TEST STATISTICS FOR A NORMAL MEAN WITH KNOWN COEFFICIENT OF VARIATION

BY

MISS NERISA THORNSRI

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE (APPLIED STATISTICS)

DEPARTMENT OF MATHEMATICS AND STATISTICS
FACULTY OF SCIENCE AND TECHNOLOGY
THAMMASAT UNIVERSITY

ACADEMIC YEAR 2015

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THESIS

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ENTITLED

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was approved as partial fulfillment of the requirements for the degree of Master of Science (Applied Statistics)

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ABSTRACT

Hypothesis testing for a normal mean when the coefficient of variation is known, is quite different from the situation when the variance is known. Mostly, the situation when the variance is known is only of theoretical interest. There are many practical situations when the coefficient of variation is known. This situation arises in medical, biological and environmental studies. In the theoretical part of the thesis, we proved that the considered estimates are unbiased estimator, minimum variance and asymptotically normal. Previously the considered estimates have not been considered as test statistics. In this thesis, we therefore construct statistical tests for the normal mean based on the best minimum variance unbiased estimators and the uniformly minimum risk estimators. Under the null hypothesis, the limiting distribution of the test statistic is derived. In the computational part, the simulation results show that all proposed test statistics perform better for a large sample and a small value of coefficient of variation. Moreover, the proposed test statistics based on the uniformly minimum risk estimators provide more efficient test procedures.

Keywords: Hypothesis testing, Mean of a normal distribution, Coefficient of variation, Type I error, Power

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Miss Nerisa Thornsri

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LIST OF ABBREVIATIONS

Symbols/Abbreviations	Terms
d	The estimator for a normal mean with a known coefficient of variation suggested by Khan,
T_{LMMS}	The estimator for a normal mean with a known coefficient of variation suggested by Gleser and Healy,
d^*	The modified estimator of d ,
T_{LMMS}^*	The modified estimator of T_{LMMS},
$\overline{X}$	The sample mean,
T_k	The test statistic based on d ,
T_{gh}	The test statistic based on T_{LMMS},
T_{km}	The test statistic based on d^*,
T_{ghm}	The test statistic based on T_{LMMS}^*,
T_b	The test statistic based on $\overline{X}$,
b	The coefficient of variation(s),
n	The sample size(s),
μ_0	The specified value(s) in the null hypothesis,
α	The significance level,
$f\left(x \mid \mu, \sigma^2\right)$	The probability density function of the normal distribution with mean μ and variance σ^2,
$f\left(x \mid \mu, b^2\mu^2\right)$	The probability density function of the normal distribution with mean μ and variance $b^2\mu^2$,

$N(0,1)$	The standard normal distribution,
$N(\mu, b^2\mu^2)$	The normal distribution with mean μ and variance $b^2\mu^2$,
$\xrightarrow{d}$	Convergence in distribution,
$\xrightarrow{p}$	Convergence in probability.

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CHAPTER 1
INTRODUCTION

1.1 Statement of the Problem and Importance of the Study

A normal distribution is the most important distribution used in mathematical statistics and are often used for statistical applications in agricultural, biological, physical and medical studies. The main part of statistical study, the Statistical inference is the process of deducing inferences or conclusions about populations from the collection and interpretation of sample data. The population is assumed to be larger than the observed data set; in other words, the observed data are assumed to be sampled from a larger population. The main classes of inferential statistics are estimation of a parameter and testing the hypotheses about the value of a parameter.

The estimation consists of a point and interval estimation. The point estimation produces an estimate of the value of parameter by estimating a single value from sample data. The interval estimation derives an estimated interval or range of possible values of parameter from sample data.

The test of the hypothesis is a method for testing a hypothesis about a parameter in a population by using data measured in a sample. There are two types of statistical hypotheses: the null hypothesis and the alternative hypothesis. Thus we decide whether to retain or reject the null hypothesis using the value of the test statistic obtained from the sample data. We can decide to retain or reject the null hypothesis, and this decision can be correct or incorrect. Two types of errors in hypothesis testing are called type I and type II errors.

A problem of making an inference about a normal mean using a prior information about the coefficient of variation $b = \sigma / \mu$, where μ and σ are the mean and standard deviation of a population, is interesting and it has been approached by many researchers. This problem appears in many practical situations in medical, biological, environmental and chemical studies. For example, in chemical studies, Bhat, & Rao (2007) asserted that “when batches of some substances (chemicals) are to be analyzed, if sufficient batches of the substances are analyzed, their coefficients of variation will be known”. In environmental studies, Niwitpong (2013) extended the work of Bhat, & Rao (2007) concerning that the standard deviation of the pollutant is related to the mean. In agricultural studies, Niwitpong, & Koonprasert (2012) illustrated the following phenomenon by conducting many trials to study. It appears that in a new study, the known coefficient of variation of the control group (treatment) is comparable with the coefficient of variation is this new study.

For many decades ago, there were several authors that have studied the estimation the mean of a normal distribution when the coefficient of variation is known. For the example, in the point estimation, Searls (1964) considered an improvement of the sample mean under the condition of a known coefficient of variation. Arnholt, & Hebert (1995) derived an optimal estimator from the estimator suggested by Searls (1964) when the coefficient of variation is known.

Later, Khan (1968) proposed the best unbiased estimator for estimating a mean with minimum variance. Furthermore, this estimator is asymptotically normal. Gleser, & Healy (1976) suggested that the estimator from Khan (1968) is inadmissible under a squared-error loss function, so the uniformly minimum risk estimator under a square error loss function is achieved. For the problem of having possible negative values from the estimators of Khan (1968) and Gleser, & Healy (1976), Khan (2013) reconsidered to modify these estimators in order to improve their efficiency. Srisodaphol, & Tongmol (2012) improved the estimators using the method of Khan (1968), Arnholt, & Hebert (1995), the jackknife technique (1974), and the Baysian estimator using the Jeffreys prior distribution.

For the interval estimation, Niwitpong (2013) proposed new confidence intervals for the normal population mean with a known coefficient of variation. The proposed confidence intervals are based on the best unbiased estimator suggested by Khan (1968), the best unbiased estimator suggested by Searls (1967), and using prior information $b = \sigma / \mu$. Panichkitkosolkul (2015a) proposed an approximate confidence interval for the ratio of the normal means with a known coefficient of variation and compared this with the exact confidence interval constructed by Niwitpong, Koonprasert, & Niwitpong (2011). This new confidence interval uses the approximation of the expectation and variance of the estimator.

For the hypothesis testing, to make an inference on the normal mean, the fact that the population variance is known, is important to derive the distribution of the estimators. This is a different situation from when a coefficient of variation is assumed to be known. There are many researches relating to the tests of hypotheses with a known coefficient of variation, such as, Bhat, & Rao (2007) derived the likelihood ratio test (LR) and the Wald tests for a normal mean with known coefficient of variation and extended the locally most powerful test (LMP) derived from Hinkley (1977). The results of simulation studies indicated the LMP test is the best test for the one-sided alternative while for the two-sided alternative, the LR or the Wald is the best test. Walid, Abu-Dayyeh, & Dorvlo (2013) constructed one-sided tests using pivotal method and compared their power functions. Banik, Kibria, & Sharma (2012) derived a test for the population coefficient of variation. The several methods existing for testing the population coefficient of variation were compared to a proposed bootstrap method. They compare the performance of test statistics in term of powers. Moreover, Panichkitkosolkul (2015b) obtained the two statistical tests for the reciprocal of a normal mean with a known coefficient of variation. The tests are developed based on the distribution of a sample mean. The first test was based on an asymptotic method. The second test was developed using the simple approximate expression in terms of expectation and variance.

This concludes our review on the work of the previous researchers on the making inferences the normal mean with a known coefficient of variation. In this thesis, we investigate the theoretical properties of the estimators, propose the test statistics for a normal mean, examine the performance of the test statistics to capacity in controlling of probability of type I error and compare the power of the test statistics. Therefore, in this study, we propose the test statistics based on the best unbiased estimator suggested by Khan (1968), the uniformly minimum risk estimator suggested by Gleser, & Healy (1976), the modified estimator by Khan (1968), the modified estimator of Gleser, & Healy (1976), and the sample mean with a known coefficient of variation. For this research, we will perform the Monte Carlo method to evaluate the proposed tests using Program R version 2.3.1.

1.2 Research Objectives

The objectives of the research are as below:

1. To investigate theoretical properties of a point estimator for the normal mean with known coefficient of variation.

2. To propose the test statistics based on the best unbiased estimator for the mean with known coefficient of variation suggested by Khan (1968), the uniformly minimum risk estimator suggested by Gleser, & Healy (1976), the modified estimator by Khan (1968), the modified estimator of Gleser, & Healy (1976), and the sample mean and examine the capacity in controlling of the probability of type I error.

3. To compare powers of test statistics for each situation in order to recommend the best test statistics based on the sample size and the value of the coefficient of variation.

1.3 Research Scope

The scope of this research consists of the following parts:

1.3.1 Theoretical part

The theoretical properties of point estimators for the normal mean with known coefficient of variation are investigated. Then, the new test statistics for a normal mean are constructed.

1.3.2 Computational Part

1. We perform a simulation study by generating random samples of size n from a normal distribution with the mean μ and variance $b^2\mu^2$, $\mu > 0$. We fix the nominal significance level α. Next, we take $\mu = \delta\mu_0$, where we choose δ = 1.0 to estimate the probability of type I error and δ = 0.85, 0.9, 0.95, 1.1, 1.2, 1.3, 1.4, 1.5 to

estimate the power of the test. Therefore, we will set the different values of n, b, μ_0, δ, and α.

We set sample sizes (n) to be 16, 25, 35,

fix the values of coefficient of variation (b) at 0.7, 1.5, 2.0, 2.5, 3.0,

fix the values of μ_0 at 2, 3, 4, 5,

fix the values of δ at 0.85, 0.9, 0.95, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5,

fix the values of α at 0.01, and 0.05.

Simulations are replicated 10,000 times for each situation.

2. The probability of type I error and power is estimated in each situation.

3. Graphs of power are plotted.

1.4 Criteria

1. First, we will consider the ability in controlling the probability of type I error. We follow the following rule by Cochran (1954) which suggested that the probability of type I error is between [0.007, 0.015] at the 0.01 significance level and the probability of type I error is between [0.040, 0.060] at the 0.05 significance level.

2. Then, we compare the power of test statistics. The test with highest power based on the coefficient of variation is recommended.

1.5 Expected Benefits of the Research

The research aims is to investigate the efficiency of the test statistics of the normal mean with known coefficient of variation, to choose the appropriate test statistic in each situation. In addition, this can be applied to other test statistics to the normal mean with known coefficient of variation.

1.6 Basic Definitions

1.6.1 Type I error

A type I error, also known as an error of the first kind, occurs when the null hypothesis (H_0) is true, but is rejected. The probability of type I error or significance level is the probability of rejecting the null hypothesis given that it is true. It is denoted by α and is also called the alpha level.

1.6.2 Type II error

A type II error, also known as an error of the second kind, occurs when the null hypothesis (H_0) is false, but erroneously fails to be rejected. The rate of the Type II error is denoted by β and related to the power of a test.

1.6.3 Power

A power in hypothesis testing is the probability of rejecting a null hypothesis when it is false and therefore should be rejected. The power of the test is calculated by subtracting of a rate of Type II error (β) from 1.0 denoted by $1-\beta$. (Vorapongsathorn, T., Taejaroenkul, S., & Viwatwongkasem, C., 2004*)*

1.6.4 Test statistic

A test statistic is a mathematical formula that allows researchers to determine the likelihood or probability of obtaining sample outcomes if the null hypothesis is true. The value of a test statistics can be used to make inferences concerning the value of population parameter stated in the null hypothesis.

1.6.5 Hypothesis testing

A hypothesis is a statement about a population parameter. The goal of a hypothesis is to decide, based on a sample from the population, which of two complementary hypotheses are true.

CHAPTER 2
THEORETICAL BACKGROUND AND REVIEW OF LITERATURE

In this chapter, the theoretical background on the notion of the coefficient of variation, of normal distribution, of hypothesis testing procedure (especially for testing of the normal population mean), estimating procedure, and asymptotic analysis is presented. Also a review of the related literature is provided.

2.1 Theoretical Background on the Coefficient of Variation

A coefficient of variation measures the variability in a series of numbers independently of the unit of measurement used for the numbers. The coefficient of variation eliminates the unit of measurement from the standard deviation of a series of number by dividing it by the mean of this series of numbers. The coefficient of variation can be used to compare distributions obtained with different units such as the variability of the weights of newborns (measured in grams) with the size of adults (measured in centimeters). The coefficient of variation should be computed only for data measured on a ratio scale and the measurements that can only take non-negative values. The coefficient of variation may not have any meaning for data on an interval scale. For example, most temperature scales (e.g., Celsius, Fahrenheit etc.) are interval scales that can take both positive and negative values, whereas the Kelvin temperature can never be less than zero, which is the complete absence of thermal energy. Hence, the Kelvin scale is a ratio scale. While the standard deviation can be derived on both the Kelvin and the Celsius scale, the coefficient of variation is only relevant as a measure of relative variability for the Kelvin scale.

The coefficient of variation is defined as the ratio of the standard deviation σ to the mean μ (or its absolute value, $|\mu|$) is given by

$$b = \frac{\sigma}{|\mu|}. \tag{2.1}$$

When only a sample of data from a population is available, the population coefficient of variation can be estimated using the ratio of the sample standard deviation s to the sample mean $\bar{x}$ (or its absolute value, $|\bar{x}|$).

Often the coefficient of variation is expressed as a percentage which corresponds to the following formula

$$\hat{b} = \frac{s}{|\bar{x}|} \times 100 \ . \tag{2.2}$$

(Abdi, 2010)

2.2 Theoretical Background on Normal Distribution

The normal distribution (sometimes called the *Gaussian distribution*) plays a central role in a large body of statistics. There are three main reasons for this. First, the normal distribution and distributions associated with it are very tractable analytically. Second, the normal distribution has the familiar bell shape, whose symmetry makes it appealing choice for many population models. Although there are many other distributions that are also bell-shaped, most do not possess analytic tractability as normal. Third, there is the Central limit Theorem, which shows that, under mild conditions, the normal distribution can used to approximate a large variety of distributions for large samples.

The normal distribution has two parameters, denoted by μ and σ^2, which are the mean and variance. The probability density function of the normal distribution with mean μ and variance σ^2, denoted by $N(\mu,\sigma^2)$ is given by

$$f(x \mid \mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \ , \ -\infty < x < \infty, \ -\infty < \mu < \infty, \ \sigma > 0 . \tag{2.3}$$

The expectation and variance of the normal distribution are

$$E(X)=\mu, \text{ and } Var(X)=\sigma^2.$$

The case where μ = 0 and σ^2 = 1 is called the standard normal distribution. The probability density function is

$$f(x)=\frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}},\ -\infty<x<\infty. \tag{2.4}$$

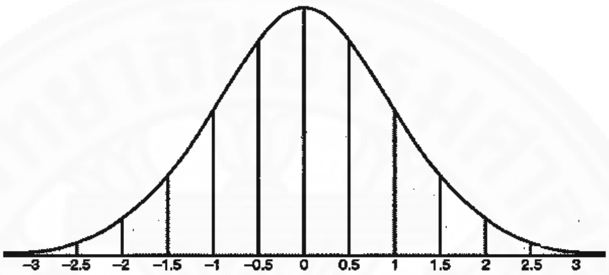

Figure 2.1: Standard normal density.

(Cassella, & Berger, 2002, p. 102)

In case of known coefficient of variation b, Walid Abu-Dayyeh, & Atsu Dorvlo (2013) suggested that the probability density function of $N(\mu,b^2\mu^2)$ where $b=\sigma/\mu$ for $b>0$ and $\mu>0$, is defined by

$$f(x\mid\mu,b^2\mu^2)=\frac{1}{b\mu\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2b^2\mu^2}},\ -\infty<x<\infty,\ \mu>0,\ b>0. \tag{2.5}$$

Then, the expectation and variance of this condition are as follows

$$E(X)=\mu, \text{ and } Var(X)=b^2\mu^2.$$

2.3 Theoretical Background on Hypothesis Testing

2.3.1 General Theory of Hypothesis Testing

The notion of a hypothesis is rather general, but the important point is that a hypothesis makes a statement about a population parameter. The goal of a hypothesis is to decide, based on a sample from the population, which of two complementary hypotheses is true. Random samples of size n are represented by independent random variables $X_1, X_2,..., X_n$ having a common probability density or mass function $f(x \mid \theta)$, $\theta \in \Theta$, where Θ is the parameter space (the set of all possible values of the parameter). We consider two competing hypotheses about possible values of θ.

The formal procedure, due to J. Neyman and E.S. Pearson (1920's) and extending earlier ideas of R.A. Fisher, is to identify a *Null hypothesis*, $H_0 : \theta \in \Theta_0$, which is contrasted with an *Alternative hypothesis*, $H_A : \theta \in \Theta_1$, where Θ_0 and Θ_1 are disjoint subsets of the parameter space Θ. Call a hypothesis *simple* if it has the form $\theta = \theta'$, a known constant and *composite* otherwise. For example, if $N(\mu, \sigma^2)$ then the hypothesis $H_0 : \mu = \mu'$ is a simple one if σ is known. However it is composite if σ is unknown because it should be expressed as $\mu = \mu', \sigma > 0$.

Definition 2.1. A *test statistic*, similarly to an estimator, is just some real-valued function $T_n = T(X_1, X_2,..., X_n)$ of the data sample $X_1, X_2,..., X_n$. Clearly, a test statistic is a random variable.

Deciding between the null and alternative hypotheses involves a *test statistic* $T_n = T(X_1, X_2,..., X_n)$ taking values in a space which is partitioned into disjoint subsets A and R, called *acceptance* and *rejection* regions, and correspond to Θ_0 and Θ_1, respectively. If an observed value of T_n, $t_n = t(X_1, X_2,..., X_n)$, then H_0 is *rejected* in favour of H_A, and if $t_n \in A$ then H_0 is *accepted*. The latter term is usually taken to mean there is too little data evidence to opt decisively for H_0.

The law of T_n depends on the unknown value of θ. A crucial role is played by *the law of* T_n *given that* H_0 *is true*. This is well-defined only if H_0 is simple. If general, A and R are chosen so that if H_0 is true, the event $\{T_n \in R\}$ occurs with a small probability. Specifically, a small number α is chosen by the statistician, and then R such that

$$P(T_n \in R \mid \theta) \le \alpha \text{ for all } \theta \in \Theta_0,$$

trying to get as close to α as possible. If $t_n \in R$ then we say that H_0 is *rejected at the* $100\alpha\%$ *level of significance*. Call α the *size* of the test. With these choices, we expect that $P(T_n \in R \mid \theta) > \alpha$ if $\theta \in \Theta_1$, i.e. the probability of rejection exceeds the chosen level of significance if H_0 is false. In fact, this property cannot be inferred from the above test structure. A test which has this property is said to be *unbiased*. We have the following rationale applicable to unbiased tests for making accept/reject decisions: If $t_n \in R$; then,

(i) *Either* H_0 is true and an event of small $(\le \alpha)$probability has occurred:

or

(ii) H_0 is false, and an event has been observed whose probability exceeds α.

Option (ii) is the better explanation of the observed outcome; it is consistent with the intuition supporting the maximum likelihood concept. This procedure gives rise to two possible errors:

Type I error: Reject H_0 when it is true, and

Type II error: Accept H_0 when it is false.

Table 2.1: Two types of errors in hypothesis testing.

Trust	**Decision**	
	Accept H_0	**Reject** H_0
H_0	Correct decision	Type I Error
H_A	Type II Error	Correct decision

Type I error is held to be more serious, explaining why the test is designed to control its probability of occurrence:

$$P(Type\,I\,error) = P(T_n \in R \mid H_0) \leq \alpha\,. \tag{2.6}$$

Computing the probability of a Type II error usually is possible only if H_A is simple.

Definition 2.2. The *power function* of a hypothesis test with a rejection region R is the function of θ defined by

$$B_{T_n}(\theta) = P(T_n \in R \mid \theta). \tag{2.7}$$

In general, we define the power function $B_{T_n}(\theta) = P(T_n \in R \mid \theta)$ for all values $\theta \in \Theta$. Thus the test is designed so that $B_{T_n}(\theta) \leq \alpha$ if $\theta \in \Theta_0$. Typically the power function is close to α if $\theta \in \Theta_1$ is close to its boundary, and increasing as θ moves away from the boundary. The sensitivity of a test can be judged in terms of how quickly $B_{T_n}(\theta)$ increases above α as $\theta \in \Theta_1$ moves away from the boundary.

Remarks: 1. This (Neyman-Pearson) testing procedure is a frequent concept: The operation meaning of the assertion 'H_0 is rejected at the $100\alpha\%$ level if significance' means that if this random experiment is independently replicated many times using the same population, then a type I error occurs in a proportion $\leq \alpha$ of such replications.

2. In ‘scientific’ contexts H_0 represents accepted wisdom or a *status quo*, and experimental data has the express purpose of refuting rather than confirming H_0. Refutation should be compelling, beyond a reasonable doubt, thus explaining the special status accorded to Type I errors, and why α is chosen to be small. It follows that H_0 and H_A are not inter-changeable. On the other hand, H_0 could represent model assumptions, such as ‘errors are normally distributed’. In quality control situations H_0 could be ‘the process is in control’, i.e. the probability p that a manufactured item is faulty is less than some very small number. For these cases, finding $t_n \in A$ gives weight to accepting H_0 as a viable working assumption, a desirable outcome.

3. There often is a difference between statistical and practical significance. Rejecting H_0 may lead to costly actions which may not be justifiable if the apparent deviation from H_0 is small. It is common to address this by quoting the probability-value, or p-value, denoted by p_v. If $t_n = t(X_1, X_2, ..., X_n)$ is observed, define D_t as that subset of $A \cup R$ representing ‘a more extreme deviation’ from H_0 than the observed one. Then,

$$p_v : P(T_n \in D_t \mid H_0). \qquad (2.8)$$

The p-value is frequently interpreted as a measure of data evidence supporting H_0, although it lacks attributes one reasonably expects of such a measure. Thus H_0 is rejected if $p_v \le \alpha$. Quoting a p-value is more informative than merely saying e.g., ‘H_0 was rejected at the 5% level’; it allows others to compare their analyses of your data. This is quite important if the data are discrete because in most cases a test cannot achieve the nominal level, e.g. $\alpha = 0.05$.

