  customer_cost == sum (i in index_i,j in i..T,k in
index_k)((P[i]+h[i][j][k])*D[j][k]*z[i][j][k])
  +sum (i in index_i,j in i..T,k in index_k)((Pc[i]+h[i][j][k])*D[j][k]*gamma[i][j][k]);

  forall ( k in index_k, j in index_j)
	sum (i in 1..j) (z[i][j][k]+gamma[i][j][k]) >= 1; }
```

2.2 Customer data file

```
/*********************************************
 * OPL 12.3 Data
 * Author: Windows7
 * Creation Date: Oct 7, 2015 at 8:27:39 PM
 *********************************************/
SheetConnection sheet("Input.xlsx");
// Change D follows the retailer experiment
D from SheetRead(sheet, "Demand1!B2:U9");
//D from SheetRead(sheet, "Demand2!B2:U9");
//D from SheetRead(sheet, "Demand3!B2:U9");
//D from SheetRead(sheet, "Demand4!B2:U9");
//D from SheetRead(sheet, "Demand5!B2:U9");
//D from SheetRead(sheet, "Demand6!B2:U9");
//D from SheetRead(sheet, "Demand7!B2:U9");
//D from SheetRead(sheet, "Demand8!B2:U9");

// Change hc follows the retailer experiment
hc from SheetRead(sheet,"Holding1!A2:D1281");
//hc from SheetRead(sheet,"Holding2!A2:D1281");
//hc from SheetRead(sheet,"Holding3!A2:D1281");
//hc from SheetRead(sheet,"Holding4!A2:D1281");
//hc from SheetRead(sheet,"Holding5!A2:D1281");
//hc from SheetRead(sheet,"Holding6!A2:D1281");

//Change Pc follows the retailer experiment
Pc = [100	100	100	90	100	100	100	100];
//Pc = [90 90 90 90 90 90 90 90];
//Pc = [100	100	90	90	90	90	100	100];
//Pc = [100	100	100	80	80	100	100	100];
//Pc = [100	70	100	100	100	70	100	100];
//Pc = [100	90	100	100	100	50	100	100];
//Pc = [100	50	100	100	100	90	100	100];
//Pc = [90	100	100	100	100	100	100	90];
//Pc = [60	100	100	100	100	100	100	60];
//Change optimal P to find customer cost
//P = [100	100	100	79.43721225	100	100	100	100];

//Change file name output here P = [];
SheetConnection fileout("8631.xlsx");

