CHANNEL ALLOCATION POLICY FOR DISTRIBUTED WIRELESS NETWORKS: DERIVATION AND ANALYSIS OF OPTIMAL INTERFERENCE

BY

MR. AMULYA BHATTARAI

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY (ENGINEERING AND TECHNOLOGY)

SIRINDHORN INTERNATIONAL INSTITUTE OF TECHNOLOGY

THAMMASAT UNIVERSITY

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DISSERTATION

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ENTITLED

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Thesis Title	CHANNEL ALLOCATION POLICY FOR DISTRIBUTED WIRELESS NETWORKS: DERIVATION AND ANALYSIS OF OPTIMAL INTERFERENCE
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ABSTRACT

Distributed wireless networks with smart (independent and rational) users are getting popular and researchers are studying decentralized equilibrium solutions to analyze and predict the converged solution of such networks. Game theoretic solutions like Nash Equilibrium (NE) is a promising means for resource (channel) allocation in such networks. In our work we study and analyze the distribution of the different network metrics like link distance, interference from/to other links, signal to interference plus noise ratio (SINR) and infer the characteristics of NE at high total throughput. It was found that at NE^{Good} (with high total throughput) communication links still have significant amount of interference and adding an interference-received term with an optimal weight (α^*_{opt}) to the link's payoff can push the distributed network to converge to NE^{Good}.

Our network consists of N links which share C channels and $C < N$ (limited resources). Each link consists of a transmitter and a receiver which are randomly

located. The transmitter of a link has a direct connection with the receiver of the link which results in several links to overlap and generally leads to a dense network with considerable amount of interference. A practical application of our work is when smart devices in a room, hall or concert arena have a direct communication with other smart devices in the area using limited bandwidth.

α^*_{opt} corresponds to the value of α which results in maximum total throughput. α^*_{opt} is obtained by running many random simulations for a wide range of α. Implementing the policy (α^*_{opt}) can enhance the total throughput of the distributed wireless networks by up to 20%.

Using definitions of NE and channel configuration constrains at NE^{Good}, we derive and propose an approximate way to mathematically express α^*_{opt} (referred to as $\alpha^{\wedge}_{opt}$) along with its probability density function (PDF). Then a generic equation for $\alpha^{\wedge}_{opt}$ is inferred for varying network size (links) and available resources (channels). Implementing the policy ($\alpha^{\wedge}_{opt}$) enhances the total throughput of the distributed wireless network by up to 15%. In a more general setting, our distributed policy can achieve up to 75% of the maximum total throughput (bench-mark value reached by centralized solution via exhaustive search) at a fraction of time and computation resources.

Keywords: Resource (Channel) Allocation, Distributed Wireless Networks, (Non) Cooperative Networks, Game Theory, Good Nash Equilibrium, Weight of Optimal Interference (α^*_{opt}). Mathematical Approximation of α^*_{opt}, ($\alpha^{\wedge}_{opt}$).

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Mr. Amulya Bhattarai

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Symbols/Abbreviations	**Terms**
DSR	Dynamic Resource Allocation
CRN	Cognitive Radio Network
PU	Primary (licensed) Users
SU	Secondary (not licensed) Users
SDR	Software Defined Radio
NE	Nash Equilibrium
N	Number of Links
C	Number of Channels
α	Weight of the interference received
α_{opt}^{*}	Optimal α
$\alpha_{opt}^{\wedge}$	Mathematically derived approx. α_{opt}^{*}
Link_j /User_j	Communication between Tx_i and Rx_j
d_{ij}	Distance between Tx_i and Rx_j
SINR_j	Signal to Noise Ratio of Link_j
I_j^{form}	Interference received by receiver j
I_j^{to}	Interference created by transmitter j
T_j	Throughput of Link_j
T_{tot}	Total throughput (sum of all links)
T_{tot}^{max}	Maximum total throughput
U_j	Payoff/Utility of Link_j
$U_{s1}, U_{s2}, \ldots, U_{sN}$	Shared utility (a channel is shared)
NE^{Good}	NE that has high T_{tot}
NE^{Medium}	NE that has medium T_{tot}
NE^{Low}	NE that has low T_{tot}
$\boldsymbol{T_{tot}^{max}, NE^{Good}, NE^{Medium}, NE^{Low}}$	Groups segregated based on data
BR (BR_Uj_Ttot)	Best Response Algorithm ($U_j = T_{tot}$)
τ seconds	Compute time for one configuration

CHAPTER 1
INTRODUCTION

1.1 Wireless Communication Trends

Communication among wireless devices is increasing at an exponential rate [1] as shown in Figure 1.1. Digital data is being generated in an escalating amount much of which is being transmitted via the wireless media. And resources especially the Radio Frequency (RF) spectrum is scarce [2]. So, researchers are looking for new techniques to enhance the throughput of wireless communication systems [3] so they can address the increased demand of wireless traffic [4].