4. An important question is how to choose a test statistic? Often the choice is made on a ‘common sense’ basis. But there are general results which can give guidance. It seems fairly obvious that we want a test to be unbiased, and to have the

property that $B_{T_n}(\theta)$ is as large as possible for all $\theta \in \Theta_1$, i.e. maximum power under H_A.

Definition 2.3. A test statistic is *uniformly most powerful (UMP)* if, for any other test statistic T_n^*, we have $B_{T_n}(\theta) \geq B_{T_n^*}(\theta)$ for all $\theta \in \Theta_1$.

If H_0 and H_A are simple hypotheses, then a fundamental result named the Neyman-Pearson lemma implies the existence of a test with UMP statistic. If H_0 is simple and H_A is composite, then H_A can be considered as a union of simple hypotheses, and it may be that a test with UMP statistic can be forged from the UMP tests for each pair $(H_0, H_A(\theta'))$, where the second component is, for each $\theta' \in \Theta_1$, the assertion that $\theta = \theta'$. If H_0 also is composite, it's not obvious how to proceed. Instead there is another route based on the 'common sense' approach which is applicable to most problems arising in practice, so-called *likelihood ratio tests*. (Helio, Dani, & Francisco (2015), pp. 221-232).

2.3.2 Hypothesis Testing of Mean for Normal distribution for Known Variance and for Known Coefficient of Variation

We note that there are two completely different situations in hypothesis testing for a mean μ of normal distribution. First one is when variance σ^2 is known. We would like to mention that mostly this situation is only for *theoretical* interest. Usually in practice we do not need to make inference about mean when variance is known. Usually the fact that variance is known implies that the mean is known, too. The second situation is when the coefficient of variation is known. This is the situation considered in my thesis. Contrary to the first situation, there are many practical problems where the coefficient of variation is known, but we need to make inference about the mean.

2.4 Theoretical Background on Statistical Test for the Mean of Normal Population

A common method of constructing a test for a population mean of a normal distribution. Let $X_1, X_2, ..., X_n$ be random samples from a normal distribution with mean μ and standard deviation σ. Suppose we wish to test the two-sided hypothesis $H_0 : \mu = \mu_0$ versus $H_A : \mu \neq \mu_0$, where μ_0 is a specified value. The common statistics are *the sample mean* $\bar{X}$ and the *minimum likelihood estimator of standard deviation* S where $\bar{X} = n^{-1}\sum_{i=1}^{n} X_i$ and $S = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}$. If the data are abnormal (not from the normal population), in many cases we can apply the Central Limit Theorem which establishes the conditions to guaranty that the limit distribution is a normal distribution. But in our situation of the data from normal population, if H_0 is true, then the statistic

$$Z = \frac{\left(\bar{X} - \mu_0\right)}{\sigma / \sqrt{n}} \sim N(0,1). \tag{2.9}$$

At the level α, the test rejects H_0 if $Z < -z_{\frac{\alpha}{2}}$ or $Z > z_{\frac{\alpha}{2}}$ where z_α is the upper percentile of the standard normal distribution.

2.5 Theoretical Background on Estimation Procedure

Definition 2.4. A *point estimator* is any fixed function $T_n = T\left(X_1, X_2, ..., X_n\right)$ of a sample; that is, any statistic is a point estimator. (Cassella, & Berger, 2002, p. 311)

Definition 2.5. The *bias* of a point estimator T_n of a parameter θ is the difference between the expected value of T_n and θ; that is, $Bias\left(T_n\right) = E\left(T_n\right) - \theta$. An estimator whose bias is equal to 0 is called *unbiased.* (Cassella, & Berger, 2002, p. 330)

2.5.1 Minimum Variance Estimators

Theorem 2.1. (Cramer-Rao Inequality). Let $X_1, X_2, ..., X_n$ be an independent and identically distributed (iid.) random variables from a distribution that has density function $f(x \mid \theta)$, where $f(x \mid \theta)$ has continuous second-order derivative by θ. Suppose the set of x for which $f(x \mid \theta) \neq 0$ does not depend on θ.

An unbiased estimator T_n of parameter θ is called a minimum variance unbiased estimator (MVUE) of θ if

$$Var(T_n) = \frac{1}{nE\left(\left(\frac{\partial}{\partial\theta}\log f(X \mid \theta)\right)^2\right)} \tag{2.10}$$

where E denotes the expected value with respect to the probability density function $f(X \mid \theta)$.

It is possible to prove that in this case

$$Var(T_n) = \frac{1}{-nE\left(\frac{\partial^2}{\partial\theta^2}\log f(X \mid \theta)\right)}, \tag{2.11}$$

The value

$$E\left(\left(\frac{\partial}{\partial\theta}\log f(X \mid \theta)\right)^2\right) = I(\theta) \tag{2.12}$$

is called the information number or the Fisher information of the sample.

The famous Cramer-Rao Inequality states that the variance of any *unbiased* estimator T_n of θ is then bounded by the reciprocal of the Fisher information $I(\theta)$ in (2.12),

$$Var_{\theta}(T_n) = \frac{1}{I(\theta)}. \tag{2.13}$$

The Cramer-Rao Lower Bound (CRLB) sets a lower bound on the variance of any unbiased estimator. It is useful as follows: First, we find an estimator that achieves the CRLB, then we know that we have found a Minimum Variance Unbiased Estimator (MVUE). Second, the CRLB can provide a benchmark against which we can compare the performance of any unbiased estimator. Next, the CRLB can be used to rule-out impossible estimators. Finally, the theory behind the CRLB can tell us if an estimator exists that achieves the lower bound

2.6 Theoretical Background on Asymptotic Analysis

2.6.1 Consistency

Definition 2.6. A sequence of estimators $T_n = T(X_1, X_2, ..., X_n)$ is said to be consistent estimator of the parameter θ if, for every $\varepsilon > 0$ and every $\theta \in \Theta$,

$$\lim_{n\to\infty} P(|T_n - \theta| < \varepsilon) = 1. \tag{2.14}$$

Informally, (2.11) says that as the sample size becomes infinite, the estimator will be arbitrarily close to the parameter with high probability, an eminently desirable probability. Or, turning things around, we can say that the probability that a consistent sequence of estimators misses the true value of the parameter is small. An equivalent statement to (2.11) is this: For every $\varepsilon > 0$ and every $\theta \in \Theta$, a consistent sequence T_n satisfies

$$\lim_{n\to\infty} P(|T_n - \theta| \geq \varepsilon) = 0. \tag{2.15}$$

Recall that, for an estimator T_n, Chebychev's Inequality states

$$P\left(\left|T_n - \theta\right| \geq \varepsilon\right) \leq \frac{E\left((T_n - \theta)^2\right)}{\varepsilon^2}.$$

So if, for every $\theta \in \Theta$, $\lim_{n\to\infty} E\left((T_n - \theta)^2\right) = 0$, then, the sequence of estimators T_n is consistent. (Cassella, & Berger, 2002, p. 468)

Theorem 2.2. If T_n is a sequence of estimators of the parameter θ satisfying

(i.) $\lim_{n\to\infty} Var(T_n) = 0$,

(ii.) $\lim_{n\to\infty} E(T_n) = \theta$, for every $\theta \in \Theta$.

Then, T_n is a consistent sequence of estimators of θ. (Cassella, & Berger, 2002, p. 469)

2.6.2 Convergence in probability

Definition 2.7 A sequence of random variables, $X_1, X_2,...$ converges in probability to a random variable X if, for every $\varepsilon > 0$,

$$\lim_{n\to\infty} P\left(\left|X_n - X\right| \geq \varepsilon\right) = 0 \quad \text{or, equivalently,}$$

$$\lim_{n\to\infty} P\left(\left|X_n - X\right| < \varepsilon\right) = 1.$$

Theorem 2.3. (Weak Law of Large Numbers (WLLN)) Let $X_1, X_2,...$ be independent and identically distributed random variables with $E(X_i) = \mu$ and $Var(X_i) = \sigma^2 < \infty$. Then, for every $\varepsilon > 0$,

$$\lim_{n\to\infty} P\left(\left|\overline{X}_n - \mu\right| < \varepsilon\right) = 1; \tag{2.16}$$

that is, $\overline{X}_n$ converges in probability to μ.

The property summarized by the WLLN, that a sequence of the "same" sample quantities approaches a constant as $n \to \infty$, is known as the *consistency*. (Cassella, & Berger, 2002, pp. 232-233)

Theorem 2.4. If the sequence of random variables $X_1, X_2,...$ converges in probability to a random variable X , the sequence converges in distribution to X .
(Cassella, & Berger, 2002, p. 236)

2.6.3 Convergence in Distribution

Definition 2.8 A sequence of random variables, $X_1, X_2,...$ converges in distribution to a random variable X if

$$\lim_{n\to\infty} F_{X_n}(x) = F_X(X) \text{ at all point } x \text{ where } F_{X_n}(x) \text{ is continuous.}$$

Theorem 2.5. (Central Limit Theorem). If the distribution of the independent and identical random sample $X_1, X_2,..., X_n$ is such that X_1 has finite expectation and variance, i.e. $|E(X_1)| < \infty$ and $Var(X_1) < \infty$, then

$$\sqrt{n}(X_1 - E(X_1)) \xrightarrow{d} N(0, \sigma^2),$$

which means that for any interval $[a,b]$,

$$P(\sqrt{n}(X_1 - E(X_1)) \in [a,b]) \to \int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} dx.$$

In other words, the random variable $\sqrt{n}(X_1 - E(X_1))$ will behave like a random variable from normal distribution when n gets large. (Cassella, & Berger, 2002, pp. 235-238)

2.6.4 Asymptotically Normal of Estimators

Definition 2.9. A sequence of random variable X_n is asymptotically normally distributed as $n \to \infty$ if there exist sequence of real constants μ_n and σ_n (with $\sigma_n > 0$) such that

$$\frac{X_n - \mu_n}{\sigma_n} \xrightarrow{d} Z \sim N(0,1).$$

Theorem 2.6. (Slutsky's Theorem) If $X_n \xrightarrow{d} X$ and $Y_n \xrightarrow{p} a$, a constant, then

(i.) $Y_n X_n \xrightarrow{d} aX$.

(ii.) $X_n + Y_n \xrightarrow{d} X + a.$

(Cassella, & Berger, 2002, pp. 239-240)

2.7 Review of the Related Literature

In this part, we start briefly describing from many authors have made an inference about the normal mean with known coefficient of variation.

2.7.1 Review of Point estimation for a Normal Mean with Known Coefficient of Variation

Khan (1968) considered a normal distribution with known coefficient of variation b and examined that the estimator $\bar{X}$ is inadmissible for estimating a mean μ when the coefficient of variation is known. Therefore, he considered a class of unbiased estimators linear in a sample mean $\bar{X}$, and a sample standard deviation S. The best estimator which has a minimum variance among these unbiased estimators has been found. The best estimator has the form $d = \omega d_1 + (1-\omega)d_2$ and $0 \le \omega \le 1$, where $d_1 = \bar{X} = n^{-1}\sum_{i=1}^{n} X_i$, $d_2 = c\sqrt{n}S$, $c = \frac{1}{2b}\Gamma\left(\frac{n-1}{2}\right) \Big/ \Gamma\left(\frac{n}{2}\right)$ and S is the minimum likelihood estimator of standard deviation. Since we consider the sample from a normal population, the estimators d_1 and d_2 are independent (and this is true only for samples from normal population). He also proved that the estimators d_1 and d_2 are unbiased estimators for normal mean μ.

Furthermore, Khan (1968) examined the asymptotic behavior of the estimator d for large n. The results are summarized as follows. First, the estimator d is asymptotically normal. Next, its asymptotic efficiency is equal to the efficiency of the maximum likelihood estimator. Finally, the asymptotic variance or $Var(d) = n^{-1}b^2\mu^2(1+2b^2)^{-1}$ is the Cramer-Rao bound. Additionally, he also showed that the estimator d is more asymptotic efficiency than the estimator $\overline{X}$ when the coefficient of variation is known.

Gleser, & Healy (1976) suggested that the estimator d is inadmissible under a squared error loss function, $L(\mu,d) = (d-\mu)^2$ and a uniformly minimum risk estimator T_{LMMS} under a square error loss function is obtained. First of all, he considered the class of estimators that is a convex combination of T_1 and T_2 in a form of $c_1T_1 + c_2T_2$. Then, he compared the risks of the estimator T_{LMMS} with the estimator d. According to Khan (1968), $T_1 = \overline{X}$, $T_2 = c_nS$ where, $c_n = \frac{\sqrt{n}}{2b}\Gamma\left(\frac{n-1}{2}\right)\Big/\Gamma\left(\frac{n}{2}\right)$. The form of the estimators obtained are $T_{LMMS} = c_1T_1 + c_2T_2$ where, $v_1 = b^2n^{-1}$, $v_2 = b^2n^{-1}(n-1)c_n^2 - 1$, $c_1 = v_2(v_1+v_2+v_1v_2)^{-1}$, $c_2 = v_1(v_1+v_2+v_1v_2)^{-1}$. The corresponding minimum risk is, $v_1v_2(v_1+v_2+v_1v_2)^{-1}\mu^2$. The results are summarized as follows. First, the estimator T_{LMMS} is more efficient than the estimator d in the terms of risk. Second, the estimator T_{LMMS} is also asymptotically normal. However, the estimators T_{LMMS} and d have the flaw of being possibly negative with positive probability for estimating a positive parameter.

Khan (2013) reconsidered the estimators d and T_{LMMS} to improve their efficiency. Furthermore, he also compared their risk with the maximum likelihood estimator. Recall, that before we considered $T_1 = \bar{X}$ and $v_1 = b^2 n^{-1}$. Let $T_1^* = B\left|\bar{X}\right|$ and $v_1^* = \left(B^2 - 1\right) + b^2 B^2 n^{-1}$ where, $B = \left(2\Phi(\lambda) - 1 + \left(2\lambda^{-1}\right)\phi(\lambda)\right)^{-1}$, $\lambda = \frac{1}{b}\sqrt{n}$. Therefore, the modified estimators for d and T_{LMMS} are $d^* = cT_1^* + (1-c)T_2$ and $T_{LMMS}^* = c_1^* T_1^* + c_2^* T_2$ where, $c = v_2\left(v_1^* + v_2\right)^{-1}$, $c_1^* = v_2\left(v_1^* + v_2 + v_1^* v_2\right)^{-1}$, $c_2^* = v_1^*\left(v_1^* + v_2 + v_1^* v_2\right)^{-1}$, $\Phi(\bullet)$ is the cumulative distribution function of standard normal distribution, and $\phi(\bullet)$ is the density function of the standard normal distribution. Next, he examined the asymptotic behavior of the modified estimators for B and v_1^*. The theoretical results are summarized as follows. First, the modified estimators have smaller asymptotic variance. Second, the modified estimators are also asymptotically normal. Simulation studies indicate that the risks of the modified estimators are a little bit different. However, the estimators may be difficult to use due to their cumbersome form. Moreover, it is not even known how much does it reduces risk.

2.7.2 Review of Literature on Statistical Tests for a Normal Mean with Known Coefficient of Variation

Bhat, & Rao (2007) derived the likelihood ratio test (LR) and the Wald tests for a normal mean with the known coefficient of variation and extended the locally most powerful test (LMP) derived from Hinkley (1977). The LR and the Wald tests are derived for a one-sided alternative and two-sided alternative, and they are compared with the tests that do not use the information on the coefficient of variation (the classical t, sign and Wilcoxon singed rank tests). The results of simulation studies indicate that the LMP test is the best test for the one-sided alternative while for the two-sided alternative, the LR or the Wald are better. Moreover, when the values of coefficients of variation are quiet large, the power of the LMP, LR, and Wald tests are almost equal to one. On the contrary, the performances of the t, sign and Wilcoxon singed rank tests are poor. Moreover, they also examined the robustness of six tests for violations of the normality assumption. The simulation was generated under three abnormal distributions to estimate type I error rates. These three abnormal distributions are uniform, lognormal and gamma distributions. The results indicate that the six tests are close to the nominal significance level ($\alpha = 0.05$) when values of the coefficient of variation are small.

Walid, Abu-Dayyeh, & Dorvlo (2013) considered tests about the normal mean with the known coefficient of variation and constructed several tests (one-sided) using the pivotal method. They derived the power functions for each test. The simulation results indicate that the power of each test increases as the sample size increases. The power of each test starts to decrease when the sample size is small and the value of the coefficient of variation is large.

Panichkitkosolkul (2015b) obtained two statistical tests for the reciprocal of a normal mean with the known coefficient of variation b. These two tests are developed based on the distribution of a sample mean. The first test is based on an asymptotic method. The second test was developed using the simple approximate expression in terms of expectation and variance. Type I errors and powers are estimated in simulation experiments. As expected, the powers of two tests decrease as the coefficient of variation increases and the powers increase as the sample size (n) increases, except for $n = 10$, and $b = 0.5$. In addition, it is also noticed that there is no difference in terms of their estimated power. Therefore, the approximate test performs as efficiently as the asymptotic test. It is suggested that although the efficiencies of the two tests are not different, the approximate test is easier to calculate.

2.7.3 Review of Literature on Interval Estimation for a Normal Mean with Known Coefficient of Variation

In 2013, Fu, Wang, & Wong extended the procedure of Bhat , & Rao (2007) and proposed the modified signed log-likelihood ratio method r^* for a normal mean when the coefficient of variation is known. They compared accuracy of confidence intervals obtained from the Wald method and the likelihood ratio test method (LR) with the confidence interval obtained by the proposed method. The simulation results indicate that in terms of the coverage probability, the lower tail error rate, the upper tail error rate and the average bias, the LR test performs the best. In addition, they compare the relative efficiency of ten point estimators of μ obtained by Anis (2008) with their proposed estimator. The results indicate that their proposed estimator has the second rank efficiency in comparison with other estimators.

Niwitpong & Niwitpong (2013) proposed new confidence intervals for the normal population mean μ with the known coefficient of variation b. The proposed confidence intervals are based on: The confidence interval (CL_s) based on the best unbiased estimator suggested by Khan (1968), the best unbiased estimator suggested by Searls (1967). The confidence interval (CL_p) based on the prior information about b. In simulation studies, the results are summarized as follows. First, coverage probabilities of three proposed confidence intervals are equal to a nominal level $1-\alpha$. Next, when the value of coefficient of variation and sample size is small, the CL_b is preferred to others. Finally, the CL_s performs better than CL_b and CL_p when $b \le 0.05$. In all other situations, CL_p performs better.

Panichkitkosolkul (2015) proposed an approximate confidence interval for the ratio of the normal means with known coefficient of variation. The new confidence interval is derived by the approximations of the expectation and variance of estimator for the ratio using Taylor series expansion. The new confidence interval is compared with the exact confidence interval constructed by Niwitpong et al. (2011) in term of coverage probability and expected length. The simulation results indicate that the new confidence interval performs as efficient as the exact confidence interval, but the approximate confidence interval is easier to compute.

CHAPTER 3
RESEARCH METHODOLOGY

The research methodology for theoretical part and computational part are discussed separately.

3.1 Theoretical Part

In this part, the objective is to investigate theoretical properties of the best unbiased estimator for the mean with known coefficient of variation suggested by Khan (1968). First, we prove the estimator is unbiased. Next, we find the Cramer-Rao Lower Bound that shows that the estimator has a minimum variance. Finally, we assert that the estimator is asymptotically normal and hence consistent.

We propose test statistics for a normal mean are based on the best unbiased estimator suggested by Khan (1968), the uniformly minimum risk estimator suggested by Gleser, & Healy (1976), the modified estimator of Khan (1968), the modified estimator of Gleser, & Healy (1976), and the sample mean.

3.2 Computational Part

In this part, the objectives are to estimate probability of type I error and power of the test of proposed test statistics. For the simulation study, we use the Monte Carlo method for generating random samples of size n from the normal distribution with mean μ and variance $b^2\mu^2$, where $b = \sigma / \mu$, $\mu > 0$ is the coefficient of variation. Therefore, we take $\mu = \delta\mu_0$, where we choose δ = 1.0 to estimate the probability of type I error and δ = 0.85, 0.9, 0.95, 1.1, 1.2, 1.3, 1.4, 1.5 to estimate the power of the test. We need to study for following combinations of n, b, μ_0, δ, and α are as follows:

n = 16, 25, 35,

b = 0.7, 1.5, 2.0, 2.5, 3.0,

μ_0 = 2, 3, 4, 5,

δ = 0.85, 0.9, 0.95, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5,

α = 0.01, and 0.05.

The simulations are repeated 10,000 times for each combination using R program version 3.1.1.

The steps for estimating the probability of type I error and power are as follows:

Step 1: State the null hypothesis and alternative hypothesis

$H_0 : \mu = \mu_0$

$H_A : \mu \neq \mu_0$

Step 2: Generate random samples of size n from $N(\mu, b^2\mu^2)$ for the given values of n, b, μ_0, δ, and α. Estimate the probability of type I error by generating random samples that satisfies H_0 and estimate the powers by generating random samples that satisfies H_A.

Step 3: Construct and calculate all test statistics.

Step 4: Set the decisive criterion for the test is to reject the null hypothesis H_0 if $Z < -z_{\frac{\alpha}{2}}$ or $Z > z_{\frac{\alpha}{2}}$ at the given significance level α.

Step 5: Count the numbers of times the null hypothesis has been rejected.

Step 6: Repeat steps 2-5, 10,000 times for each combination.

Step 7: Compute probability of type I error and powers by counting the numbers of times the null hypothesis is rejected from step 5 divided the number of 10,000 replications.

Step 8: Compare the estimated probability of type I error of each test statistic to the criterion of Cochran (1954).

Step 9: Compare the powers of the test statistic and recommend the best test statistics for this situation.

Step 10: Plot the graphs of powers to compare.