customer_cost to SheetWrite(fileout, "Result!c18");
```

Appendix C
Input data

1. Example of Demand data

    a. Base Case

Djk	j1	j2	j3	j4	j5	j6	j7	j8
k1	0	0	0	0	0	0	0	0
k2	0	0	0	0	0	0	0	0
k3	1	1	0	1	1	1	1	0
k4	1	1	1	1	1	1	2	1
k5	1	2	2	1	2	1	1	2
k6	3	3	3	2	3	2	3	3
k7	4	4	4	4	3	4	4	4
k8	5	5	5	5	5	4	5	6
k9	7	6	6	6	6	5	7	5
k10	6	7	7	7	6	6	7	7
k11	7	7	6	7	6	6	7	6
k12	5	6	6	6	6	6	6	6
k13	5	5	5	6	5	5	5	5
k14	3	4	3	4	4	4	4	4
k15	3	3	2	3	2	3	3	3
k16	2	1	2	2	2	2	2	2
k17	1	1	1	1	1	1	1	1
k18	0	1	1	1	1	0	0	1
k19	0	0	0	0	0	0	0	0
k20	0	0	0	0	0	0	0	0

b. Case D1

Djk	j1	j2	j3	j4	j5	j6	j7	j8
k1	0	0	0	0	0	0	0	0
k2	0	0	0	0	0	0	0	0
k3	1	1	0	1	1	1	1	0
k4	1	1	1	1	1	1	2	1
k5	1	2	2	1	2	1	1	2
k6	4	4	4	2	4	2	4	4
k7	5	5	5	5	4	5	5	5
k8	6	6	6	6	6	5	6	7
k9	8	7	7	7	7	6	8	6
k10	7	8	8	8	7	7	8	8
k11	8	8	7	8	7	7	8	7
k12	6	7	7	7	7	7	7	7
k13	6	6	6	7	6	6	6	6
k14	4	5	4	5	5	5	5	5
k15	4	4	2	4	2	4	4	4
k16	2	1	2	2	2	2	2	2
k17	1	1	1	1	1	1	1	1
k18	0	1	1	1	1	0	0	1
k19	0	0	0	0	0	0	0	0
k20	0	0	0	0	0	0	0	0

c. Case D2

Djk	j1	j2	j3	j4	j5	j6	j7	j8
k1	0	0	0	0	0	0	0	0
k2	0	0	0	0	0	0	0	0
k3	1	1	0	1	1	1	1	0
k4	1	1	1	1	1	1	2	1
k5	1	2	2	1	2	1	1	2
k6	2	2	2	2	2	2	2	2
k7	3	3	3	3	2	3	3	3
k8	4	4	4	4	4	3	4	5
k9	6	5	5	5	5	4	6	4
k10	5	6	6	6	5	5	6	6
k11	6	6	5	6	5	5	6	5
k12	4	5	5	5	5	5	5	5
k13	4	4	4	5	4	4	4	4
k14	2	3	2	3	3	3	3	3
k15	2	2	2	2	2	2	2	2
k16	2	1	2	2	2	2	2	2
k17	1	1	1	1	1	1	1	1
k18	0	1	1	1	1	0	0	1
k19	0	0	0	0	0	0	0	0
k20	0	0	0	0	0	0	0	0

d. Case D3

Djk	j1	j2	j3	j4	j5	j6	j7	j8
k1	2	2	1	1	2	2	2	2
k2	1	0	1	1	1	1	1	1
k3	1	1	1	1	1	1	1	1
k4	2	3	2	2	2	2	2	2
k5	2	1	1	2	2	1	1	2
k6	4	5	5	5	5	4	4	5
k7	3	2	3	3	3	2	3	2
k8	4	3	3	3	3	3	3	3
k9	4	4	4	5	4	4	5	5
k10	3	3	3	3	3	3	3	4
k11	6	5	5	5	6	5	5	5
k12	2	3	3	3	3	3	2	4
k13	3	3	3	3	3	4	4	4
k14	3	3	3	4	3	3	3	3
k15	3	3	4	3	3	4	3	4
k16	2	1	2	1	2	2	1	1
k17	4	4	4	3	4	5	4	4
k18	3	3	3	4	2	3	2	3
k19	2	2	2	2	2	1	1	2
k20	2	2	2	1	2	2	2	2

e. Case D4

Djk	j1	j2	j3	j4	j5	j6	j7	j8
k1	1	2	2	2	1	2	2	2
k2	1	1	1	1	0	1	0	1
k3	1	1	1	1	1	1	1	1
k4	2	3	2	2	3	2	2	2
k5	2	2	2	1	1	2	2	1
k6	5	5	5	5	4	4	5	5
k7	3	3	3	3	3	3	2	3
k8	3	3	4	4	4	3	3	3
k9	4	4	5	4	5	4	4	4
k10	3	3	3	3	3	3	4	3
k11	5	6	5	6	5	6	5	5
k12	3	3	3	3	3	3	3	2
k13	3	3	4	4	3	3	3	3
k14	3	3	3	4	3	4	3	3
k15	4	3	4	3	4	3	3	3
k16	1	2	2	1	2	2	2	1
k17	4	3	4	3	4	4	3	4
k18	3	3	3	2	3	3	2	3
k19	2	2	2	2	1	1	2	1
k20	1	2	1	2	1	2	2	2

f. Case D5

Djk	j1	j2	j3	j4	j5	j6	j7	j8
k1	2	1	1	2	2	2	2	2
k2	0	1	1	1	1	1	1	1
k3	1	1	1	1	1	1	1	1
k4	2	2	2	2	2	2	3	2
k5	2	1	2	2	1	2	2	2
k6	5	4	4	4	5	5	4	4
k7	3	3	3	2	3	3	3	3
k8	3	3	3	3	3	3	3	3
k9	5	4	5	4	4	4	4	4
k10	3	3	3	3	3	3	3	3
k11	6	6	5	5	6	5	6	6
k12	3	2	3	3	2	3	3	3
k13	3	3	3	4	3	3	3	3
k14	3	3	4	3	3	3	3	3
k15	3	3	3	3	3	3	3	3
k16	2	2	2	2	2	1	2	2
k17	4	4	4	4	4	4	4	4
k18	3	2	3	3	3	3	3	3
k19	2	2	2	2	2	1	1	2
k20	2	2	2	2	2	2	2	2

2. Example of customer's holding cost

consumer	base	0.4	-0.4	sd 10	sd 0
1	0.520311	0.728436	0.312187	0.203339	0.207821
2	0.526127	0.736577	0.315676	0.225276	0.226478
3	0.4572	0.640079	0.27432	0.244174	0.248291
4	0.524077	0.733708	0.314446	0.265677	0.270293
5	0.436181	0.610653	0.261709	0.281404	0.293975
6	0.347333	0.486267	0.2084	0.30348	0.312075
7	0.39532	0.553448	0.237192	0.323209	0.338838
8	0.370396	0.518554	0.222237	0.342847	0.361452
9	0.222549	0.311568	0.133529	0.363351	0.384395
10	0.436895	0.611653	0.262137	0.37533	0.404061
11	0.33749	0.472486	0.202494	0.402576	0.426361
12	0.457032	0.639845	0.274219	0.416933	0.444144
13	0.460583	0.644816	0.27635	0.433923	0.47434
14	0.370079	0.51811	0.222047	0.464087	0.492232
15	0.524546	0.734364	0.314727	0.479226	0.512841
16	0.367238	0.514133	0.220343	0.506472	0.529963
17	0.197797	0.276916	0.118678	0.511528	0.553058
18	0.562788	0.787903	0.337673	0.536967	0.574556
19	0.37072	0.519007	0.222432	0.557194	0.591477
20	0.245455	0.343637	0.147273	0.586466	0.623021