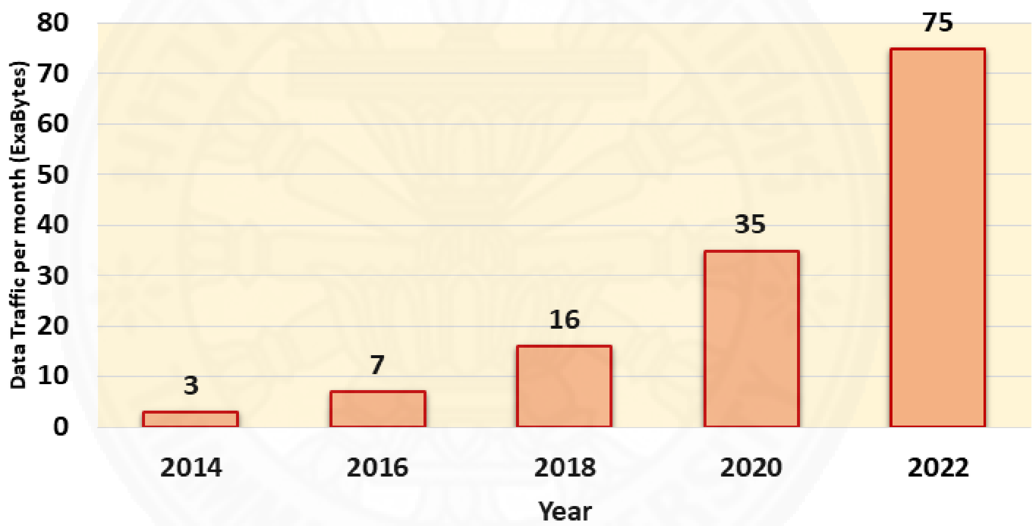

Figure 1.1 Exponential rate of data generated and communicated via the wireless media (Global Mobile Data Traffic) over the years, 2014 to 2022 [62].

1.2 Contemporary Wireless Technologies

Most of the current technologies that are catering to the needs of the present wireless communications are using static resource allocation. In fixed resource allocation, power, spectrum, modulation techniques and other transmission parameters are generally predetermined and do not change as the wireless network changes. Hence, the fixed resource allocation is inefficient as only a fraction of the allocated resources is utilized by the designated users [5] as illustrated in Figure 1.2. Some portions of spectrum bandwidth are heavily used whereas other bands have medium or low usage [63]. Research has established that an average of 15 % to 20 % of the licensed bands are being utilized [6].

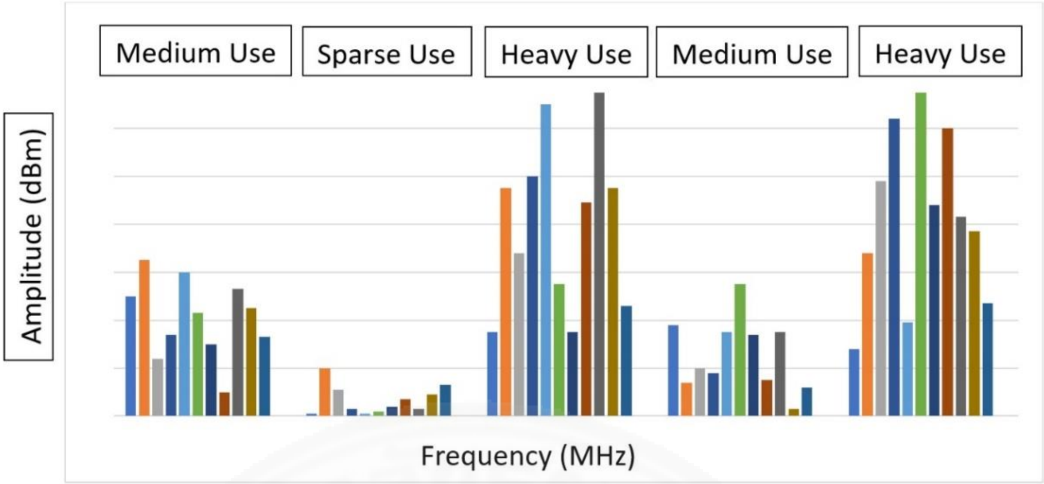

Figure 1.2 Spectrum utilization of the bandwidth shows sporadic use of the assigned spectrum [64]. Y-axis represents the average signal strength received and x-axis shows the five spectrum bands in the MHz range. Portrays the inefficient use of fixed spectrum assignment.