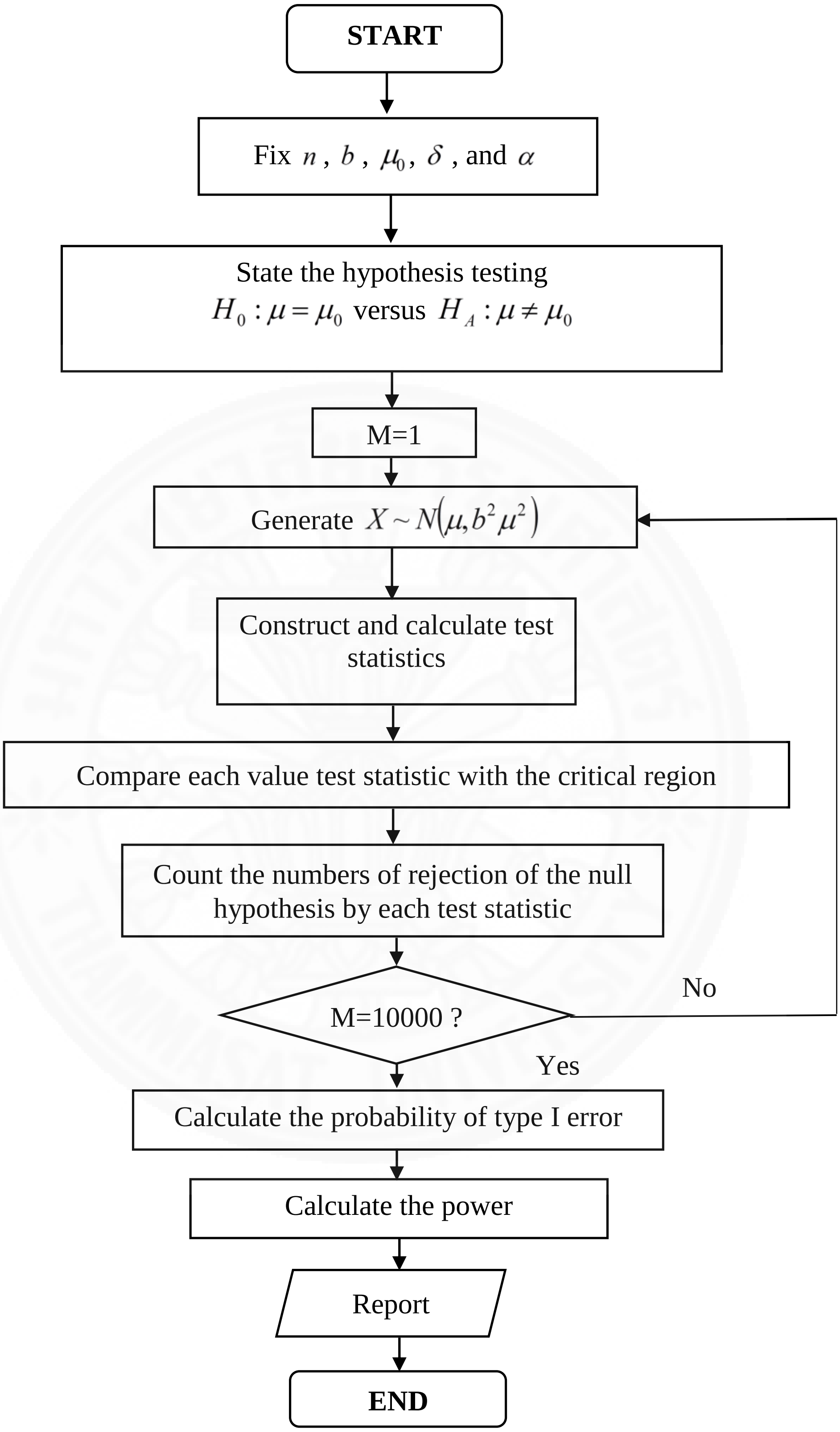

Figure 3.1: Programming Flowchart

CHAPTER 4
THEORETICAL RESULTS

4.1 Introduction

In this thesis, after we investigate the theoretical properties of Khan's (1968) point estimate, we construct the novel test statistics for a mean of normal distribution. First of all, we consider a point estimator for the normal mean under the assumption of known coefficient of variation $b = \sigma / \mu$. Hence, the probability density function, denoted by $N\left(\mu, b^2\mu^2\right)$ can be written as

$$f\left(x \mid \mu, b^2\mu^2\right) = \frac{1}{b\mu\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2b^2\mu^2}} , \tag{4.1}$$

where $-\infty < x < \infty,\ \mu > 0,\ b > 0$.

Let $X_i \sim N\left(\mu, b^2\mu^2\right)$, $i = 1,2,\ldots,n$ be independent identically distributed normal random variables with mean μ and variance $b^2\mu^2$. Khan (1968) suggested the point estimator d of the mean with the known coefficient of variation b as

$d = \omega d_1 + (1-\omega)d_2$, $0 \le \omega \le 1$

Where $d_1 = \overline{X} = n^{-1}\sum_{i=1}^{n} X_i$, $d_2 = c\sqrt{n}S$, $c = \frac{1}{2b}\Gamma\left(\frac{n-1}{2}\right)\Big/\Gamma\left(\frac{n}{2}\right)$, and S is the minimum likelihood estimator of standard deviation.

We investigate the theoretical properties of this point estimator**.** Khan **(**1968) stated that it is an unbiased estimator for the mean with minimum variance. Furthermore, this estimate is asymptotically normal.

Consequently, the structure of this chapter is organized as follows. In Section 4.2 it is shown that the estimator is unbiased. In Section 4.3 it is shown the estimator has minimum variance among all unbiased estimators. The important properties of asymptotically normal of the estimator and hence its consistency are established in Section 4.4. In Section 4.5, the new test statistics for a normal mean are constructed.

4.2 Unbiasedness of the Estimator

From definition 2.5 the estimator d is unbiased if $E(d) = \mu$ for all μ. We prove these properties for estimators d_1 , d_2, and d separately.

First, we take expected value to d_1 and get

$$E(d_1) = E(\overline{X}) = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \frac{1}{n}\sum_{i=1}^{n}\mu \text{ (because } E(X_i) = \mu\text{)}$$

$$= \frac{1}{n}(n\mu) = \mu .$$

Therefore,

$$E(d_1) = \mu .$$

Then, we find the expected value of d_2 in the indirect way. Let

$$U = \frac{nS^2}{\sigma^2} = \frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{\sigma^2} \text{, then } U \sim \chi^2_{n-1} ,$$

where χ^2_{n-1} denotes the chi-square distribution with $n-1$ degrees of freedom. Hence the probability density function of U is

$$f(u) = \frac{1}{\Gamma\left(\frac{n-1}{2}\right)2^{\left(\frac{n-1}{2}\right)}} u^{\left(\frac{n-1}{2}\right)-1} e^{-\frac{u}{2}}, \quad u > 0 .$$

From this

$$E(\sqrt{U}) = \int_0^\infty \sqrt{u}\, f(u)du = \frac{1}{\Gamma\left(\frac{n-1}{2}\right)2^{\left(\frac{n-1}{2}\right)}} \int_0^\infty u^{\left(\frac{n}{2}\right)-1} e^{-\frac{u}{2}} du . \qquad (4.2)$$

In order to evaluate the last integral, we make a substitution $u = 2t \rightarrow t = \frac{u}{2}$ and $du = 2dt \rightarrow dt = \frac{du}{2}$. Hence

$$\int_0^\infty u^{\left(\frac{n}{2}\right)-1} e^{-\frac{u}{2}} du = 2^{\left(\frac{n}{2}\right)} \int_0^\infty \left(\frac{u}{2}\right)^{\left(\frac{n}{2}\right)-1} e^{-\frac{u}{2}} d\frac{u}{2}$$

$$= 2^{\left(\frac{n}{2}\right)} \int_0^\infty t^{\left(\frac{n}{2}\right)-1} e^{-t} dt .$$

From the definition of the Gamma function: $\int_0^\infty x^{\alpha-1}e^{-x}dx = \Gamma(\alpha)$ we derive that

$$\int_0^\infty u^{\left(\frac{n}{2}\right)-1} e^{-\frac{u}{2}} du = 2^{\left(\frac{n}{2}\right)} \Gamma\left(\frac{n}{2}\right). \tag{4.3}$$

From (4.2) and (4.3), we get

$$E(\sqrt{U}) = \frac{1}{\Gamma\left(\frac{n-1}{2}\right)2^{\left(\frac{n-1}{2}\right)}} 2^{\left(\frac{n}{2}\right)} \Gamma\left(\frac{n}{2}\right) = \frac{1}{\Gamma\left(\frac{n-1}{2}\right)2^{\left(\frac{n-1}{2}\right)}} 2^{\left(\frac{n}{2}\right)} \Gamma\left(\frac{n}{2}\right).$$

Thus,

$$E(\sqrt{U}) = E\left(\sqrt{\frac{nS^2}{\sigma^2}}\right) = \frac{\sqrt{2}\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}. \tag{4.4}$$

This equation can be rewritten as $E\left(\frac{1}{\sqrt{2}}\Gamma\left(\frac{n-1}{2}\right) \Big/ \Gamma\left(\frac{n}{2}\right)\sqrt{n}S\right) = \sigma$. Taking into consideration that $\sigma = b\mu$, we obtain

$$E\left(\frac{1}{\sqrt{2}}\Gamma\left(\frac{n-1}{2}\right) \Big/ \Gamma\left(\frac{n}{2}\right)\sqrt{n}S\right) = b\mu ,$$

or

$$E\left(\frac{1}{\sqrt{2}b}\Gamma\left(\frac{n-1}{2}\right) \Big/ \Gamma\left(\frac{n}{2}\right)\sqrt{n}S\right) = \mu .$$

From the fact that $c = \frac{1}{2b}\Gamma\left(\frac{n-1}{2}\right)\Big/\Gamma\left(\frac{n}{2}\right)$ and $d_2 = c\sqrt{n}S$. We conclude that $E\left(c\sqrt{n}S\right) = \mu$.

Therefore, $E(d_2) = \mu$. Consequently, we obtained $E(d_1) = E(d_2) = \mu$.

Finally, we find expected value of d as

$$E(d) = E(\omega d_2 + (1-\omega)d_1) = \omega E(d_2) + (1-\omega)E(d_1) = \omega\mu + (1-\omega)\mu = \mu .$$

Therefore, $E(d) = \mu$.

The property that d is an unbiased estimator is obtained. In the next section we prove that d is a minimum variance unbiased estimator.

4.3 Minimum Variance of the Estimator

From theorem 2.1, the estimator d has minimum variance when its variance attains the Cramer-Rao Lower Bound or the reciprocal of the Fisher information $I(\mu)$.

The Fisher information is given by

$$I(\mu) = -nE\left(\frac{\partial^2}{\partial\mu^2}\log f\left(X \mid \mu\right)\right). \tag{4.5}$$

The formula of Cramer-Rao Lower Bound is following

$$I^{-1}(\mu) = \frac{1}{-nE\left(\frac{\partial^2}{\partial\mu^2}\log f\left(X \mid \mu\right)\right)} . \tag{4.6}$$

First, we find the variance of d

$$Var(d) = Var(\omega d_2) + Var((1-\omega)d_1) \text{ (because } d_1 \text{ and } d_2 \text{ are independent)}$$

$$= \omega^2 Var(d_2) + (1-\omega)^2 Var(d_1). \tag{4.7}$$

From (4.7), we will find an optimum value of ω by minimizing $Var(d)$ in (4.7) with respected to ω. First, we find the first-order partial derivatives and set this partial is equal to 0

$$\frac{d}{d(\omega)} Var(d) = \frac{d}{d(\omega)}\left(\omega^2 Var(d_2) + (1-\omega)^2 Var(d_1)\right) = 0\,. \tag{4.8}$$

From (4.8), we solve the solution to find the optimal value of ω we obtain

$$2\omega Var(d_2) - 2(1-\omega)Var(d_1) = 0\,.$$

Then, we get the optimum value of ω, that is

$$\omega = \frac{Var(d_1)}{Var(d_1) + Var(d_2)}\,. \tag{4.9}$$

From (4.9) we can get

$$\omega = \frac{1}{1 + \left(\frac{n(n-1)}{2b^2}\left[\frac{\Gamma^2\left(\frac{n-1}{2}\right)}{\Gamma^2\left(\frac{n}{2}\right)}\right] - 1\right)}\,. \tag{4.10}$$

Where, $Var(d_1) = \frac{\mu^2 b^2}{n}$ and $Var(d_2) = \mu^2\left(\left(\frac{n-1}{2}\right)\Gamma^2\left(\frac{n-1}{2}\right) \Big/ \Gamma^2\left(\frac{n}{2}\right) - 1\right)$.

Next, the optimum value of ω has the asymptotic behavior for large n as

$$\omega \approx \frac{2b^2}{\left(1+2b^2\right)}, n \to \infty , \tag{4.11}$$

we can show that

$$Var\left(d_2\right) \approx \frac{\mu^2}{2n}, n \to \infty , \tag{4.12}$$

it is known fact that

$$Var\left(d_1\right) = Var\left(\bar{X}\right) = \frac{\mu^2 b^2}{n} . \tag{4.13}$$

From (4.11), (4.12) and (4.13) we get

$$\begin{aligned}
Var\left(d\right) &= \left(\frac{2b^2}{1+2b^2}\right)^2 \left(\frac{\mu^2}{2n}\right) + \left(\frac{1}{1+2b^2}\right)^2 \left(\frac{b^2\mu^2}{n}\right) \\
&= \left(\frac{4b^4}{\left(1+2b^2\right)^2}\right)\left(\frac{\mu^2}{2n}\right) + \left(\frac{1}{\left(1+2b^2\right)^2}\right)\left(\frac{b^2\mu^2}{n}\right) \\
&= \frac{2b^4\mu^2}{n\left(1+2b^2\right)^2} + \frac{b^2\mu^2}{n\left(1+2b^2\right)^2} \\
&= \frac{2b^4\mu^2 + b^2\mu^2}{n\left(1+2b^2\right)^2} = \frac{\left(1+2b^2\right)b^2\mu^2}{n\left(1+2b^2\right)^2} = \frac{b^2\mu^2}{\left(1+2b^2\right)n} .
\end{aligned}$$

Thus we obtain,

$$Var\left(d\right) = \frac{b^2\mu^2}{\left(1+2b^2\right)n} . \tag{4.14}$$

Recall again, the probability density function of the normal distribution with known coefficient of variation b , see (4.1)

$$f\left(x \mid \mu, b^2\mu^2\right) = \frac{1}{b\mu\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2b^2\mu^2}} .$$

We take the logarithm of the probability density function,

$$\ln f\left(x; \mu, b^2\mu^2\right) = \ln\left\{ \frac{1}{b\mu\sqrt{2\pi b^2\mu^2}} e^{-(x-\mu)^2 / 2b^2\mu^2} \right\}$$

$$= -\frac{1}{2}\ln\left(2\pi b^2\right) - \frac{1}{2}\ln\left(\mu^2\right) - \frac{1}{2}\left(\frac{x-\mu}{b^2\mu^2}\right)^2 .$$

We take the first derivatives of this equation with respect to μ and

$$\frac{\partial}{\partial\mu}\ln f(x; \mu, b^2\mu^2) = -\frac{1}{\mu} - \frac{1}{2b^2\mu^4}\left(-2\mu^2(x-\mu) - 2\mu(x-\mu)^2\right)$$

$$= -\frac{1}{\mu} - \frac{1}{b^2\mu^4}\left(-x\mu^2 + \mu^3 - \mu x^2 + 2x\mu^2 - \mu^3\right).$$

The second derivative is

$$\frac{\partial^2}{\partial\mu^2}\ln f(x; \mu, b^2\mu^2) = \frac{1}{\mu^2} + \frac{2x}{b^2\mu^3} - \frac{3x^2}{b^2\mu^4}.$$

Then, we take expected value of the second derivatives as

$$E\left(\frac{\partial^2}{\partial\mu^2}\ln f(X; \mu, b^2\mu^2)\right) = \frac{1}{\mu^2} + \frac{2}{b^2\mu^3}E(X) - \frac{3}{b^2\mu^4}E\left(X^2\right).$$

From the previous results, we have

$E(X) = \mu$, and $Var(X) = b^2\mu^2$. (4.15)

After that, we need to find $E\left(X^2\right)$ from

$$Var(X) = E\left(X^2\right) - (E(X))^2 . \qquad (4.16)$$

From (4.14) and (4.15)

$$E\left(X^2\right)=Var\left(X\right)+\left(E\left(X\right)\right)^2=b^2\mu^2+\mu^2. \tag{4.17}$$

Substituting $E(X)=\mu$ form (4.15) and $E\left(X^2\right)=\left(b^2+1\right)\mu^2$ from (4.16) into $E\left(\frac{\partial^2}{\partial\mu^2}\ln f(X;\mu,b^2\mu^2)\right)$ we obtain

$$-E\left(\frac{\partial^2}{\partial\mu^2}\ln f(X;\mu,b^2\mu^2)\right)=-\frac{1}{\mu^2}-\frac{2}{b^2\mu^2}+\frac{3\left(b^2+1\right)}{b^2\mu^2}=\frac{-\left(1+2b^2\right)}{b^2\mu^2}$$

and the Fisher information is

$$-nE\left(\frac{\partial^2}{\partial\mu^2}\ln f(X;\mu,b^2\mu^2)\right)=\frac{\left(1+2b^2\right)n}{b^2\mu^2}=I\left(\mu\right)$$

Thus, the Cramer-Rao Bound is

$$\frac{1}{I\left(\mu\right)}=\frac{1}{-nE\left(\frac{\partial^2}{\partial\mu^2}\ln f(x;\mu,b^2\mu^2)\right)}=\frac{b^2\mu^2}{\left(1+2b^2\right)n}. \tag{4.18}$$

From (4.14) and (4.18), we can see that

$$Var\left(d\right)=\frac{b^2\mu^2}{\left(1+2b^2\right)n}=\frac{1}{I\left(\mu\right)}.$$

Since $Var(d)$ attains the Cramer-Rao Lower Bound, d is a minimum variance estimator of μ. Hence, from theorem 2.1 conclude that d is a minimum variance unbiased estimator (MVUE) of μ.

The next section, the asymptotically normal of d is established.

4.4 Asymptotically Normal and Consistency

Based on theorem 2.3 and definition 2.7, we will show that the estimator d converges in probability to μ.

For every $\varepsilon > 0$, we get

$$P(|d-\mu|<\varepsilon)=\int_{\mu-\varepsilon}^{\mu+\varepsilon}\frac{\sqrt{(1+2b^2)n}}{\sqrt{2\pi}b\mu}e^{-\frac{1}{2\left(\frac{b^2\mu^2}{(1+2b^2)n}\right)}(d-\mu)^2}\,\mathrm{d}(d) \qquad (4.19)$$

If we let $y=d-\mu$ then $\mathrm{d}y=\mathrm{d}(d)$. Substituting in (4.19), we get

$$P(|d-\mu|<\varepsilon)=\int_{-\varepsilon}^{\varepsilon}\frac{\sqrt{(1+2b^2)n}}{\sqrt{2\pi b\mu}}e^{-\frac{1}{2\left(\frac{b^2\mu^2}{(1+2b^2)n}\right)}y^2}\,dz$$

Let

$$z=\frac{y}{\left(\frac{b\mu}{\sqrt{(1+2b^2)n}}\right)},\ \text{then}\ dz=\frac{dy}{\left(\frac{b\mu}{\sqrt{(1+2b^2)n}}\right)}. \qquad (4.20)$$

Substituting (4.20), we obtain

$$P(|d-\mu|<\varepsilon)=\int_{-\frac{\varepsilon\sqrt{(1+2b^2)n}}{b\mu}}^{\frac{\varepsilon\sqrt{(1+2b^2)n}}{b\mu}}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}\,dz$$

$$=P\left(-\frac{\varepsilon\sqrt{(1+2b^2)n}}{b\mu}<Z<\frac{\varepsilon\sqrt{(1+2b^2)n}}{b\mu}\right)$$

Thus $\lim_{n\to\infty}P(|d-\mu|<\varepsilon)=1$. Therefore, d converges in probability to μ. From theorem 2.4 we can infer that d also converges in distribution to μ.

Based on theorem 2.6 (Slutsky's Theorem), we illustrate that asymptotic normality implies consistency. Suppose that

$$\sqrt{n}\,\frac{d-\mu}{\left(\frac{b\mu}{\sqrt{(1+2b^2)}}\right)} \xrightarrow{d} Z \sim N(0,1).$$

By applying Slutsky's Theorem we conclude

$$\left(\frac{bu(1+2b^2)^{-\frac{1}{2}}}{\sqrt{n}}\right)\left(\sqrt{n}\,\frac{(d-\mu)}{bu(1+2b^2)^{-\frac{1}{2}}}\right) \to \lim_{n\to\infty}\left(\frac{bu(1+2b^2)^{-\frac{1}{2}}}{\sqrt{n}}\right)Z = 0.$$

Therefore, $d-\mu \xrightarrow{d} 0$. From theorem 2.4, we know that convergence in distribution to a point is equivalent to the convergence in probability, so d is a consistent estimator of μ.

4.5 Tests for Normal Mean with Known Coefficient of Variation

When the coefficient of variation is known, for the hypothesis testing $H_0 : \mu = \mu_0$ versus $H_A : \mu \neq \mu_0$, we propose the following tests for a normal mean

Test 1: T_k -test is based on the best unbiased estimator proposed by Khan (1968), $d = \omega d_1 + (1-\omega)d_2$. Under H_0, we apply the central limit theorem to find the limiting distribution of the test statistic:

$$T_k = \frac{(v_1+v_2)^{-\frac{1}{2}}(d-\mu_0)}{\mu_0\sqrt{v_1 v_2}} \sim N(0,1). \tag{4.21}$$

Where, $v_1 = b^2 n^{-1}$, $v_2 = b^2 n^{-1}(n-1)c_n^2 - 1$, and $c_n = \frac{\sqrt{n}}{2b}\Gamma\left(\frac{n-1}{2}\right)\Big/\Gamma\left(\frac{n}{2}\right)$.

Test 2: T_{gh}-test is based on the uniformly minimum risk estimator proposed by Gleser, & Healy (1976), $T_{LMMS} = c_1 T_1 + c_2 T_2$. Under H_0, we apply the central limit theorem central limit theorem to find the limiting distribution of the test statistic:

$$T_{gh} = \frac{\left(\frac{T_{LMMS}}{c_1 + c_2} - \mu_0\right)}{\mu_0 \sqrt{c_1^2 v_1 + c_2^2 v_2}} \sim N(0,1). \qquad (4.22)$$

Where, $T_1 = \overline{X}$, $T_2 = c_n S$, $c_1 = v_2(v_1 + v_2 + v_1 v_2)^{-1}$, and $c_2 = v_1(v_1 + v_2 + v_1 v_2)^{-1}$.

Test 3: T_{km} -test is based on the modified estimator of Khan (1968) $d^* = cT_1^* + (1-c)T_2$. Under H_0, we apply the central limit theorem to find the limiting distribution of the test statistic:

$$T_{km} = \frac{\left(v_1^* + v_2\right)^{-\frac{1}{2}}\left(d^* - \mu_0\right)}{\mu_0 \sqrt{v_1^* v_2}} \sim N(0,1). \qquad (4.23)$$

Where, $T_1^* = B\left|\overline{X}\right|$, $v_1^* = \left(B^2 - 1\right) + b^2 B^2 n^{-1}$, $c = v_2\left(v_1^* + v_2\right)^{-1}$,

$B = \left(2\Phi(\lambda) - 1 + \left(2\lambda^{-1}\right)\phi(\lambda)\right)^{-1}$, and $\lambda = \frac{\sqrt{n}}{b}$.

Test 4: T_{ghm}-test is based on the modified estimator $T_{LMMS}^* = c_1^* T_1^* + c_2^* T_2$ of Gleser, & Healy (1976). Under H_0, we apply central limit theorem to find the limiting distribution of the test statistic:

$$T_{ghm} = \frac{\left(\frac{T_{LMMS}^*}{c_1^* + c_2^*} - \mu_0\right)}{\mu_0 \sqrt{c_1^{*2} v_1^* + c_2^{*2} v_2}} \sim N(0,1). \qquad (4.24)$$

Where, $c_1^* = v_2\left(v_1^* + v_2 + v_1^* v_2\right)^{-1}$, and $c_2^* = v_1^*\left(v_1^* + v_2 + v_1^* v_2\right)^{-1}$.

Test 5: T_b -test is based on the sample mean, $\overline{X} = \sum_{i=1}^{n} X_i$. Under H_0, we apply the central limit theorem to find the limiting distribution of the test statistic:

$$T_b = \frac{n^{-\frac{1}{2}}\left(\overline{X} - \mu_0\right)}{b\mu_0} \sim N(0,1). \qquad (4.25)$$

Table 4.1: The rejection criterion for testing $H_0 : \mu = \mu_0$ versus $H_A : \mu \neq \mu_0$ at the significance level α.

Test statistics	**Rejection criterion**
T_k	$T_k < -z_{\frac{\alpha}{2}}$ or $T_k > z_{\frac{\alpha}{2}}$
T_{gh}	$T_{gh} < -z_{\frac{\alpha}{2}}$ or $T_{gh} > z_{\frac{\alpha}{2}}$
T_{km}	$T_{km} < -z_{\frac{\alpha}{2}}$ or $T_{km} > z_{\frac{\alpha}{2}}$
T_{ghm}	$T_{ghm} < -z_{\frac{\alpha}{2}}$ or $T_{ghm} > z_{\frac{\alpha}{2}}$
T_b	$T_b < -z_{\frac{\alpha}{2}}$ or $T_b > z_{\frac{\alpha}{2}}$

Where z_α is the upper α^{th} quantile of the standard normal distribution.

CHAPTER 5
COMPUTATIONAL RESULTS

5.1 Introduction

In this chapter, a Monte Carlo simulation study is used to construct the procedure of the proposed test. In Section 5.2, we discuss the results of Monte Carlo simulations for calculating the probability of type I error and power performances of the proposed tests. We fix type I error rates and compare powers of previously discussed test statistics in order to recommend the best test statistics based on the coefficient of variation. In Section 5.4, we plot graphs of the powers of the tests in order to visualize the comparisons. The computational results are conducted in R program version 3.1.1.

5.2 Results of Monte Carlo Simulations

Monte Carlo simulations are performed to evaluate the performance of the proposed test statistics. First, we generate random samples of size n from the normal distribution with mean μ and variance $b^2\mu^2$, $b=\sigma/\mu$, $\mu>0$. We fix the nominal significance level α. Next, we take $\mu=\delta\mu_0$, where we choose δ = 1.0 to estimate the probability of type I error and δ = 0.85, 0.9, 0.95, 1.1, 1.2, 1.3, 1.4, 1.5 to estimate the power of the test. Then, we conduct the tests and count the numbers of times when the null hypotheses has been rejected.

For this, we need to study the following combinations of n, b, μ_0, δ, and α.

n = 16, 25, 35,

b = 0.7, 1.5, 2.0, 2.5, 3.0,

μ_0 = 2, 3, 4, 5,

δ = 0.85, 0.9, 0.95, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5,

α = 0.01, and 0.05.

The simulation procedure repeats 10,000 times for each combination.

After that, we estimate the probability of type I error and power for each test. Next, we consider the tests that are able control type I error (the probability of type I error should be close to the nominal α) with the criterion of Cochran (1954). Finally, we compare powers of test statistics for each situation in order to recommend the test statistics based on the sample size and the coefficient of variation.

In this section, we divide the performance of simulation results into 2 parts. Part 1 is titled as ability to controlling the probability of type I error. The results are reported in Tables 5.1-5.2. Part 2 is titled as power of the tests. The results are reported in Tables 5.3- 5.18.

5.2.1 Ability to Control the Probability of type I error

5.2.1.1 Estimated probability of type I error at $\alpha = 0.01$.

From Tables 5.1, we see that the T_k procedure could control the probability of type I error for almost all values of μ_0 and b , except when μ_0= 2, b = 2.0, n = 16, and μ_0= 2, b = 3.0, n = 25.

The T_{gh} procedures could not control the probability of type I error for a small sample size and large values of b . For n = 25, the T_{gh} procedure could control the probability of type I error for almost all cases, except when μ_0= 2, b = 3.0; μ_0= 4, b = 1.5, and μ_0= 4, b = 2.0 while for n = 35, the T_{gh} procedures also could not control the probability of type I error when μ_0= 3, b = 2.5, and μ_0= 3, b = 3.0.

The T_{km} procedure could control the probability of type I error for almost all values of μ_0 and b , except when μ_0= 2, b = 2.0 , n = 16 ; μ_0= 2, b = 3.0, n = 16.

The T_{ghm} procedure could not control the probability of type I error for a small sample size and large values of b . Whereas in n = 25 and 35, the T_{ghm} test could control the probability of type I error for almost all values of μ_0 and b , except when

μ_0 = 2, b = 3.0 ; μ_0 = 4, b = 1.5, and μ_0 = 4, b = 2.0. The results for T_{ghm} procedures are similar to the results for the procedure of T_{gh}.

For the T_b procedure, this test could control the probability of type I error for all combinations.

Table 5.1: Estimated probability of type I error at $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	**0.0108**	**0.012**	**0.0108**	**0.012**	**0.0097**
		1.5	**0.0137**	0.0169	**0.0136**	0.017	**0.0092**
		2.0	0.0178	0.0208	0.0178	0.0205	**0.0092**
		2.5	**0.0147**	0.017	**0.0149**	0.0169	**0.0098**
		3.0	**0.015**	0.0188	0.0157	0.0187	**0.0108**
	3	0.7	**0.013**	**0.0147**	**0.013**	**0.0147**	**0.0112**
		1.5	**0.0145**	0.017	**0.0144**	0.0169	**0.0104**
		2.0	**0.0137**	0.0169	**0.0134**	0.0168	**0.0097**
		2.5	**0.0138**	0.0165	**0.0144**	0.0164	**0.0106**
		3.0	**0.015**	0.0183	**0.015**	0.0186	**0.0117**
	4	0.7	**0.0129**	**0.0144**	**0.0129**	**0.0144**	**0.0082**
		1.5	**0.0118**	**0.0142**	**0.0117**	**0.0141**	**0.0083**
		2.0	**0.0132**	**0.015**	**0.0131**	**0.015**	**0.0104**
		2.5	**0.013**	0.0163	**0.0126**	0.0163	**0.0081**
		3.0	**0.0146**	0.0173	**0.0141**	0.0166	**0.0088**
	5	0.7	**0.013**	**0.0145**	**0.013**	**0.0145**	**0.009**
		1.5	**0.0135**	0.0161	**0.0134**	**0.015**	**0.0097**
		2.0	**0.0144**	0.017	**0.0147**	0.0169	**0.0103**
		2.5	**0.015**	0.0184	**0.015**	0.0179	**0.0108**
		3.0	**0.0145**	0.0174	**0.0146**	0.0171	**0.01**
25	2	0.7	**0.0113**	**0.0121**	**0.0113**	**0.0121**	**0.0079**
		1.5	**0.0136**	**0.015**	**0.0135**	**0.015**	**0.0096**
		2.0	**0.0131**	**0.0149**	**0.0132**	**0.015**	**0.0111**
		2.5	**0.0127**	**0.0142**	**0.0129**	**0.0142**	**0.0119**
		3.0	0.0151	0.0168	**0.0148**	0.0163	**0.0109**
	3	0.7	**0.0122**	**0.013**	**0.0122**	**0.013**	**0.0091**
		1.5	**0.0118**	**0.0133**	**0.0117**	**0.0133**	**0.0091**
		2.0	**0.0112**	**0.0129**	**0.0111**	**0.0129**	**0.0102**
		2.5	**0.012**	**0.0143**	**0.0113**	**0.014**	**0.0097**
		3.0	**0.0124**	**0.014**	**0.0124**	**0.0141**	**0.01**
	4	0.7	**0.0115**	**0.012**	**0.0115**	**0.012**	**0.0091**
		1.5	**0.0149**	0.0173	**0.0148**	0.0173	**0.0094**
		2.0	**0.015**	0.0178	**0.015**	0.0179	**0.0086**
		2.5	**0.0122**	**0.0143**	**0.012**	**0.0141**	**0.0104**
		3.0	**0.0141**	**0.015**	**0.0138**	**0.015**	**0.0113**
	5	0.7	**0.0108**	**0.0114**	**0.0108**	**0.0114**	**0.0102**
		1.5	**0.0134**	**0.0146**	**0.0134**	**0.0145**	**0.0108**
		2.0	**0.0134**	**0.015**	**0.0135**	**0.015**	**0.0116**
		2.5	**0.0136**	**0.015**	**0.0136**	**0.015**	**0.0112**
		3.0	**0.0135**	**0.015**	**0.0137**	**0.015**	**0.0107**

Table 5.1 (Continued): Estimated probability of type I error at $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	**0.0114**	**0.0119**	**0.0114**	**0.0119**	**0.0093**
		1.5	**0.0119**	**0.0127**	**0.0119**	**0.0127**	**0.0104**
		2.0	**0.0132**	**0.0146**	**0.0132**	**0.0144**	**0.0083**
		2.5	**0.0123**	**0.0136**	**0.0125**	**0.0139**	**0.0095**
		3.0	**0.0114**	**0.0123**	**0.0112**	**0.0127**	**0.01**
	3	0.7	**0.0073**	**0.0078**	**0.0073**	**0.0078**	**0.0094**
		1.5	**0.0125**	**0.0132**	**0.0125**	**0.0132**	**0.0109**
		2.0	**0.0107**	**0.012**	**0.0108**	**0.012**	**0.0097**
		2.5	**0.0146**	0.0155	**0.0146**	0.0155	**0.0086**
		3.0	**0.0148**	0.0163	**0.0146**	0.016	**0.0123**
	4	0.7	**0.0106**	**0.0112**	**0.0106**	**0.0112**	**0.0101**
		1.5	**0.0123**	**0.013**	**0.0123**	**0.013**	**0.0099**
		2.0	**0.0129**	**0.014**	**0.0129**	**0.0141**	**0.0117**
		2.5	**0.0128**	**0.014**	**0.0128**	**0.0139**	**0.0098**
		3.0	**0.0129**	**0.0143**	**0.013**	**0.0141**	**0.0101**
	5	0.7	**0.0096**	**0.01**	**0.0096**	**0.01**	**0.0084**
		1.5	**0.0121**	**0.0128**	**0.0121**	**0.0127**	**0.0097**
		2.0	**0.0124**	**0.0139**	**0.0124**	**0.014**	**0.0097**
		2.5	**0.0123**	**0.0132**	**0.0119**	**0.0131**	**0.0117**
		3.0	**0.0107**	**0.012**	**0.0107**	**0.0117**	**0.0106**

The bold font is the tests that are able to control the probability of type I error.

5.2.1.2 Estimated probability of type I error at $\alpha = 0.05$.

From Table 5.2, we see that the T_k procedure could control the probability of type I error for all values of μ_0 and b when n = 25, and 35. While n = 16, the T_k test could control the probability of type I error, except when μ_0 = 2, b = 2.0; μ_0 = 2, b = 2.5; μ_0 = 3, b = 2.0; μ_0 = 3, b = 2.5; μ_0 = 4, b = 2.0; μ_0 = 5, b = 2.0, and μ_0 = 5, b = 3.0.

The T_{gh} procedure could not control the probability of type I error for a small sample size and large values of b. Whereas in n = 25, the T_{gh} procedure could control the probability of type I error, except when μ_0 = 3, b = 2.0; μ_0 = 3, b = 2.5; μ_0 = 3, b = 3.0; μ_0 = 5, b = 1.5; μ_0 = 5, b = 2.0, and μ_0 = 5, b = 2.5. When n = 35, the T_{gh} procedure could control the probability of type I error for almost all values of μ_0 and b, except when μ_0 = 2, b = 2.5.

The T_{km} procedure could control the probability of type I error for all values of μ_0 and b for n = 25 and 35, except when μ_0 = 5, b = 2.5. While n = 16, the T_{km} test could control the probability of type I error, except when μ_0 = 2, b = 2.5; μ_0 = 3, b = 2.0; μ_0 = 3, b = 2.5; μ_0 = 4, b = 2.0; μ_0 = 5, b = 2.0, and μ_0 = 5, b = 3.0. This shows that the T_{km} procedure is similar to the procedure T_k.

For the procedure T_{ghm}, the results are similar to the procedure T_{ghm}.

For the T_b procedure, this test still could control the probability of type I error for all combinations.

Table 5.2: Estimated probability of type I error at $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	**0.0537**	**0.057**	**0.0537**	**0.057**	**0.05**
		1.5	**0.0572**	**0.0544**	**0.0571**	**0.0542**	**0.049**
		2.0	0.0616	0.069	**0.06**	0.068	**0.0505**
		2.5	0.062	0.0697	0.0616	0.0603	**0.0518**
		3.0	**0.0583**	**0.0568**	**0.0577**	**0.0564**	**0.0499**
	3	0.7	**0.0573**	**0.0511**	**0.0573**	**0.0511**	**0.0508**
		1.5	**0.0571**	**0.0558**	**0.0575**	**0.0557**	**0.0504**
		2.0	0.0604	0.0689	0.0607	0.0692	**0.0505**
		2.5	0.0638	0.0724	0.0619	0.0707	**0.048**
		3.0	**0.0583**	**0.0575**	**0.0577**	**0.057**	**0.0421**
	4	0.7	**0.0563**	**0.0518**	**0.0563**	**0.0518**	**0.0537**
		1.5	**0.0558**	**0.0529**	**0.0558**	**0.0528**	**0.0543**
		2.0	0.0602	0.0694	0.0602	0.0677	**0.0506**
		2.5	**0.0599**	0.0682	**0.0599**	0.0662	**0.0502**
		3.0	**0.0588**	0.0673	**0.0591**	0.0664	**0.0482**
	5	0.7	**0.056**	**0.0589**	**0.056**	**0.0589**	**0.0483**
		1.5	**0.0591**	**0.056**	**0.0594**	**0.0559**	**0.0494**
		2.0	0.0616	0.0711	0.0614	0.0697	**0.0498**
		2.5	**0.0571**	0.0653	**0.0569**	0.0648	**0.0516**
		3.0	0.0643	0.0713	0.0614	0.0701	**0.0498**
25	2	0.7	**0.0523**	**0.0539**	**0.0523**	**0.0539**	**0.0446**
		1.5	**0.0516**	**0.0562**	**0.0517**	**0.0562**	**0.0477**
		2.0	**0.052**	**0.0574**	**0.0521**	**0.0572**	**0.0484**
		2.5	**0.0534**	**0.0594**	**0.0546**	**0.0593**	**0.047**
		3.0	**0.055**	**0.06**	**0.054**	**0.0596**	**0.0515**
	3	0.7	**0.0523**	**0.0553**	**0.0523**	**0.0553**	**0.0524**
		1.5	**0.0537**	**0.0585**	**0.0538**	**0.0587**	**0.0529**
		2.0	**0.0597**	0.0642	**0.0594**	0.0639	**0.0474**
		2.5	**0.0577**	0.0617	**0.0577**	0.0614	**0.0481**
		3.0	**0.0592**	0.0649	**0.0598**	0.0655	**0.0464**
	4	0.7	**0.0505**	**0.0524**	**0.0505**	**0.0524**	**0.0489**
		1.5	**0.0549**	**0.06**	**0.0551**	**0.06**	**0.049**
		2.0	**0.0565**	**0.06**	**0.0567**	**0.06**	**0.0508**
		2.5	**0.0509**	**0.0555**	**0.0507**	**0.0558**	**0.0519**
		3.0	**0.0531**	**0.058**	**0.0525**	**0.0572**	**0.0508**
	5	0.7	**0.0523**	**0.0544**	**0.0523**	**0.0544**	**0.0503**
		1.5	**0.0578**	0.0621	**0.0578**	0.0624	**0.0529**
		2.0	**0.0581**	0.0625	**0.058**	0.0627	**0.0459**
		2.5	**0.0596**	0.0647	0.0611	0.0656	**0.0495**
		3.0	**0.0519**	**0.0572**	**0.0523**	**0.0572**	**0.0489**

Table 5.2 (Continued): Estimated probability of type I error at $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	**0.0529**	**0.0545**	**0.0529**	**0.0545**	**0.047**
		1.5	**0.0519**	**0.0544**	**0.0519**	**0.0545**	**0.0512**
		2.0	**0.0542**	**0.0576**	**0.0544**	**0.0574**	**0.0538**
		2.5	**0.0595**	0.0624	**0.0589**	0.0629	**0.0536**
		3.0	**0.0553**	**0.0582**	**0.0545**	**0.0581**	**0.0517**
	3	0.7	**0.0556**	**0.0579**	**0.0556**	**0.0579**	**0.051**
		1.5	**0.0485**	**0.0509**	**0.0485**	**0.0509**	**0.051**
		2.0	**0.0563**	**0.0599**	**0.0561**	**0.0598**	**0.0499**
		2.5	**0.0516**	**0.0554**	**0.052**	**0.0557**	**0.0513**
		3.0	**0.0506**	**0.0547**	**0.0504**	**0.0541**	**0.0533**
	4	0.7	**0.0532**	**0.0549**	**0.0532**	**0.0549**	**0.0526**
		1.5	**0.0557**	**0.059**	**0.0558**	**0.059**	**0.0518**
		2.0	**0.0545**	**0.0587**	**0.0548**	**0.0587**	**0.0506**
		2.5	**0.0564**	**0.0589**	**0.0564**	**0.0588**	**0.0523**
		3.0	**0.0562**	**0.0591**	**0.0566**	**0.0596**	**0.056**
	5	0.7	**0.0531**	**0.0545**	**0.0531**	**0.0545**	**0.0467**
		1.5	**0.0536**	**0.0566**	**0.0536**	**0.0566**	**0.0478**
		2.0	**0.0515**	**0.0552**	**0.0516**	**0.0553**	**0.0474**
		2.5	**0.0529**	**0.057**	**0.0524**	**0.0561**	**0.0512**
		3.0	**0.0542**	**0.0578**	**0.055**	**0.0578**	**0.0528**

The bold font is the tests that are able to control the probability of type I error.

5.2.2 The power of the test

5.2.2.1 Estimated power at $\alpha = 0.01$.

The results are concluded in Tables 5.3-5.10. From the previous part we know which test statistics could control the probability of type I error, and hence we paid most of the attention to them. We found that the power of all tests tends to get higher when the sample size increases and the value of the coefficient of variation decreases.

The results from Tables 5.3-5.10 show that when n = 16, $b \leq$ 1.5, the T_{gh} test has the same power as the T_{ghm} test and the T_k test has the same power as the T_{km} test.

For n = 16, $b \geq$ 1.5, the T_{km} procedure seems to be the most powerful and we see that when $b \geq$ 1.5 the power of all tests starts decreasing. For example, in Table 5.10, for n = 35, μ_0 = 4, δ = 1.5, the powers of T_{km} are 0.9893, 0.9273, 0.9039, 0.8879 and 0.8908 for the values of b = 0.7, 1.5, 2.0, 2.5 and 3.0, respectively.

When n = 25, $b \leq$ 1.5, the T_{gh} test the same power as the T_{ghm} test and the T_k test has the same power as the T_{km} test. For n = 25, $b \geq$ 1.5, the T_{ghm} procedure appears to be the most powerful when compared to others.

For n = 35, $b \leq$ 1.5, the T_{gh} test has the same high power as the T_{ghm} test and the T_k test has the same power as the T_{km} test. For n = 35, $b \geq$ 1.5, the T_{ghm} procedure appears to be the most powerful when compared to others. In addition, the power of all tests is close to 1 for b = 0.7, δ = 1.5.

For the T_b test, although this test could control the probability of type I error for all combinations, but in term of powers, this test has got the lowest power for all combinations.