Three prominent wireless technologies, WiFi, Bluetooth and Mobile Networks (2G, 3G and 4G) are discussed as follows:

1.2.1 WiFi

WiFi is a popular technology for connecting local area network devices via wireless medium. It uses the IEEE 802.11 standards [7]. Devices such as personal computers, video-game consoles, phones/tablets, digital cameras, smart TVs, digital audio players and modern printers use WiFi. These devices connect with each other and to the Internet via a wireless Access Point (AP). The operating frequency is mostly at Industrial Scientific and Medical (ISM) band; 2.4 GHz and 5.8 GHz. There are four non-overlapping channels in the 2.4 GHz band each with a bandwidth of around 20 MHz; and a device establishes a connection with an AP in one of these channels [8] which is illustrated in Figure 1.3. The modulation used is mostly orthogonal frequency-division multiplexing (OFDM).

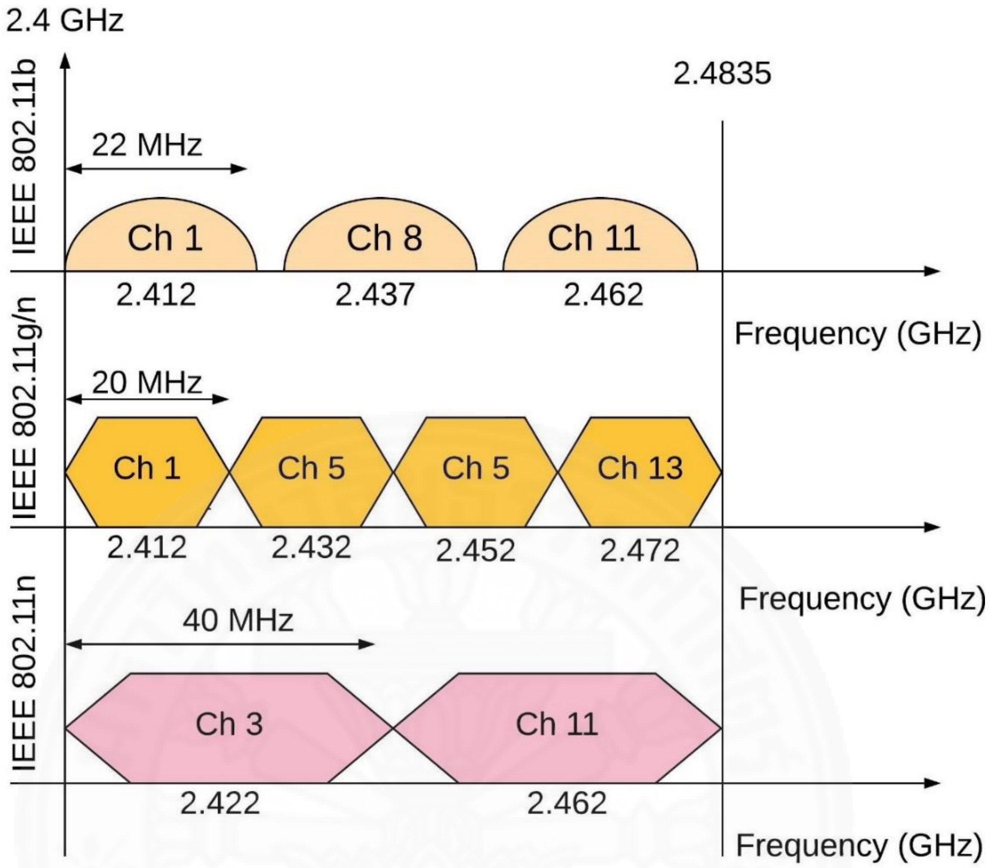

Figure 1.3 Channels in Wireless Local Area Network (WLAN) using IEEE 802.11 protocols (trademark WiFi) shows the central frequency, the number of channels, bandwidth of each channel for different standards [65].