Table 5.3: Estimated power for $\delta = 0.85$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.0405	0.0448	0.0405	0.0448	0.0205
		1.5	0.0158	*	0.0157	*	0.006
		2.0	*	*	*	*	0.0043
		2.5	0.0118	*	0.0123	*	0.0031
		3.0	0.013	*	*	*	0.0028
	3	0.7	0.0419	0.0461	0.0419	0.0461	0.0203
		1.5	0.0156	*	0.0154	*	0.0051
		2.0	0.0118	*	0.0114	*	0.0037
		2.5	0.011	*	0.0102	*	0.0032
		3.0	0.0136	*	0.0139	*	0.0039
	4	0.7	0.0369	0.0412	0.0369	0.0412	0.0228
		1.5	0.0159	0.0203	0.0154	0.0205	0.0056
		2.0	0.0134	0.0171	0.0135	0.0171	0.0043
		2.5	0.0116	*	0.012	*	0.0041
		3.0	0.0095	*	0.0097	*	0.0041
	5	0.7	0.0369	0.042	0.0369	0.042	0.0189
		1.5	0.0125	*	0.0124	0.0162	0.0054
		2.0	0.013	*	0.0133	*	0.0035
		2.5	0.0122	*	0.0126	*	0.0025
		3.0	0.0112	*	0.0118	*	0.0033
25	2	0.7	0.0853	0.0895	0.0853	0.0895	0.0398
		1.5	0.0354	0.0395	0.0355	0.0395	0.0086
		2.0	0.0306	0.0354	0.0307	0.0353	0.0069
		2.5	0.0281	0.0316	0.0282	0.0319	0.0035
		3.0	*	*	0.0271	*	0.0049
	3	0.7	0.0847	0.0906	0.0847	0.0906	0.0432
		1.5	0.0342	0.0387	0.0341	0.0388	0.007
		2.0	0.0287	0.0325	0.0288	0.0325	0.0059
		2.5	0.0293	0.0333	0.0293	0.034	0.0028
		3.0	0.0281	0.0326	0.028	0.0342	0.0032
	4	0.7	0.087	0.0913	0.087	0.0913	0.0377
		1.5	0.0353	*	0.0354	*	0.0082
		2.0	0.0297	*	0.0297	*	0.0038
		2.5	0.0244	0.0286	0.0242	0.0278	0.0038
		3.0	0.0243	0.0283	0.0256	0.03	0.004
	5	0.7	0.0832	0.0887	0.0832	0.0887	0.0363
		1.5	0.0354	0.0395	0.0353	0.0394	0.0059
		2.0	0.0274	0.0318	0.0273	0.0319	0.0054
		2.5	0.0272	0.0321	0.0276	0.0323	0.0036
		3.0	0.025	0.0291	0.0246	0.0289	0.0039

Table 5.3 (Continued): Estimated power for $\delta = 0.85$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
	2	0.7	0.1538	0.1591	0.1538	0.1591	0.0583
		1.5	0.0627	0.0675	0.0627	0.0675	0.0111
		2.0	0.0502	0.0538	0.0502	0.0541	0.0066
		2.5	0.0524	0.0568	0.0514	0.0568	0.0054
		3.0	0.0456	0.0514	0.0466	0.051	0.0041
	3	0.7	0.1527	0.1575	0.1527	0.1575	0.0623
		1.5	0.065	0.0709	0.065	0.0709	0.0084
		2.0	0.0514	0.0558	0.0513	0.056	0.0065
		2.5	0.0503	*	0.0509	*	0.0061
		3.0	0.0485	*	0.0487	*	0.0039
	4	0.7	0.1507	0.1556	0.1507	0.1556	0.0616
		1.5	0.0591	0.0653	0.0591	0.0653	0.0098
		2.0	0.0535	0.0588	0.0537	0.0589	0.0052
		2.5	0.0496	0.0546	0.0497	0.0552	0.006
		3.0	0.0456	0.0504	0.0466	0.0507	0.0049
	5	0.7	0.1526	0.1593	0.1526	0.1593	0.0629
		1.5	0.0641	0.0679	0.0641	0.0679	0.0105
		2.0	0.0492	0.053	0.0494	0.053	0.0066
		2.5	0.0534	0.059	0.0538	0.06	0.0044
		3.0	0.0468	0.0519	0.0477	0.0518	0.0041

* indicates that the test could not control the probability of type I error.

Table 5.4: Estimated power for $\delta = 0.9$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.0166	0.0184	0.0166	0.0184	0.0141
		1.5	0.01	*	0.0099	*	0.0062
		2.0	*	*	*	*	0.0037
		2.5	0.0073	*	0.0067	*	0.0039
		3.0	0.0065	*	*	*	0.0047
	3	0.7	0.0183	0.0196	0.0183	0.0196	0.0114
		1.5	0.0078	*	0.0075	*	0.0063
		2.0	0.0085	*	0.0085	*	0.0055
		2.5	0.0072	*	0.0065	*	0.0049
		3.0	0.0074	*	0.0069	*	0.0041
	4	0.7	0.0179	0.0201	0.0179	0.0201	0.0127
		1.5	0.0079	0.0106	0.0079	0.0103	0.0055
		2.0	0.0067	0.0086	0.0066	0.0087	0.007
		2.5	0.0082	*	0.0084	*	0.0053
		3.0	0.0077	*	0.0083	*	0.0055
	5	0.7	0.0178	0.0194	0.0178	0.0194	0.0145
		1.5	0.0085	*	0.0084	0.0101	0.0057
		2.0	0.0084	*	0.0082	*	0.0054
		2.5	0.0079	*	0.0079	*	0.0044
		3.0	0.0076	*	0.0085	*	0.0041
25	2	0.7	0.0342	0.0356	0.0342	0.0356	0.0199
		1.5	0.016	0.0185	0.016	0.0185	0.0069
		2.0	0.0153	0.0175	0.015	0.0171	0.0051
		2.5	0.011	0.0131	0.0109	0.0133	0.0047
		3.0	*	*	0.0124	*	0.0053
	3	0.7	0.0351	0.0373	0.0351	0.0373	0.0223
		1.5	0.0167	0.0188	0.0168	0.0187	0.0065
		2.0	0.0111	0.0142	0.0119	0.0141	0.0048
		2.5	0.0128	0.0147	0.0125	0.0152	0.0039
		3.0	0.0127	0.0147	0.0127	0.0144	0.0042
	4	0.7	0.031	0.0332	0.031	0.0332	0.0206
		1.5	0.0167	*	0.0166	*	0.0069
		2.0	0.0142	*	0.0142	*	0.006
		2.5	0.0136	0.015	0.0134	0.0153	0.0041
		3.0	0.0111	0.014	0.0116	0.0139	0.0053
	5	0.7	0.0327	0.0347	0.0327	0.0347	0.0204
		1.5	0.0156	0.0173	0.0156	0.0173	0.0064
		2.0	0.0149	0.0175	0.015	0.0178	0.0046
		2.5	0.0127	0.0153	0.0133	0.0151	0.0056
		3.0	0.0117	0.0132	0.012	0.0141	0.0042

Table 5.4 (Continued): Estimated power for $\delta = 0.9$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	0.0509	0.0531	0.0509	0.0531	0.0279
		1.5	0.0229	0.0246	0.0229	0.0246	0.0076
		2.0	0.0195	0.0212	0.0194	0.0212	0.0071
		2.5	0.0194	0.0221	0.0195	0.0225	0.0053
		3.0	0.0219	0.0242	0.0222	0.0248	0.0051
	3	0.7	0.0533	0.0557	0.0533	0.0557	0.0262
		1.5	0.0242	0.0256	0.0242	0.0256	0.0081
		2.0	0.0244	0.0268	0.0242	0.0266	0.0065
		2.5	0.0195	*	0.0192	*	0.0048
		3.0	0.0189	*	0.02	*	0.0052
	4	0.7	0.0483	0.0509	0.0483	0.0509	0.0269
		1.5	0.0242	0.0263	0.0242	0.0263	0.0073
		2.0	0.0205	0.0219	0.0205	0.022	0.0063
		2.5	0.0204	0.0221	0.02	0.0218	0.005
		3.0	0.0195	0.0213	0.0191	0.0203	0.0051
	5	0.7	0.0555	0.0573	0.0555	0.0573	0.026
		1.5	0.0216	0.0234	0.0215	0.0233	0.0071
		2.0	0.0197	0.0219	0.0196	0.0218	0.0074
		2.5	0.0207	0.0226	0.0205	0.0225	0.0062
		3.0	0.019	0.0218	0.0194	0.0224	0.0039

* indicates that the test could not control the probability of type I error.

Table 5.5: Estimated power for $\delta = 0.95$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.0093	0.0107	0.0093	0.0107	0.008
		1.5	0.0074	*	0.0074	*	0.0077
		2.0	*	*	*	*	0.0081
		2.5	0.0097	*	0.0097	*	0.0072
		3.0	0.0081	*	*	*	0.0059
	3	0.7	0.0081	0.0097	0.0081	0.0097	0.0084
		1.5	0.0084	*	0.008	*	0.0077
		2.0	0.0093	*	0.009	*	0.0069
		2.5	0.0078	*	0.0075	*	0.0063
		3.0	0.0095	*	0.0092	*	0.0066
	4	0.7	0.0084	0.0095	0.0084	0.0095	0.0088
		1.5	0.0074	0.0105	0.0075	0.0106	0.0074
		2.0	0.0075	0.0098	0.0075	0.0099	0.0067
		2.5	0.0075	*	0.0078	*	0.0075
		3.0	0.0085	*	0.012	*	0.0078
	5	0.7	0.0094	0.0108	0.0094	0.0108	0.008
		1.5	0.0071	*	0.007	0.0085	0.0083
		2.0	0.0092	*	0.0091	*	0.0063
		2.5	0.0067	*	0.0068	*	0.0079
		3.0	0.0086	*	0.0084	*	0.0071
25	2	0.7	0.01	0.0112	0.01	0.0112	0.0114
		1.5	0.009	0.0096	0.009	0.0096	0.0072
		2.0	0.0081	0.0095	0.0082	0.0094	0.0076
		2.5	0.0085	0.01	0.0085	0.01	0.0072
		3.0	*	*	0.009	*	0.008
	3	0.7	0.012	0.0129	0.012	0.0129	0.0113
		1.5	0.0072	0.0082	0.0072	0.0082	0.006
		2.0	0.0089	0.0109	0.0092	0.0107	0.0071
		2.5	0.0082	0.0096	0.0084	0.0097	0.0068
		3.0	0.008	0.0094	0.0083	0.0096	0.0066
	4	0.7	0.0133	0.0141	0.0133	0.0141	0.0097
		1.5	0.0088	*	0.0087	*	0.0067
		2.0	0.0083	*	0.0079	*	0.0074
		2.5	0.01	0.0112	0.0099	0.011	0.006
		3.0	0.0084	0.0101	0.0081	0.0098	0.0076
	5	0.7	0.0114	0.0124	0.0114	0.0124	0.0098
		1.5	0.0099	0.0111	0.01	0.0111	0.0093
		2.0	0.0086	0.0092	0.0084	0.009	0.0082
		2.5	0.0075	0.009	0.0077	0.0089	0.0068
		3.0	0.0078	0.0088	0.0079	0.0094	0.0093

Table 5.5 (Continued): Estimated power for $\delta = 0.95$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	0.0158	0.0167	0.0158	0.0167	0.0128
		1.5	0.0095	0.0104	0.0095	0.0104	0.009
		2.0	0.0083	0.0089	0.0085	0.009	0.006
		2.5	0.0091	0.01	0.0093	0.0103	0.007
		3.0	0.0093	0.0105	0.0094	0.0103	0.0066
	3	0.7	0.0155	0.0161	0.0155	0.0161	0.011
		1.5	0.0098	0.0106	0.0098	0.0106	0.0093
		2.0	0.0088	0.01	0.0088	0.0099	0.0063
		2.5	0.0104	*	0.0104	*	0.0051
		3.0	0.0085	*	0.0085	*	0.0075
	4	0.7	0.0153	0.0158	0.0153	0.0158	0.01
		1.5	0.0094	0.0101	0.0094	0.0101	0.0096
		2.0	0.009	0.01	0.0089	0.0099	0.0075
		2.5	0.0091	0.01	0.0091	0.0099	0.0061
		3.0	0.0108	0.0115	0.0104	0.0108	0.0062
	5	0.7	0.0144	0.0155	0.0144	0.0155	0.0118
		1.5	0.0116	0.0126	0.0116	0.0126	0.0082
		2.0	0.0094	0.0102	0.0094	0.0102	0.0071
		2.5	0.0094	0.0106	0.0093	0.0108	0.0079
		3.0	0.0074	0.0084	0.0072	0.0084	0.0063

* indicates that the test could not control the probability of type I error.

Table 5.6: Estimated power for $\delta = 1.1$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.0709	0.077	0.0709	0.077	0.0354
		1.5	0.0628	*	0.0631	*	0.0231
		2.0	*	*	*	*	0.0181
		2.5	0.0627	*	0.0645	*	0.0213
		3.0	0.0628	*	*	*	0.0209
	3	0.7	0.0647	0.071	0.0647	0.071	0.0343
		1.5	0.0611	*	0.0617	*	0.0219
		2.0	0.0619	*	0.0629	*	0.0211
		2.5	0.0584	*	0.0579	*	0.0182
		3.0	0.0616	*	0.06	*	0.021
	4	0.7	0.0699	0.0751	0.0699	0.0751	0.0369
		1.5	0.0629	0.0704	0.0631	0.0702	0.0222
		2.0	0.0598	0.0675	0.0602	0.0671	0.0222
		2.5	0.0612	*	0.0626	*	0.0219
		3.0	0.0566	*	0.0576	*	0.0212
	5	0.7	0.0726	0.0774	0.0726	0.0774	0.0377
		1.5	0.0606	*	0.0607	0.0664	0.022
		2.0	0.0581	*	0.0582	*	0.0205
		2.5	0.0611	*	0.0608	*	0.0213
		3.0	0.0623	*	0.0618	*	0.0174
25	2	0.7	0.0943	0.0989	0.0943	0.0989	0.0445
		1.5	0.0645	0.0694	0.0645	0.0694	0.0215
		2.0	0.0653	0.0704	0.0654	0.0707	0.0215
		2.5	0.0708	0.077	0.0709	0.0771	0.0185
		3.0	*	*	0.0732	*	0.0206
	3	0.7	0.0998	0.104	0.0998	0.104	0.0505
		1.5	0.0683	0.0745	0.0684	0.0746	0.023
		2.0	0.0686	0.0746	0.0693	0.0745	0.0244
		2.5	0.0684	0.0734	0.0683	0.0738	0.0198
		3.0	0.0668	0.0711	0.0672	0.0725	0.0221
	4	0.7	0.094	0.0982	0.094	0.0982	0.0473
		1.5	0.0749	*	0.0749	*	0.0274
		2.0	0.0689	*	0.0693	*	0.0255
		2.5	0.0645	0.0686	0.0641	0.068	0.0212
		3.0	0.0686	0.0755	0.0686	0.0749	0.0211
	5	0.7	0.0919	0.096	0.0919	0.096	0.0497
		1.5	0.0678	0.0725	0.0678	0.0725	0.0261
		2.0	0.0701	0.0757	0.0707	0.0761	0.023
		2.5	0.0637	0.0697	0.0645	0.0704	0.0244
		3.0	0.0673	0.0722	0.0661	0.0719	0.0193

Table 5.6 (Continued): Estimated power for $\delta = 1.1$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	0.1241	0.1274	0.1241	0.1274	0.0588
		1.5	0.0841	0.0896	0.0841	0.0896	0.0266
		2.0	0.0814	0.0862	0.0814	0.0863	0.0248
		2.5	0.0749	0.0807	0.0757	0.0808	0.0193
		3.0	0.0795	0.084	0.0797	0.084	0.0215
	3	0.7	0.1229	0.1269	0.1229	0.1269	0.0591
		1.5	0.0884	0.0916	0.0884	0.0916	0.0279
		2.0	0.0854	0.0903	0.0851	0.0904	0.023
		2.5	0.0815	*	0.0816	*	0.0237
		3.0	0.0834	*	0.0836	*	0.0222
	4	0.7	0.1178	0.1218	0.1178	0.1218	0.0606
		1.5	0.0827	0.0861	0.0827	0.0861	0.0243
		2.0	0.0841	0.0891	0.0841	0.0889	0.0257
		2.5	0.082	0.0848	0.0817	0.0847	0.0238
		3.0	0.0775	0.0827	0.0781	0.0831	0.0198
	5	0.7	0.1225	0.1256	0.1225	0.1256	0.0627
		1.5	0.086	0.0897	0.086	0.0897	0.0284
		2.0	0.0794	0.0857	0.0793	0.0855	0.0226
		2.5	0.0812	0.0853	0.0814	0.0851	0.0222
		3.0	0.0861	0.0909	0.0865	0.0911	0.0212

* indicates that the test could not control the probability of type I error.

Table 5.7: Estimated power for $\delta = 1.2$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.2394	0.2496	0.2394	0.2496	0.1162
		1.5	0.1677	*	0.1677	*	0.0502
		2.0	*	*	*	*	0.0398
		2.5	0.1611	*	0.1632	*	0.0377
		3.0	0.1539	*	*	*	0.034
	3	0.7	0.2387	0.2483	0.2387	0.2483	0.1214
		1.5	0.1682	*	0.169	*	0.0513
		2.0	0.1661	*	0.1675	*	0.0421
		2.5	0.1642	*	0.167	*	0.041
		3.0	0.1589	*	0.16	*	0.0383
	4	0.7	0.2411	0.2531	0.2411	0.2531	0.117
		1.5	0.1724	0.1857	0.1723	0.1853	0.0512
		2.0	0.1586	0.1747	0.1607	0.176	0.0393
		2.5	0.1526	*	0.1553	*	0.0371
		3.0	0.1625	*	0.1647	*	0.0384
	5	0.7	0.2411	0.2528	0.2411	0.2528	0.1215
		1.5	0.1647	*	0.1651	0.1787	0.0485
		2.0	0.1648	*	0.1659	*	0.0402
		2.5	0.1589	*	0.1607	*	0.0388
		3.0	0.1509	*	0.1515	*	0.0361
25	2	0.7	0.3479	0.3562	0.3479	0.3562	0.1763
		1.5	0.232	0.2422	0.2322	0.2423	0.0583
		2.0	0.2138	0.2248	0.2135	0.2251	0.0493
		2.5	0.2065	0.2177	0.2082	0.2193	0.0377
		3.0	*	*	0.2106	*	0.0412
	3	0.7	0.3516	0.3603	0.3516	0.3603	0.1676
		1.5	0.2367	0.2474	0.2368	0.2474	0.0604
		2.0	0.2109	0.2215	0.2112	0.222	0.0467
		2.5	0.2139	0.2243	0.2163	0.2255	0.0421
		3.0	0.2154	0.2265	0.2166	0.2287	0.0372
	4	0.7	0.3366	0.3454	0.3366	0.3454	0.1667
		1.5	0.2327	*	0.2328	*	0.059
		2.0	0.2176	*	0.2178	*	0.0476
		2.5	0.2125	0.2215	0.213	0.2233	0.0412
		3.0	0.2135	0.2258	0.2158	0.2276	0.0391
	5	0.7	0.3404	0.3477	0.3404	0.3477	0.1718
		1.5	0.2203	0.2303	0.2202	0.2302	0.061
		2.0	0.2225	0.2334	0.2224	0.234	0.0496
		2.5	0.2035	0.2132	0.2049	0.215	0.0465
		3.0	0.2091	0.2192	0.2107	0.2207	0.0369

Table 5.7 (Continued): Estimated power for $\delta = 1.2$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	0.4584	0.4647	0.4584	0.4647	0.2267
		1.5	0.2895	0.2985	0.2895	0.2986	0.0676
		2.0	0.2805	0.2899	0.2808	0.2902	0.0568
		2.5	0.2731	0.2822	0.2734	0.2817	0.0453
		3.0	0.2679	0.2785	0.2687	0.2783	0.0419
	3	0.7	0.4601	0.4665	0.4601	0.4665	0.2369
		1.5	0.2995	0.3094	0.2995	0.3094	0.0715
		2.0	0.2808	0.2883	0.2808	0.2887	0.0548
		2.5	0.2764	*	0.2768	*	0.0466
		3.0	0.2605	*	0.2609	*	0.0422
	4	0.7	0.4613	0.4672	0.4613	0.4672	0.2289
		1.5	0.2988	0.3075	0.2989	0.3075	0.0717
		2.0	0.284	0.2933	0.2843	0.2933	0.0525
		2.5	0.2727	0.28	0.2736	0.2816	0.0451
		3.0	0.2641	0.2734	0.2662	0.2762	0.0445
	5	0.7	0.4525	0.4587	0.4525	0.4587	0.2267
		1.5	0.296	0.3035	0.296	0.3035	0.0735
		2.0	0.2836	0.2936	0.2835	0.2937	0.0538
		2.5	0.2688	0.2787	0.2697	0.2788	0.0434
		3.0	0.264	0.2734	0.2653	0.2742	0.0392

* indicates that the test could not control the probability of type I error.

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Table 5.8: Estimated power for $\delta = 1.3$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.4829	0.4971	0.4829	0.4971	0.2514
		1.5	0.3409	*	0.341	*	0.0892
		2.0	*	*	*	*	0.0741
		2.5	0.3037	*	0.308	*	0.0606
		3.0	0.3061	*	*	*	0.0577
	3	0.7	0.4746	0.4889	0.4746	0.4889	0.2522
		1.5	0.3353	*	0.3359	*	0.0956
		2.0	0.3134	*	0.3152	*	0.0707
		2.5	0.315	*	0.3175	*	0.062
		3.0	0.305	*	0.3102	*	0.0547
	4	0.7	0.4875	0.4991	0.4875	0.4991	0.2605
		1.5	0.3284	0.3471	0.3288	0.3479	0.0916
		2.0	0.311	0.331	0.3111	0.3316	0.0665
		2.5	0.3015	*	0.3049	*	0.0611
		3.0	0.2932	*	0.2978	*	0.0612
	5	0.7	0.4847	0.4953	0.4847	0.4953	0.2622
		1.5	0.3368	*	0.3373	0.3556	0.0912
		2.0	0.321	*	0.3241	*	0.0722
		2.5	0.3071	*	0.3094	*	0.0588
		3.0	0.3066	*	0.3129	*	0.0573
25	2	0.7	0.6532	0.6604	0.6532	0.6604	0.3705
		1.5	0.4586	0.4721	0.4587	0.4721	0.1135
		2.0	0.4345	0.4488	0.4362	0.4497	0.087
		2.5	0.427	0.4409	0.4288	0.4437	0.0716
		3.0	*	*	0.4167	*	0.0653
	3	0.7	0.6497	0.6579	0.6497	0.6579	0.3673
		1.5	0.4659	0.4778	0.4659	0.478	0.1188
		2.0	0.4344	0.4485	0.435	0.4493	0.0862
		2.5	0.4263	0.4405	0.4298	0.4429	0.0687
		3.0	0.415	0.4277	0.417	0.4292	0.0639
	4	0.7	0.6552	0.6629	0.6552	0.6629	0.3667
		1.5	0.4606	*	0.4609	*	0.1227
		2.0	0.427	*	0.4277	*	0.086
		2.5	0.4282	0.4423	0.4301	0.4438	0.0691
		3.0	0.3974	0.4115	0.3996	0.4138	0.0605
	5	0.7	0.6508	0.6605	0.6508	0.6605	0.3692
		1.5	0.4615	0.475	0.4616	0.475	0.1129
		2.0	0.4285	0.444	0.4296	0.4447	0.0861
		2.5	0.4131	0.4268	0.4154	0.4272	0.0737
		3.0	0.4111	0.4257	0.4159	0.4296	0.0612

Table 5.8 (Continued): Estimated power for $\delta = 1.3$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	0.7907	0.7953	0.7907	0.7953	0.4894
		1.5	0.5775	0.5881	0.5776	0.5881	0.1462
		2.0	0.5578	0.5679	0.5581	0.568	0.1024
		2.5	0.5332	0.5433	0.5343	0.5437	0.0836
		3.0	0.527	0.5382	0.528	0.5408	0.07
	3	0.7	0.7933	0.7979	0.7933	0.7979	0.4889
		1.5	0.5916	0.5998	0.5917	0.5998	0.1481
		2.0	0.5594	0.5707	0.5599	0.5714	0.0993
		2.5	0.5237	*	0.5248	*	0.0777
		3.0	0.5294	*	0.5297	*	0.0698
	4	0.7	0.7853	0.7889	0.7853	0.7889	0.4901
		1.5	0.5762	0.5855	0.5762	0.5855	0.143
		2.0	0.541	0.5538	0.5408	0.5542	0.0956
		2.5	0.5348	0.5453	0.5355	0.5463	0.0775
		3.0	0.5196	0.53	0.52	0.5311	0.0715
	5	0.7	0.7941	0.7984	0.7941	0.7984	0.4819
		1.5	0.5778	0.5873	0.5778	0.5873	0.1439
		2.0	0.5503	0.5598	0.5505	0.5601	0.0975
		2.5	0.5354	0.5467	0.5366	0.5479	0.0811
		3.0	0.5241	0.5363	0.5258	0.538	0.0684

* indicates that the test could not control the probability of type I error.

Table 5.9: Estimated power for $\delta = 1.4$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.7017	0.7125	0.7017	0.7125	0.4154
		1.5	0.5178	*	0.5184	*	0.1498
		2.0	*	*	*	*	0.1096
		2.5	0.466	*	0.4708	*	0.0981
		3.0	0.4658	*	*	*	0.0849
	3	0.7	0.6945	0.7048	0.6945	0.7048	0.4163
		1.5	0.5168	*	0.517	*	0.145
		2.0	0.4861	*	0.4896	*	0.1102
		2.5	0.4672	*	0.4735	*	0.0923
		3.0	0.4623	*	0.4683	*	0.0927
	4	0.7	0.7009	0.711	0.7009	0.711	0.4212
		1.5	0.5159	0.5329	0.5165	0.5333	0.1536
		2.0	0.4772	0.499	0.4789	0.5004	0.1091
		2.5	0.4782	*	0.4831	*	0.096
		3.0	0.478	*	0.4836	*	0.0841
	5	0.7	0.6946	0.7036	0.6946	0.7036	0.4168
		1.5	0.5083	*	0.5092	0.5235	0.1406
		2.0	0.4829	*	0.4855	*	0.1108
		2.5	0.469	*	0.4751	*	0.0961
		3.0	0.4614	*	0.4689	*	0.0836
25	2	0.7	0.8591	0.8636	0.8591	0.8636	0.5781
		1.5	0.673	0.6837	0.673	0.6835	0.1905
		2.0	0.6411	0.6539	0.6426	0.6548	0.1383
		2.5	0.6269	0.6392	0.6284	0.6402	0.1117
		3.0	*	*	0.6202	*	0.0943
	3	0.7	0.8594	0.8639	0.8594	0.8639	0.5735
		1.5	0.6746	0.6863	0.6746	0.6863	0.1797
		2.0	0.6407	0.6528	0.6411	0.6539	0.1397
		2.5	0.6239	0.6378	0.6273	0.6394	0.1042
		3.0	0.6154	0.6288	0.6189	0.6332	0.0959
	4	0.7	0.8592	0.863	0.8592	0.863	0.574
		1.5	0.675	*	0.6751	*	0.1867
		2.0	0.6412	*	0.6413	*	0.1353
		2.5	0.6358	0.6474	0.6379	0.6496	0.1146
		3.0	0.6168	0.6306	0.6188	0.6328	0.093
	5	0.7	0.851	0.8554	0.851	0.8554	0.5772
		1.5	0.6762	0.687	0.6763	0.6871	0.1922
		2.0	0.6441	0.6573	0.6451	0.6571	0.1295
		2.5	0.6181	0.6302	0.6204	0.634	0.1078
		3.0	0.6101	0.6229	0.6128	0.6246	0.0942

Table 5.9 (Continued): Estimated power for $\delta = 1.4$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	0.9467	0.9484	0.9467	0.9484	0.718
		1.5	0.8013	0.8093	0.8013	0.8093	0.2408
		2.0	0.7777	0.7853	0.7779	0.7855	0.1562
		2.5	0.7592	0.7681	0.76	0.7686	0.121
		3.0	0.7514	0.7591	0.7532	0.761	0.1061
	3	0.7	0.9472	0.9491	0.9472	0.9491	0.7202
		1.5	0.8067	0.813	0.8067	0.813	0.2388
		2.0	0.7703	0.7762	0.7705	0.7765	0.1625
		2.5	0.7571	*	0.7583	*	0.1302
		3.0	0.747	*	0.7494	*	0.1081
	4	0.7	0.9475	0.9485	0.9475	0.9485	0.7227
		1.5	0.8019	0.8085	0.8019	0.8085	0.2442
		2.0	0.77	0.7762	0.7701	0.7763	0.164
		2.5	0.7591	0.7686	0.7602	0.7688	0.1302
		3.0	0.7499	0.7573	0.7514	0.7595	0.114
	5	0.7	0.946	0.9471	0.946	0.9471	0.7137
		1.5	0.8001	0.8061	0.8001	0.8061	0.2417
		2.0	0.7694	0.7773	0.7693	0.7772	0.1603
		2.5	0.7546	0.7643	0.756	0.7649	0.1358
		3.0	0.7428	0.7507	0.7452	0.752	0.1111

* indicates that the test could not control the probability of type I error.

Table 5.10: Estimated power for $\delta = 1.5$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.8448	0.8515	0.8448	0.8515	0.5778
		1.5	0.6644	*	0.6648	*	0.2107
		2.0	*	*	*	*	0.1532
		2.5	0.6178	*	0.6229	*	0.1304
		3.0	0.6081	*	*	*	0.1163
	3	0.7	0.8495	0.8559	0.8495	0.8559	0.5731
		1.5	0.6615	*	0.6615	*	0.2038
		2.0	0.6302	*	0.6331	*	0.1579
		2.5	0.6217	*	0.6282	*	0.1325
		3.0	0.6111	*	0.63	*	0.1144
	4	0.7	0.842	0.8485	0.842	0.8485	0.5717
		1.5	0.6644	0.68	0.6644	0.6811	0.2014
		2.0	0.6312	0.6486	0.6338	0.6508	0.1609
		2.5	0.623	*	0.6286	*	0.1329
		3.0	0.6135	*	0.623	*	0.1118
	5	0.7	0.8462	0.8533	0.8462	0.8533	0.5788
		1.5	0.6615	*	0.662	0.6781	0.2087
		2.0	0.6422	*	0.6462	*	0.1622
		2.5	0.6308	*	0.6358	*	0.1283
		3.0	0.6108	*	0.6177	*	0.1176
25	2	0.7	0.9561	0.9577	0.9561	0.9577	0.7502
		1.5	0.8231	0.8315	0.8233	0.8317	0.2725
		2.0	0.8057	0.8145	0.8065	0.8155	0.1836
		2.5	0.787	0.7971	0.7895	0.7999	0.1549
		3.0	*	*	0.7869	*	0.1354
	3	0.7	0.9531	0.9554	0.9531	0.9554	0.7462
		1.5	0.8242	0.8322	0.8241	0.8323	0.2754
		2.0	0.799	0.8082	0.8006	0.8087	0.1923
		2.5	0.7867	0.7959	0.7886	0.7977	0.1585
		3.0	0.7728	0.783	0.7774	0.7878	0.1367
	4	0.7	0.9573	0.9583	0.9573	0.9583	0.7348
		1.5	0.8266	*	0.8266	*	0.2683
		2.0	0.7994	*	0.8003	*	0.1892
		2.5	0.7883	0.7984	0.7904	0.799	0.151
		3.0	0.7721	0.7835	0.7747	0.7852	0.135
	5	0.7	0.9502	0.9515	0.9502	0.9515	0.7412
		1.5	0.8251	0.8321	0.8249	0.8321	0.2762
		2.0	0.7973	0.8051	0.7982	0.8062	0.1928
		2.5	0.7804	0.7891	0.7821	0.7898	0.1533
		3.0	0.7737	0.7824	0.7756	0.7859	0.1365

Table 5.10 (Continued): Estimated power for $\delta = 1.5$ and $\alpha = 0.01$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	0.9902	0.9906	0.9902	0.9906	0.8654
		1.5	0.9238	0.9269	0.9238	0.9269	0.3478
		2.0	0.8994	0.9039	0.8995	0.9041	0.2302
		2.5	0.8917	0.8974	0.8934	0.8977	0.1819
		3.0	0.8848	0.8899	0.8873	0.8915	0.1545
	3	0.7	0.9877	0.9883	0.9877	0.9883	0.8604
		1.5	0.9232	0.9264	0.9232	0.9262	0.3548
		2.0	0.9023	0.9051	0.9023	0.905	0.241
		2.5	0.8928	*	0.8931	*	0.1794
		3.0	0.881	*	0.8831	*	0.1598
	4	0.7	0.9893	0.9895	0.9893	0.9895	0.861
		1.5	0.9273	0.9304	0.9273	0.9304	0.342
		2.0	0.9036	0.9076	0.9039	0.9073	0.2333
		2.5	0.8874	0.8929	0.8879	0.8936	0.1767
		3.0	0.8896	0.8944	0.8908	0.8956	0.1568
	5	0.7	0.9891	0.9893	0.9891	0.9893	0.863
		1.5	0.921	0.9246	0.921	0.9246	0.3453
		2.0	0.8967	0.9003	0.8966	0.9002	0.2373
		2.5	0.8877	0.8915	0.8879	0.8917	0.1899
		3.0	0.885	0.8895	0.8865	0.8922	0.1507

* indicates that the test could not control the probability of type I error.

From Table 5.3-5.7 we see that, the power increases with b, n and δ but not depended on the choices of μ_0. There is a little difference in the powers of T_k, T_{km} tests and T_{gh}, T_{ghm} tests when the values of the coefficient of variation are small. Otherwise, the T_{ghm} procedure appears to be most powerful.

5.2.2.2 Estimated power at $\alpha = 0.05$.

The results are concluded in Tables 5.11-5.18. From the previous part we know which test statistics could control the probability of type I error, and hence we paid most of the attention to them. We found that the power of all tests tends to get higher when the sample size increases and the value of the coefficient of variation decrease.

The results from Tables 5.11-5.18 show that when n = 16, $b \leq$ 1.5, the T_{gh} test has the same power as the T_{ghm} test and the T_k test has the same power as the T_{km} test.

For n = 16, $b \geq$ 1.5, the T_{km} procedure seems to be the most powerful and we see that when $b \geq$ 1.5 the power of all tests starts to decrease. For example, in Table 5.18 for n = 35, μ_0 = 5, δ = 1.5, the estimated powers of T_{km} are 0.9975, 0.9669, 0.9587, 0.9536 and 0.9493 for the values of b = 0.7, 1.5, 2.0, 2.5 and 3.0, respectively.

When n = 25, $b \leq$ 1.5, the T_{gh} test has the same power as the T_{ghm} test and the T_k test has the same power as the T_{km} test. For n = 25, $b \geq$ 1.5, the T_{ghm} procedure appears to be the most powerful when compared to others.

For n = 35, $b \leq$ 1.5, the T_{gh} test has the same power as the T_{ghm} test and the T_k test has the same power as the T_{km} test. For n = 35, $b \geq$ 1.5, the T_{ghm} procedure appears to be the most powerful when compared to others. In addition, the power of all tests is close to 1 for b = 0.7, δ = 1.5.

Although the T_b test could control the probability of type I error for all combinations, but this test has the lowest power for all combinations.

Table 5.11: Estimated power for $\delta = 0.85$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.1568	0.166	0.1568	0.166	0.0975
		1.5	0.0863	0.096	0.0867	0.0965	0.0359
		2.0	*	*	0.0771	*	0.0296
		2.5	*	*	*	*	0.0304
		3.0	0.0665	0.078	0.0687	0.0795	0.024
	3	0.7	0.1505	0.1606	0.1505	0.1606	0.1013
		1.5	0.0809	0.0911	0.0811	0.092	0.0352
		2.0	*	*	*	*	0.0294
		2.5	*	*	*	*	0.0229
		3.0	0.0713	0.0821	0.0734	0.0833	0.0247
	4	0.7	0.153	0.1626	0.153	0.1626	0.0953
		1.5	0.0826	0.0921	0.0823	0.092	0.0369
		2.0	*	*	*	*	0.0314
		2.5	0.0783	*	0.0797	*	0.0282
		3.0	0.0708	*	0.0732	*	0.0254
	5	0.7	0.1484	0.1577	0.1484	0.1577	0.0966
		1.5	0.0808	0.0912	0.0812	0.0917	0.0342
		2.0	*	*	*	*	0.0278
		2.5	0.0699	*	0.0711	*	0.0261
		3.0	*	*	*	*	0.025
25	2	0.7	0.2653	0.2739	0.2653	0.2739	0.1502
		1.5	0.1431	0.153	0.1432	0.153	0.045
		2.0	0.1343	0.1442	0.1346	0.1444	0.0315
		2.5	0.1286	0.1374	0.1295	0.1382	0.0284
		3.0	0.1219	0.1326	0.1239	0.1341	0.0268
	3	0.7	0.2584	0.2655	0.2584	0.2655	0.1449
		1.5	0.1454	0.1544	0.1455	0.1545	0.0416
		2.0	0.1308	*	0.1303	*	0.0321
		2.5	0.1296	*	0.1315	*	0.0291
		3.0	0.1191	*	0.1202	*	0.0255
	4	0.7	0.2633	0.2701	0.2633	0.2701	0.1575
		1.5	0.149	0.1585	0.149	0.1586	0.0447
		2.0	0.1301	0.1404	0.1306	0.1402	0.0356
		2.5	0.1239	0.1328	0.1236	0.1334	0.0303
		3.0	0.1193	0.1301	0.1228	0.132	0.0273
	5	0.7	0.2618	0.2682	0.2618	0.2682	0.1519
		1.5	0.1438	*	0.1439	*	0.0454
		2.0	0.1263	*	0.1267	*	0.0321
		2.5	0.1199	*	*	*	0.0283
		3.0	0.124	0.1329	0.1241	0.1339	0.0288

Table 5.11 (Continued): Estimated power for $\delta = 0.85$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	0.3774	0.383	0.3774	0.383	0.2072
		1.5	0.2157	0.2248	0.2157	0.2248	0.0529
		2.0	0.1973	0.2062	0.1976	0.2063	0.0433
		2.5	0.1865	*	0.1877	*	0.0325
		3.0	0.1783	0.1869	0.1792	0.1869	0.0273
	3	0.7	0.3826	0.3902	0.3826	0.3902	0.2042
		1.5	0.2145	0.2217	0.2145	0.2217	0.0526
		2.0	0.1938	0.2029	0.1941	0.2027	0.0421
		2.5	0.1806	0.1885	0.1807	0.1883	0.0325
		3.0	0.1755	0.1873	0.177	0.1878	0.0286
	4	0.7	0.3859	0.3911	0.3859	0.3911	0.2076
		1.5	0.2159	0.2248	0.216	0.2248	0.0548
		2.0	0.1976	0.2071	0.1976	0.2074	0.0382
		2.5	0.1838	0.1935	0.1855	0.1938	0.0339
		3.0	0.1822	0.1915	0.1845	0.1931	0.0286
	5	0.7	0.383	0.3896	0.383	0.3896	0.2081
		1.5	0.2136	0.2222	0.2136	0.2222	0.0541
		2.0	0.1981	0.2061	0.1983	0.2064	0.0393
		2.5	0.1801	0.1887	0.1808	0.1892	0.03
		3.0	0.1813	0.1888	0.1817	0.1902	0.0295

* indicates that the test could not control the probability of type I error.

Table 5.12: Estimated power for $\delta = 0.9$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.0818	0.0874	0.0818	0.0874	0.062
		1.5	0.0556	0.0639	0.0555	0.0636	0.037
		2.0	*	*	0.0496	*	0.0322
		2.5	*	*	*	*	0.0296
		3.0	0.0524	0.0605	0.0524	0.061	0.0328
	3	0.7	0.0818	0.0876	0.0818	0.0876	0.065
		1.5	0.0561	0.0648	0.0563	0.0652	0.0365
		2.0	*	*	*	*	0.0352
		2.5	*	*	*	*	0.0329
		3.0	0.0514	0.0602	0.0513	0.0595	0.0336
	4	0.7	0.0822	0.0883	0.0822	0.0883	0.0638
		1.5	0.0548	0.0624	0.0549	0.0621	0.0398
		2.0	*	*	*	*	0.0348
		2.5	0.0476	*	0.0472	*	0.0288
		3.0	0.0518	*	0.0526	*	0.03
	5	0.7	0.0814	0.0881	0.0814	0.0881	0.0662
		1.5	0.05	0.0575	0.0503	0.058	0.034
		2.0	*	*	*	*	0.0315
		2.5	0.05	*	0.0503	*	0.0289
		3.0	*	*	*	*	0.03
25	2	0.7	0.1253	0.1291	0.1253	0.1291	0.0815
		1.5	0.0753	0.0821	0.0753	0.0821	0.0397
		2.0	0.0682	0.0755	0.0688	0.0756	0.0359
		2.5	0.0689	0.0763	0.0694	0.076	0.0322
		3.0	0.0648	0.071	0.0671	0.0723	0.0317
	3	0.7	0.1244	0.1302	0.1244	0.1302	0.0846
		1.5	0.0749	0.0812	0.075	0.0814	0.0402
		2.0	0.0712	*	0.0712	*	0.0377
		2.5	0.066	*	0.0659	*	0.0344
		3.0	0.0646	*	0.0644	*	0.0343
	4	0.7	0.1197	0.1234	0.1197	0.1234	0.0843
		1.5	0.0746	0.079	0.0746	0.079	0.0404
		2.0	0.0698	0.0746	0.0695	0.075	0.0365
		2.5	0.0671	0.0723	0.0669	0.0722	0.0365
		3.0	0.0636	0.0703	0.065	0.0721	0.0301
	5	0.7	0.1206	0.1249	0.1206	0.1249	0.0874
		1.5	0.0751	*	0.0752	*	0.0425
		2.0	0.0723	*	0.0727	*	0.0361
		2.5	0.0644	*	*	*	0.032
		3.0	0.0636	0.0711	0.0653	0.0718	0.0316

Table 5.12 (Continued): Estimated power for $\delta = 0.9$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	0.1761	0.1805	0.1761	0.1805	0.1112
		1.5	0.1073	0.1113	0.1074	0.1113	0.0465
		2.0	0.0942	0.0992	0.0944	0.0993	0.0374
		2.5	0.089	*	0.0895	*	0.0317
		3.0	0.0927	0.0981	0.0922	0.0981	0.0336
	3	0.7	0.1718	0.176	0.1718	0.176	0.1109
		1.5	0.1033	0.1092	0.1033	0.1093	0.0486
		2.0	0.0954	0.1005	0.0954	0.1007	0.0387
		2.5	0.0916	0.0973	0.092	0.0978	0.0337
		3.0	0.084	0.0893	0.0844	0.0898	0.0297
	4	0.7	0.1736	0.1767	0.1736	0.1767	0.1068
		1.5	0.1077	0.1117	0.1078	0.1116	0.0457
		2.0	0.0914	0.0962	0.0914	0.0963	0.0352
		2.5	0.0924	0.0982	0.0925	0.0979	0.0351
		3.0	0.0846	0.0909	0.0846	0.0908	0.0341
	5	0.7	0.1669	0.17	0.1669	0.17	0.1063
		1.5	0.1009	0.1065	0.1009	0.1065	0.0444
		2.0	0.0976	0.103	0.0978	0.1031	0.0398
		2.5	0.093	0.0982	0.0929	0.0981	0.0352
		3.0	0.091	0.0958	0.0914	0.0964	0.0323

* indicates that the test could not control the probability of type I error.

Table 5.13: Estimated power for $\delta = 0.95$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.0511	0.0559	0.0511	0.0559	0.047
		1.5	0.0463	0.0532	0.0464	0.0529	0.0439
		2.0	*	*	0.0428	*	0.0408
		2.5	*	*	*	*	0.0394
		3.0	0.0438	0.0517	0.0436	0.0497	0.0382
	3	0.7	0.0477	0.0519	0.0477	0.0519	0.0467
		1.5	0.0458	0.0518	0.0459	0.0521	0.0397
		2.0	*	*	*	*	0.0401
		2.5	*	*	*	*	0.0417
		3.0	0.0465	0.0531	0.0457	0.0535	0.0403
	4	0.7	0.0512	0.0551	0.0512	0.0551	0.0494
		1.5	0.0444	0.0499	0.0445	0.0496	0.0403
		2.0	*	*	*	*	0.0395
		2.5	0.0442	*	0.0446	*	0.0403
		3.0	0.0506	*	0.0499	*	0.0376
	5	0.7	0.048	0.0517	0.048	0.0517	0.0448
		1.5	0.0448	0.0506	0.0449	0.0505	0.0421
		2.0	*	*	*	*	0.048
		2.5	0.0421	*	0.0427	*	0.0404
		3.0	*	*	*	*	0.039
25	2	0.7	0.0616	0.0647	0.0616	0.0647	0.0501
		1.5	0.0526	0.056	0.0527	0.0559	0.0419
		2.0	0.0467	0.0507	0.047	0.0511	0.0421
		2.5	0.0487	0.0545	0.0485	0.0543	0.0403
		3.0	0.0487	0.0529	0.0489	0.0536	0.0379
	3	0.7	0.0555	0.0584	0.0555	0.0584	0.0532
		1.5	0.0524	0.0563	0.0523	0.0562	0.0404
		2.0	0.0474	*	0.0474	*	0.0418
		2.5	0.0473	*	0.0469	*	0.0392
		3.0	0.0464	*	0.0468	*	0.0387
	4	0.7	0.057	0.0602	0.057	0.0602	0.0542
		1.5	0.0509	0.0548	0.0508	0.055	0.041
		2.0	0.0483	0.052	0.0478	0.0518	0.0415
		2.5	0.0477	0.0526	0.0478	0.0523	0.0409
		3.0	0.0469	0.0539	0.0485	0.0531	0.0379
	5	0.7	0.0591	0.0615	0.0591	0.0615	0.0562
		1.5	0.0524	*	0.0524	*	0.0396
		2.0	0.047	*	0.047	*	0.0455
		2.5	0.0436	*	*	*	0.0376
		3.0	0.0485	0.0534	0.0488	0.0528	0.0416

Table 5.13 (Continued): Estimated power for $\delta = 0.95$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	0.0717	0.0738	0.0717	0.0738	0.0582
		1.5	0.0561	0.0594	0.0561	0.0595	0.0466
		2.0	0.0561	0.0593	0.0562	0.0591	0.0402
		2.5	0.0526	*	0.0529	*	0.0391
		3.0	0.051	0.0548	0.0514	0.0554	0.0387
	3	0.7	0.068	0.0699	0.068	0.0699	0.0605
		1.5	0.0559	0.0589	0.0559	0.0589	0.0434
		2.0	0.0474	0.0507	0.0476	0.0508	0.0437
		2.5	0.0527	0.0552	0.0525	0.055	0.0375
		3.0	0.0487	0.0523	0.0479	0.0516	0.039
	4	0.7	0.0701	0.0723	0.0701	0.0723	0.0586
		1.5	0.0543	0.0583	0.0543	0.0583	0.0427
		2.0	0.0509	0.0528	0.0508	0.0526	0.0421
		2.5	0.051	0.0551	0.0514	0.0552	0.0405
		3.0	0.0523	0.0556	0.0523	0.0557	0.0383
	5	0.7	0.0703	0.0722	0.0703	0.0722	0.0591
		1.5	0.0575	0.059	0.0575	0.059	0.0439
		2.0	0.0537	0.0575	0.0538	0.0574	0.0422
		2.5	0.0487	0.0526	0.0499	0.0529	0.0419
		3.0	0.0492	0.0529	0.0492	0.0538	0.0391

* indicates that the test could not control the probability of type I error.

Table 5.14: Estimated power for $\delta = 1.1$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.1798	0.1883	0.1798	0.1883	0.1181
		1.5	0.1607	0.172	0.161	0.1725	0.0832
		2.0	*	*	0.134	*	0.0796
		2.5	*	*	*	*	0.0726
		3.0	0.1523	0.1645	0.1527	0.1646	0.0724
	3	0.7	0.1818	0.1905	0.1818	0.1905	0.1135
		1.5	0.1497	0.1631	0.15	0.1629	0.0875
		2.0	*	*	*	*	0.0821
		2.5	*	*	*	*	0.0744
		3.0	0.1433	0.1561	0.1431	0.1554	0.0777
	4	0.7	0.1791	0.1855	0.1791	0.1855	0.1138
		1.5	0.1556	0.1672	0.1558	0.1672	0.0813
		2.0	*	*	*	*	0.083
		2.5	0.1486	*	0.1504	*	0.0747
		3.0	0.151	*	0.1516	*	0.0795
	5	0.7	0.1828	0.1915	0.1828	0.1915	0.1155
		1.5	0.152	0.1634	0.1511	0.1626	0.0816
		2.0	*	*	*	*	0.0819
		2.5	0.1535	*	0.1553	*	0.0738
		3.0	*	*	*	*	0.0764
25	2	0.7	0.2214	0.2261	0.2214	0.2261	0.1335
		1.5	0.1787	0.1868	0.1789	0.1869	0.0892
		2.0	0.1701	0.1769	0.1696	0.1768	0.0794
		2.5	0.1689	0.1778	0.1684	0.1778	0.079
		3.0	0.1626	0.1707	0.1623	0.1696	0.0736
	3	0.7	0.228	0.2333	0.228	0.2333	0.1426
		1.5	0.1765	0.185	0.1766	0.1849	0.0863
		2.0	0.1668	*	0.1671	*	0.0793
		2.5	0.17	*	0.1702	*	0.0806
		3.0	0.1674	*	0.1681	*	0.0786
	4	0.7	0.2188	0.2238	0.2188	0.2238	0.1348
		1.5	0.1717	0.1804	0.1718	0.1801	0.0936
		2.0	0.1661	0.1755	0.1661	0.1754	0.0835
		2.5	0.1644	0.1729	0.165	0.1726	0.087
		3.0	0.1695	0.179	0.1708	0.1795	0.0806
	5	0.7	0.2221	0.2273	0.2221	0.2273	0.1397
		1.5	0.1757	*	0.1759	*	0.0909
		2.0	0.1716	*	0.1717	*	0.0822
		2.5	0.1658	*	*	*	0.0768
		3.0	0.1716	0.1789	0.1715	0.1815	0.0779

Table 5.14 (Continued): Estimated power for $\delta = 1.1$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	0.2659	0.2705	0.2659	0.2705	0.1648
		1.5	0.1994	0.2058	0.1995	0.2058	0.0906
		2.0	0.1915	0.1986	0.1917	0.1987	0.0871
		2.5	0.1869	*	0.1877	*	0.0847
		3.0	0.1892	0.1962	0.1896	0.1968	0.0785
	3	0.7	0.2689	0.2729	0.2689	0.2729	0.166
		1.5	0.2008	0.2067	0.2008	0.2067	0.0942
		2.0	0.1986	0.2052	0.1987	0.2052	0.0913
		2.5	0.19	0.1949	0.1903	0.1953	0.0864
		3.0	0.1882	0.1948	0.189	0.1948	0.0811
	4	0.7	0.2654	0.2685	0.2654	0.2685	0.15
		1.5	0.1979	0.2037	0.1978	0.2037	0.0933
		2.0	0.1888	0.1954	0.1888	0.1954	0.0868
		2.5	0.1899	0.1958	0.19	0.1962	0.0771
		3.0	0.1772	0.1847	0.1787	0.1846	0.0792
	5	0.7	0.2726	0.2778	0.2726	0.2778	0.1649
		1.5	0.2065	0.2116	0.2065	0.2116	0.0935
		2.0	0.2019	0.2083	0.2018	0.2086	0.0835
		2.5	0.1924	0.1987	0.1923	0.1986	0.0868
		3.0	0.1904	0.1962	0.1901	0.1975	0.0794

* indicates that the test could not control the probability of type I error.

Table 5.15: Estimated power for $\delta = 1.2$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.4198	0.4302	0.4198	0.4302	0.2479
		1.5	0.3252	0.3393	0.3256	0.3392	0.1373
		2.0	*	*	0.3092	*	0.1165
		2.5	*	*	*	*	0.1103
		3.0	0.2898	0.3061	0.2911	0.3098	0.1065
	3	0.7	0.4217	0.4308	0.4217	0.4308	0.2495
		1.5	0.3252	0.3407	0.3253	0.3407	0.1418
		2.0	*	*	*	*	0.1237
		2.5	*	*	*	*	0.1122
		3.0	0.2886	0.3063	0.2917	0.3101	0.1121
	4	0.7	0.4128	0.4245	0.4128	0.4245	0.2554
		1.5	0.3222	0.3383	0.3227	0.3381	0.1383
		2.0	*	*	*	*	0.1154
		2.5	0.2951	*	0.2987	*	0.1138
		3.0	0.299	*	0.302	*	0.1077
	5	0.7	0.4216	0.4313	0.4216	0.4313	0.2541
		1.5	0.3186	0.3334	0.3184	0.3336	0.1334
		2.0	*	*	*	*	0.12
		2.5	0.3002	*	0.3032	*	0.1135
		3.0	*	*	*	*	0.1089
25	2	0.7	0.5484	0.5551	0.5484	0.5551	0.3307
		1.5	0.4096	0.4204	0.4098	0.4203	0.1564
		2.0	0.3848	0.398	0.386	0.3982	0.133
		2.5	0.3716	0.3848	0.3735	0.3854	0.1171
		3.0	0.3684	0.38	0.3697	0.383	0.1117
	3	0.7	0.5406	0.548	0.5406	0.548	0.3321
		1.5	0.4012	0.412	0.4014	0.4121	0.1527
		2.0	0.3802	*	0.3804	*	0.1315
		2.5	0.385	*	0.386	*	0.1246
		3.0	0.3769	*	0.3803	*	0.1128
	4	0.7	0.545	0.5519	0.545	0.5519	0.3327
		1.5	0.4062	0.4182	0.4063	0.4183	0.1579
		2.0	0.3913	0.4015	0.3916	0.4016	0.1277
		2.5	0.3817	0.3904	0.3827	0.3917	0.1226
		3.0	0.3668	0.3784	0.3687	0.3795	0.1153
	5	0.7	0.5406	0.5463	0.5406	0.5463	0.3337
		1.5	0.4077	*	0.4078	*	0.1537
		2.0	0.3866	*	0.3868	*	0.1351
		2.5	0.3735	*	*	*	0.127
		3.0	0.3723	0.3829	0.3732	0.3846	0.1146

Table 5.15 (Continued): Estimated power for $\delta = 1.2$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	0.6519	0.6574	0.6519	0.6574	0.4164
		1.5	0.4978	0.5055	0.4978	0.5055	0.1757
		2.0	0.4695	0.4787	0.4697	0.4789	0.141
		2.5	0.4576	*	0.4579	*	0.1297
		3.0	0.4461	0.4555	0.4469	0.4561	0.1172
	3	0.7	0.6684	0.673	0.6684	0.673	0.4144
		1.5	0.496	0.5022	0.496	0.5023	0.1745
		2.0	0.473	0.4809	0.4729	0.4812	0.1441
		2.5	0.4667	0.4768	0.4683	0.4773	0.1327
		3.0	0.4516	0.4615	0.4545	0.4627	0.1224
	4	0.7	0.6508	0.6553	0.6508	0.6553	0.4104
		1.5	0.4866	0.4947	0.4866	0.4947	0.1718
		2.0	0.4665	0.474	0.4664	0.474	0.1387
		2.5	0.4569	0.4662	0.4577	0.4666	0.1259
		3.0	0.4561	0.4651	0.4574	0.4659	0.12
	5	0.7	0.6671	0.671	0.6671	0.671	0.4216
		1.5	0.4867	0.4938	0.4867	0.4938	0.174
		2.0	0.4679	0.4766	0.4679	0.477	0.1423
		2.5	0.4504	0.4588	0.4504	0.4589	0.1298
		3.0	0.4473	0.4557	0.4501	0.4578	0.1231

* indicates that the test could not control the probability of type I error.

Table 5.16: Estimated power for $\delta = 1.3$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.6641	0.6732	0.6641	0.6732	0.4396
		1.5	0.5116	0.5291	0.5124	0.5297	0.2066
		2.0	*	*	0.49	*	0.1764
		2.5	*	*	*	*	0.152
		3.0	0.4737	0.4893	0.4772	0.4942	0.1471
	3	0.7	0.6693	0.6784	0.6693	0.6784	0.4297
		1.5	0.5051	0.5204	0.5049	0.5213	0.1978
		2.0	*	*	*	*	0.1654
		2.5	*	*	*	*	0.1605
		3.0	0.4656	0.482	0.4713	0.488	0.144
	4	0.7	0.6649	0.6724	0.6649	0.6724	0.4316
		1.5	0.5212	0.5359	0.5214	0.5362	0.2065
		2.0	*	*	*	*	0.1756
		2.5	0.4726	*	0.4759	*	0.1526
		3.0	0.4709	*	0.4788	*	0.1558
	5	0.7	0.6624	0.6717	0.6624	0.6717	0.4213
		1.5	0.5068	0.5208	0.5069	0.5209	0.2043
		2.0	*	*	*	*	0.1718
		2.5	0.477	*	0.4813	*	0.1587
		3.0	*	*	*	*	0.1457
25	2	0.7	0.8103	0.8147	0.8103	0.8147	0.5545
		1.5	0.6385	0.6471	0.6389	0.6471	0.2453
		2.0	0.6224	0.6335	0.6236	0.6334	0.2007
		2.5	0.6074	0.6164	0.6088	0.6188	0.1762
		3.0	0.5899	0.6006	0.5936	0.6051	0.1662
	3	0.7	0.8051	0.8098	0.8051	0.8098	0.553
		1.5	0.6392	0.6482	0.6395	0.6482	0.2515
		2.0	0.6148	*	0.6156	*	0.1927
		2.5	0.5955	*	0.5988	*	0.1759
		3.0	0.5899	*	0.594	*	0.1646
	4	0.7	0.8072	0.8117	0.8072	0.8117	0.5636
		1.5	0.6379	0.6465	0.6379	0.6464	0.2442
		2.0	0.6163	0.626	0.6171	0.6261	0.1996
		2.5	0.6009	0.6125	0.6029	0.6139	0.17
		3.0	0.6019	0.614	0.605	0.6159	0.163
	5	0.7	0.8037	0.8093	0.8037	0.8093	0.5566
		1.5	0.6463	*	0.6465	*	0.2412
		2.0	0.6153	*	0.616	*	0.1953
		2.5	0.5984	*	*	*	0.1626
		3.0	0.6008	0.612	0.6065	0.6175	0.1536

Table 5.16 (Continued): Estimated power for $\delta = 1.3$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	0.9018	0.9035	0.9018	0.9035	0.6727
		1.5	0.7513	0.7576	0.7513	0.7576	0.279
		2.0	0.7293	0.7363	0.7292	0.7363	0.2252
		2.5	0.7132	*	0.7134	*	0.1905
		3.0	0.6979	0.7051	0.7007	0.7066	0.1701
	3	0.7	0.9044	0.9057	0.9044	0.9057	0.6736
		1.5	0.7559	0.7615	0.7559	0.7615	0.2861
		2.0	0.7255	0.7328	0.7259	0.7326	0.2225
		2.5	0.7071	0.715	0.7086	0.7159	0.1875
		3.0	0.6965	0.7032	0.6979	0.7056	0.1705
	4	0.7	0.8977	0.9002	0.8977	0.9002	0.6743
		1.5	0.7508	0.7568	0.7508	0.7568	0.2919
		2.0	0.7248	0.7325	0.7251	0.7328	0.2211
		2.5	0.7163	0.7224	0.7163	0.7235	0.1814
		3.0	0.7137	0.7195	0.7142	0.7206	0.1707
	5	0.7	0.9022	0.9042	0.9022	0.9042	0.6727
		1.5	0.7547	0.7583	0.7547	0.7583	0.2825
		2.0	0.7171	0.7235	0.7175	0.724	0.2161
		2.5	0.7037	0.7116	0.706	0.7122	0.1932
		3.0	0.7027	0.7102	0.7051	0.7108	0.1707

* indicates that the test could not control the probability of type I error.

Table 5.17: Estimated power for $\delta = 1.4$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.8307	0.8367	0.8307	0.8367	0.5858
		1.5	0.6719	0.683	0.6724	0.684	0.28
		2.0	*	*	0.6492	*	0.2303
		2.5	*	*	*	*	0.2071
		3.0	0.6288	0.6456	0.6344	0.6508	0.1942
	3	0.7	0.8289	0.835	0.8289	0.835	0.5978
		1.5	0.6715	0.6842	0.6726	0.6851	0.2782
		2.0	*	*	*	*	0.2291
		2.5	*	*	*	*	0.2074
		3.0	0.6354	0.6523	0.642	0.6555	0.1968
	4	0.7	0.8311	0.8378	0.8311	0.8378	0.5946
		1.5	0.6736	0.688	0.6747	0.6882	0.2752
		2.0	*	*	*	*	0.2267
		2.5	0.6356	*	0.6431	*	0.2053
		3.0	0.6295	*	0.636	*	0.1885
	5	0.7	0.8246	0.8304	0.8246	0.8304	0.5882
		1.5	0.6711	0.6841	0.6712	0.6848	0.2731
		2.0	*	*	*	*	0.2282
		2.5	0.6365	*	0.6403	*	0.208
		3.0	*	*	*	*	0.1903
25	2	0.7	0.9382	0.9402	0.9382	0.9402	0.7438
		1.5	0.8216	0.8286	0.8216	0.8286	0.3335
		2.0	0.7964	0.8024	0.7971	0.8031	0.2709
		2.5	0.7829	0.79	0.784	0.7923	0.2297
		3.0	0.7619	0.7693	0.7638	0.7706	0.21
	3	0.7	0.9404	0.9422	0.9404	0.9422	0.7387
		1.5	0.8114	0.819	0.8115	0.8191	0.3318
		2.0	0.7922	*	0.7922	*	0.2699
		2.5	0.775	*	0.7779	*	0.2351
		3.0	0.7668	*	0.7682	*	0.2023
	4	0.7	0.9397	0.9414	0.9397	0.9414	0.7406
		1.5	0.8155	0.8229	0.8156	0.8229	0.3375
		2.0	0.7922	0.7996	0.7922	0.7999	0.2642
		2.5	0.7769	0.7857	0.7797	0.7869	0.2278
		3.0	0.7815	0.7892	0.7854	0.792	0.2105
	5	0.7	0.9413	0.9429	0.9413	0.9429	0.7392
		1.5	0.8106	*	0.8106	*	0.3444
		2.0	0.7875	*	0.7886	*	0.2595
		2.5	0.7766	*	*	*	0.2247
		3.0	0.7681	0.7766	0.7731	0.78	0.2068

Table 5.17 (Continued): Estimated power for $\delta = 1.4$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	0.9808	0.9813	0.9808	0.9813	0.8473
		1.5	0.9034	0.9064	0.9034	0.9064	0.4042
		2.0	0.8841	0.8875	0.8841	0.8877	0.2923
		2.5	0.8748	*	0.8759	*	0.2579
		3.0	0.866	0.8696	0.867	0.8701	0.225
	3	0.7	0.9836	0.9837	0.9836	0.9837	0.8473
		1.5	0.9067	0.9097	0.9067	0.9098	0.4048
		2.0	0.8857	0.8894	0.8856	0.8893	0.3042
		2.5	0.8789	0.8825	0.8793	0.8827	0.2579
		3.0	0.8685	0.8723	0.87	0.8744	0.2286
	4	0.7	0.9813	0.9816	0.9813	0.9816	0.8448
		1.5	0.9078	0.9107	0.908	0.9107	0.3948
		2.0	0.8773	0.8812	0.8775	0.8812	0.3004
		2.5	0.8695	0.8736	0.8702	0.8747	0.2461
		3.0	0.8653	0.8693	0.8664	0.8703	0.2296
	5	0.7	0.9814	0.9815	0.9814	0.9815	0.8521
		1.5	0.9063	0.9108	0.9063	0.9108	0.3985
		2.0	0.8899	0.8929	0.8902	0.893	0.3037
		2.5	0.8813	0.8845	0.8821	0.8848	0.2478
		3.0	0.8699	0.8743	0.8723	0.8757	0.2314

* indicates that the test could not control the probability of type I error.

Table 5.18: Estimated power for $\delta = 1.5$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
16	2	0.7	0.9208	0.9232	0.9208	0.9232	0.7245
		1.5	0.8014	0.8097	0.802	0.8105	0.358
		2.0	*	*	0.7767	*	0.2798
		2.5	*	*	*	*	0.2529
		3.0	0.7579	0.7702	0.7632	0.7785	0.233
	3	0.7	0.9268	0.9304	0.9268	0.9304	0.7312
		1.5	0.795	0.8042	0.7951	0.8042	0.3512
		2.0	*	*	*	*	0.2915
		2.5	*	*	*	*	0.256
		3.0	0.7548	0.7666	0.7615	0.7724	0.232
	4	0.7	0.9241	0.9265	0.9241	0.9265	0.7255
		1.5	0.7971	0.8072	0.798	0.8076	0.3494
		2.0	*	*	*	*	0.2798
		2.5	0.7653	*	0.7702	*	0.2517
		3.0	0.7486	*	0.757	*	0.2326
	5	0.7	0.9252	0.9275	0.9252	0.9275	0.7369
		1.5	0.7947	0.8043	0.7953	0.8045	0.3494
		2.0	*	*	*	*	0.2801
		2.5	0.7584	*	0.7633	*	0.2531
		3.0	*	*	*	*	0.2345
25	2	0.7	0.9803	0.9809	0.9803	0.9809	0.8576
		1.5	0.9091	0.9115	0.9091	0.9115	0.4162
		2.0	0.8969	0.9009	0.8978	0.9011	0.3398
		2.5	0.8889	0.8928	0.8903	0.8937	0.283
		3.0	0.8823	0.887	0.8854	0.8899	0.2534
	3	0.7	0.9828	0.9834	0.9828	0.9834	0.8626
		1.5	0.9204	0.9235	0.9205	0.9236	0.428
		2.0	0.8979	*	0.8983	*	0.3271
		2.5	0.8827	*	0.8841	*	0.2901
		3.0	0.8781	*	0.881	*	0.2585
	4	0.7	0.9829	0.9836	0.9829	0.9836	0.8566
		1.5	0.9181	0.9215	0.9181	0.9213	0.43
		2.0	0.8924	0.8982	0.8933	0.8986	0.3266
		2.5	0.8906	0.8964	0.8923	0.897	0.2882
		3.0	0.8788	0.8839	0.882	0.8871	0.2611
	5	0.7	0.9815	0.9819	0.9815	0.9819	0.8597
		1.5	0.9129	*	0.9131	*	0.4304
		2.0	0.8934	*	0.8942	*	0.334
		2.5	0.8815	*	*	*	0.277
		3.0	0.8829	0.8886	0.8861	0.8903	0.2563

Table 5.18 (Continued): Estimated power for $\delta = 1.5$ and $\alpha = 0.05$.

n	μ_0	b	Test statistics				
			T_k	T_{gh}	T_{km}	T_{ghm}	T_b
35	2	0.7	0.9973	0.9974	0.9973	0.9974	0.9329
		1.5	0.9675	0.9689	0.9675	0.9689	0.5125
		2.0	0.9584	0.9601	0.9587	0.9602	0.3854
		2.5	0.9542	*	0.9552	*	0.3218
		3.0	0.9486	0.9511	0.9495	0.9522	0.281
	3	0.7	0.997	0.997	0.997	0.997	0.9335
		1.5	0.9694	0.9702	0.9694	0.9702	0.5073
		2.0	0.9593	0.9605	0.9594	0.9604	0.3829
		2.5	0.952	0.9537	0.9526	0.9544	0.3169
		3.0	0.9535	0.9553	0.9548	0.9564	0.2834
	4	0.7	0.9959	0.9959	0.9959	0.9959	0.9293
		1.5	0.9705	0.9713	0.9705	0.9713	0.5069
		2.0	0.9594	0.961	0.9594	0.9608	0.378
		2.5	0.9501	0.952	0.9506	0.9525	0.3186
		3.0	0.9523	0.9539	0.953	0.9551	0.281
	5	0.7	0.9975	0.9975	0.9975	0.9975	0.9367
		1.5	0.9669	0.9682	0.9669	0.9682	0.5049
		2.0	0.9584	0.9598	0.9587	0.9601	0.3867
		2.5	0.9535	0.9552	0.9536	0.9555	0.3206
		3.0	0.9488	0.9503	0.9493	0.9511	0.2863

* indicates that the test could not control the probability of type I error.

From Tables 5.11-5.18, we can see that the power increases with b , n and δ but it does not depended on the choices of μ_0. There is a little difference in the powers of the T_k , T_{km} tests and T_{gh} , T_{ghm} tests when the values of the coefficient of variation are small. Otherwise, the T_{ghm} procedure appears to be most powerful.

5.3 Graph of Power Comparison

The graphs of power comparisons are presented by the selected values of n, b, μ_0, δ, and α.

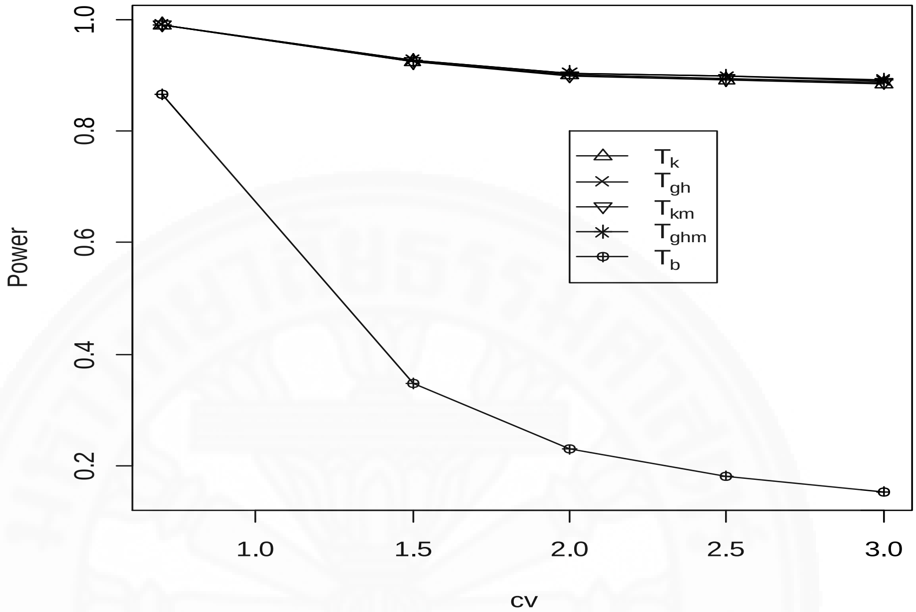

Figure 5.1: Estimated power comparison of the five tests when n = 35, μ_0 = 2, δ = 1.5, and α = 0.01.

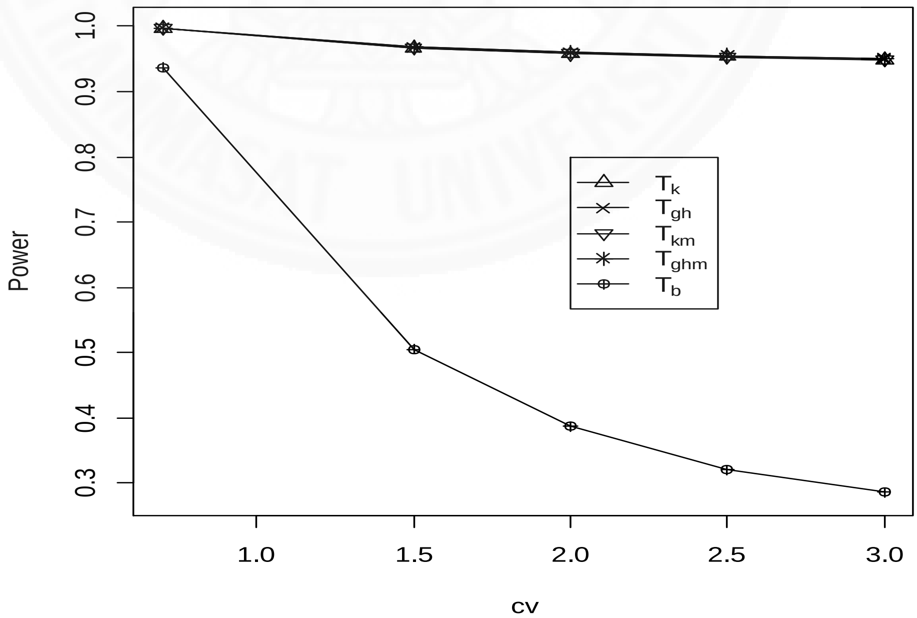

Figure 5.2: Estimated power comparison of the five tests when n = 35, μ_0 = 5, δ = 1.5, and α = 0.05.

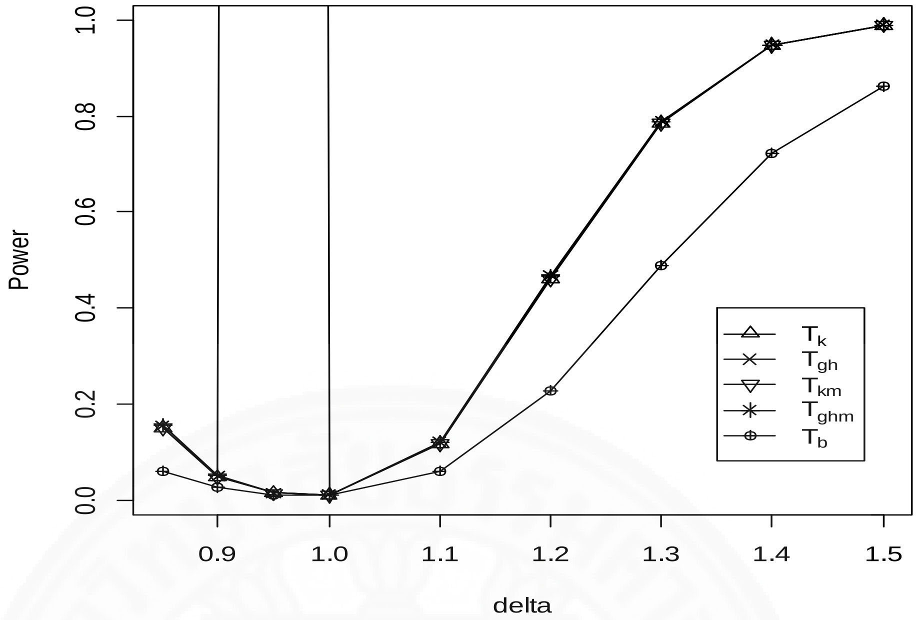

Figure 5.3: Estimated power comparison of the five tests when $n = 35$, $\mu_0 = 4$, $b = 0.7$, and $\alpha = 0.01$.

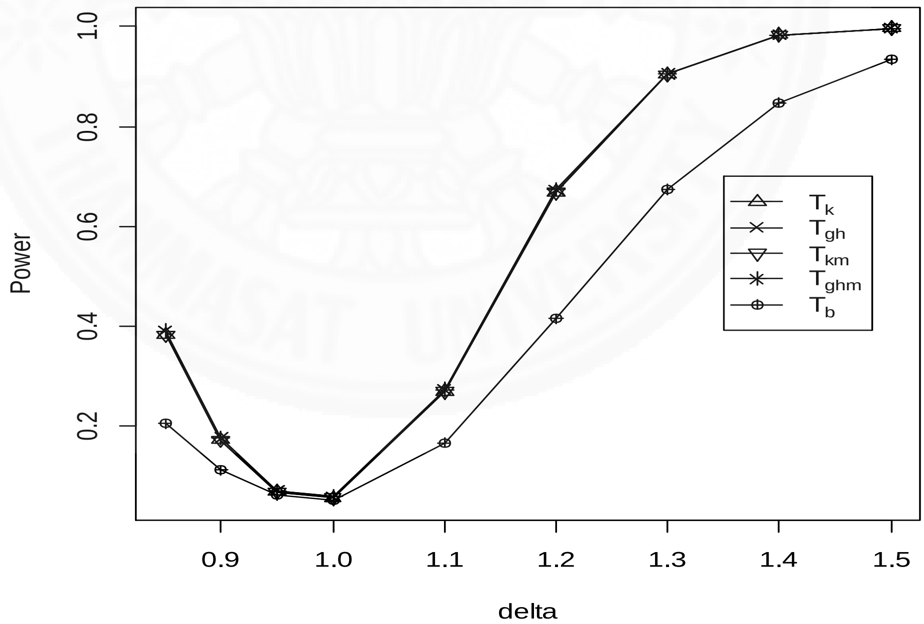

Figure 5.4: Estimated power comparison of the five tests when $n = 35$, $\mu_0 = 3$, $b = 0.7$, and $\alpha = 0.05$.

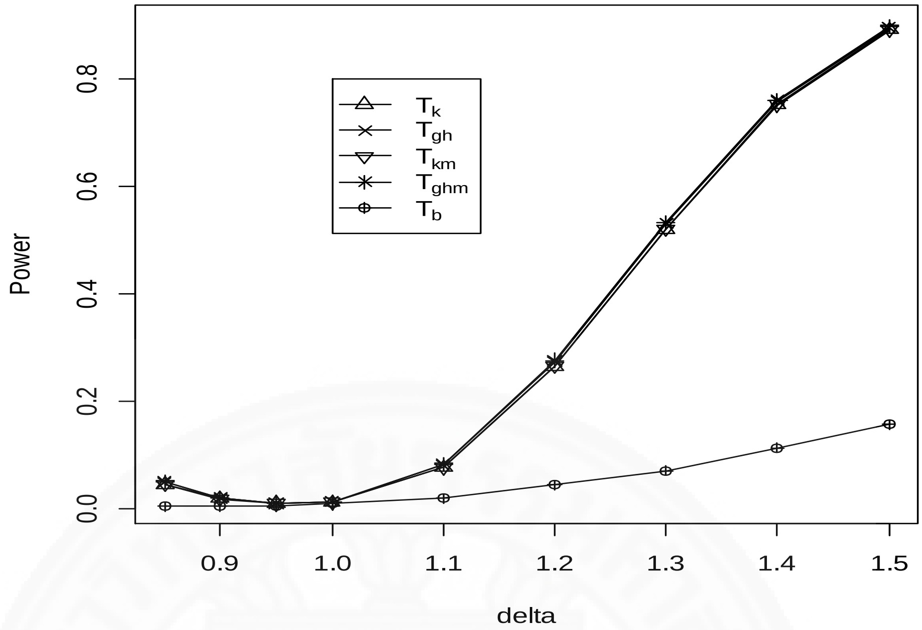

Figure 5.5: Estimated power comparison of the five tests when $n = 35$, $\mu_0 = 4$, b = 3.0, and $\alpha = 0.01$.

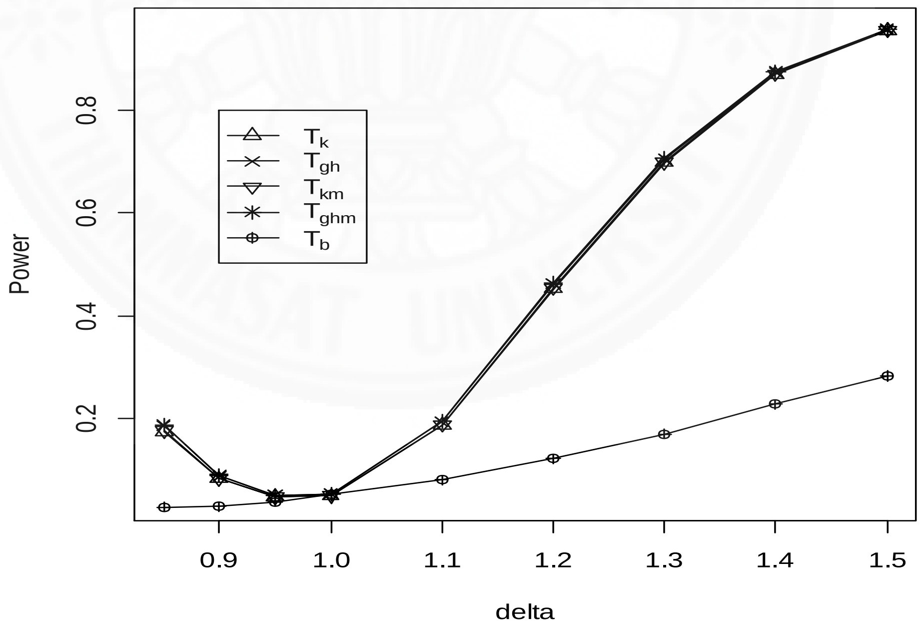

Figure 5.6: Estimated power comparison of the five tests when $n = 35$, $\mu_0 = 3$, b = 3.0, and $\alpha = 0.01$.

CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The objectives of this research are to investigate theoretical properties of particular test statistics, propose a test statistic for normal mean using information on the coefficient of variation, estimate the probability of probability of type I error and the power of the tests.

For the theoretical part, we investigate Khan's (1968) test statistic. We consider the properties of the point estimator for normal mean when the coefficient of variation is known. First, we prove that this point estimator is unbiased. Second, we examined this point estimator has minimum variance among all unbiased estimators. Next, we argue that this estimator is asymptotically normal and hence consistent. Most importantly, we propose tests based on the best unbiased estimator suggested by Khan (1968), the uniformly minimum risk estimator suggested by Gleser, & Healy (1976), the modified estimator improved by Khan (1968), the modified estimator of Gleser, & Healy (1976), and the sample mean with known coefficient of variation.

In the computation part, we calculate the probability of type I error and the power performance of the proposed tests. We choose the criterion of Cochran (1954) to consider the capacity in controlling the probability of type I error of the proposed tests regarding the nominal level α. The powers of the tests that pass the criterion of Cochran (1954) are compared in each situation. The normal distribution random samples with known coefficient of variation b are generated for the various values of sample size n, mean μ_0, δ and the nominal significance level α using Monte Carlo simulation.

By the Central Limit Theorem we know that the sampling distribution of the mean is approximately normal if the sample size is large enough ($n \geq 30$). In this thesis, we study the efficiency of these test statistics when the sample is small and compare simulation results when the sample size is large ($n = 35$).

For the computational part, the results indicated that the proposed tests tend to be able to control the probability of type I error in a small value of b and a large value of n. The power comparison indicates that when the sample size is small and the coefficient of variation is large, there is little different powers of T_k, T_{km} tests and T_{gh}, T_{ghm} tests. Otherwise, the T_{ghm} test becomes the most powerful. And the T_b test is not recommended in almost all situations except for n = 16, μ_0 = 5 and α = 0.05.

Furthermore, we notice that, the powers of all tests are affected by the values of n, b and δ. Namely, the powers of all tests are higher when the values of n, δ increase and the value of b decrease. In contrast, the powers are lower when the values of b decrease, but the values of n and δ increase. In addition, the powers of all tests are close to each other for a small value of b (b = 0.7) and the large values of n and δ (δ = 1.5, n = 35).

6.2 Future Research

For the future research, we could mention the following ideas:

1. In this thesis, we studied test statistics only. For the future study, we suggest to construct confidence intervals for these proposed tests.

2. We can construct the other tests for a normal mean with the known coefficient of variation and use the alternative approach to construct the tests.

3. It is interesting to apply the bootstrap procedure for the estimation of the mean of a normal population with the known coefficient of variation.

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APPENDICES

APPENDIX A
REFERENCE OF THEORITICAL PART

Definition A.1 The *expected value* or mean of a random variable $g(X)$, denoted by $E(g(X))$, is

$$E(g(X)) = \int_{-\infty}^{\infty} g(x) f(x) dx \text{, if } X \text{ is continuous or}$$

$$E(g(X)) = \sum_{x \in X} g(x) P(X = x) \text{, if } X \text{ is discrete,}$$

provided that the integral or sum exist. If $E|g(X)| = \infty$, we say that $E(g(X))$ does not exist.

Theorem A.1 Let X be a random variable and let a, b, and c be constants. Then for any functions $g_1(x)$ and $g_2(x)$ whose expectations exist,

$$E(ag_1(x) + bg_2(x) + c) = a(Eg_1(x)) + bE(g_2(x)) + c .$$

Definition A.2 The *variance* of a random variable X is its second central moment, $Var(X) = E(X - E(X))^2$. The positive square root of $Var(X)$ is the *standard deviation* of X .

Theorem A.2 If X is a random variable with finite variance, then for any constant a and b , $Var(aX + b) = a^2 Var(X)$. It is sometimes easier to use an alternative formula for the variance, given by

$$Var(X) = E(X^2) - E(X)^2 .$$

(Cassella, & Berger, 2002, pp. 57-61)

APPENDIX B
PROVING THE EXPECTATION AND VARIANCE

Recall again, a point estimator $T_{LMMS} = c_1T_1 + c_2T_2$; then, we take the expected value to T_{LMMS} , we get

$$E(T_{LMMS}) = E(c_1T_1 + c_2T_2) = c_1E(T_1) + c_2E(T_2).$$

We know that $E(T_1) = \mu$ and $E(T_2) = \mu$. Thus,

$$E(T_{LMMS}) = c_1\mu + c_2\mu .$$

Therefore, we obtain

$$E(T_{LMMS}) = (c_1 + c_2)\mu . \tag{B.1}$$

Next, we take the variance to T_{LMMS} , we get

$$Var(T_{LMMS}) = Var(c_1T_1 + c_2T_2) = c_1^2Var(T_1) + c_2^2Var(T_2).$$

Since,

$$Var(T_1) = v_1\mu^2 \text{ and } Var(T_2) = v_2\mu^2 .$$

Thus,

$$Var(T_{LMMS}) = c_1^2v_1\mu^2 + c_2^2v_2\mu^2 .$$

Therefore,

$$Var(T_{LMMS}) = (c_1^2v_1 + c_2^2v_2)\mu^2 . \tag{B.2}$$

Similarity, we also obtain

$$E(T_{LMMS}^*) = (c_1^* + c_2^*)\mu , \tag{B.3}$$

and

$$Var(T_{LMMS}) = \left(c_1^{*2} v_1^* + c_2^{*2} v_2\right)\mu^2 . \tag{B.4}$$

APPENDIX C
R RPOGRAM FOR SIMULATIONS

R Code for generating random variables to estimate the probability of Type I errors and powers of the test statistics for a normal mean with known coefficient of variation.

```
Results<-function (delta,alpha,M)
{
n <- c(16,25,35)
b <- c(0.7, 1.5, 2, 2.5, 3.0)
mu0 <-c(2,3,4,5)
a <- b^2

temp1 <- rep(0,M)
temp2 <- rep(0,M)
temp3 <- rep(0,M)
temp4 <- rep(0,M)
temp5 <- rep(0,M)

for (i in 1:length(n)){
for (j in 1:length(a)){

cn <- (gamma((n[i]-1)/2)/gamma(n[i]/2))*sqrt(n[i]/(2*a[j]))
lambda<-sqrt(n[i]/a[j])
cum.lambda <-(2*pnorm(lambda)-1+(2/lambda)*dnorm(lambda))
beta <-1/cum.lambda

v1 <- a[j]/n[i]
v1.mod <-((beta^2)-1)+((beta^2)*v1)
v2 <-((v1*(n[i]-1))*(cn^2))-1
```

```
alpha1 <-v2/(v1+v2)
c <- v2/(v1.mod+v2)

c1 <-v2/(v1+v2+(v1*v2))
c2<- v1/(v1+v2+(v1*v2))

c1.mod <- v2/(v1.mod+v2+(v1.mod*v2))
c2.mod <- v1.mod/(v1.mod+v2+(v1.mod*v2))

for (k in 1:length(mu0)){
  for (l in 1:M){
      mu.x <- delta* mu0[k]
      x <- rnorm(n[i], mu.x, mu.x*sqrt(a[j]))
      x.bar <- mean(x)
      x.sd <- sd(x)

      T1 <- x.bar
      T1.mod <- beta*abs(x.bar)
      T2 <- cn*x.sd

  d <- (alpha1*T1)+(1-alpha1)*T2
  TLMMS <- (c1*T1)+(c2*T2)
  dstar <-(c*T1.mod)+(1-c)*T2
  Tmod <- (c1.mod*T1.mod)+(c2.mod*T2)
  xbar <- T1

var.d <-((v1*v2)/(v1+v2))*(mu0[k]^2)
var.TLMMS <-((c1^2)*v1+(c2^2)*v2)*(mu0[k]^2)
var.dstar <- ((v1.mod*v2)/(v1.mod+v2))*(mu0[k]^2)
var.Tmod<-(((c1.mod^2)*v1.mod)+(c2.mod^2)*v2)*(mu0[k]^2)
var.xbar <-v1*(mu0[k]^2)
```

```
Tk <-(d-mu0[k])/sqrt(var.d)
Tgh<-((TLMMS/(c1+c2))-mu0[k])/sqrt(var.TLMMS)
Tkm<-(dstar-mu0[k])/sqrt(var.dstar)
Tghm <-((Tmod/(c1.mod+c2.mod))-mu0[k])/sqrt(var.Tmod)
Tb <- (xbar-mu0[k])/sqrt(var.xbar)

if((Tk<=qnorm(alpha/2))||(Tk>=qnorm(1-alpha/2))){temp1[l]<-1}
else{temp1[l]<-0}
if((Tgh <=qnorm(alpha/2))||( Tgh >=qnorm(1-alpha/2))){temp2[l]<-1}
else{temp2[l]<-0}
if((Tkm <=qnorm(alpha/2))||( Tkm >=qnorm(1-alpha/2))){temp3[l]<-1}
else{temp3[l]<-0}
if((Tghm <=qnorm(alpha/2))||( Tghm >=qnorm(1-alpha/2))){temp4[l]<-1}
else{temp4[l]<-0}
if((Tb <=qnorm(alpha/2))||( Tb >=qnorm(1-alpha/2))){temp5[l]<-1}
else{temp5[l]<-0}

} #end loop l

cat("n =",n[i],", a =",a[j],", mu0 =",mu0[k],"\n")

if (delta ==1) {Label <- "Type I error of Tk = "}
else {Label <- "Power of Tk = "}
out1<- mean(temp1)
cat(Label, out1)
cat("\n")

if (delta ==1) {Label <- "Type I error of Tgh = "}
else {Label <- "Power of Tgh = "}
out2<- mean(temp2)
cat(Label,out2)
```

```
cat("\n")

if (delta ==1) {Label <- "Type I error of Tkm = "}
else {Label <- "Power of Tkm = "}
out3<- mean(temp3)
cat(Label, out3)
cat("\n")

if (delta ==1) {Label <- "Type I error of Tghm = "}
else {Label <- "Power of Tghm = "}
out4<- mean(temp4)
cat(Label, out4)
cat("\n")

if (delta ==1) {Label <- "Type I error of Tb = "}
else {Label <- "Power of Tb = "}
out5<- mean(temp5)
cat(Label, out5)
cat("\n\n")

} #end loop k
} #end loop j
} #end loop i
}
Results (delta,alpha,M)
```

BIOGRAPHY

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