1.2.2 Bluetooth

Bluetooth is a wireless technology for building personal area networks (PANs), which range over short distances (less than around 10m). For example, devices in the personal area such as mouse, key-board, screen, speaker, etc. are connected via wireless media to the computer using Bluetooth. It operates in the ISM band from 2.4 to 2.485 GHz [9] as illustrated in Figure 1.4. There are around 40 channels and the bandwidth of each channels is 2 MHz (including the guard band) and Bluetooth uses Frequency-Hopping Spread Spectrum (FHSS) technology for transmitting the packets [10].

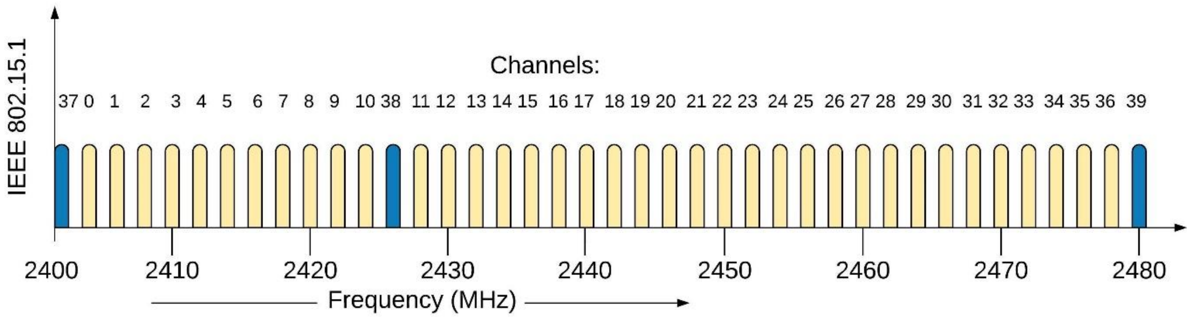

Figure 1.4 Channels in Bluetooth standard (IEEE 802.11) shows the channel bandwidth and the distribution in the ISM band, 2.4-2.48GHz [66].

1.2.3 Mobile Networks (1G, 2G, 3G and 4G)

Mobile networks have evolved from 1G to 4G. In 1G radio signals were transmitted in analog form. The operating frequency was 800 MHz and it was mainly used for making calls in a city or within a country by connecting the Base Station (BS) with the Mobile Station (MS) [11]. The uplink and downlink bandwidth are 20 MHz (with over 600 channels) each.

2G networks were based on digital networks; signals were transmitted via digital format. IS-95 and GSM were the prominent technology used. The operating frequency were 900 MHz and 1800 MHz. Each 200 kHz channel is further divided into 8 time slots to cater to more users [12]. The prominent operating frequency of the 3G technology such as UMTS is 2100 MHz. The bandwidth of each channel is much larger, 1.25 MHz to 5 MHz wherein many users simultaneously communicate [13].

Table. 1.1 Operating Frequency and Channel Bandwidth in 1G, 2G, 3G and 4G Cellular Networks [11].

Technology	Operating Frequency	Bandwidth of Channels
1G	800 MHz	30 kHz
2G (GSM, IS95)	800-900 MHz or 1.9 GHz	30-200 kHz or 1.25 MHz
3G (UMTS, WCDMA)	2.1 GHz	5 MHz
4G (LTE, WiMax)	1.8- 2.5 GHz	5-20 MHz

Table 1.1 shows the operating frequency and details of the channel in 1G, 2G, 3G and 4G technology. 4G technology such as LTE's operating frequency could be up to 5 GHz and the channel bandwidth can be up to 20 MHz; allowing higher data rates and flexibility. Efficient channel allocation schemes are implemented which enhances the communication between users [12].

Current technologies explained above represent static resource allocation; they are not able to scan and adopt to changing dynamics of the wireless network [5]. Intelligent Radios that can dynamic allocation resources as per the changing environment is an upcoming trend [14]; which shall be discussed in detail in the upcoming section.

1.3 Cognitive Radio Network (CRN)

CRN is a prospective scheme in future wireless communications [6]. CRN assumes intelligent radios that can sense the surrounding area and find vacant frequency bands, interference levels, modulations schemes used by other users [17] and make use of the used resources to enhance the efficiency of the system.

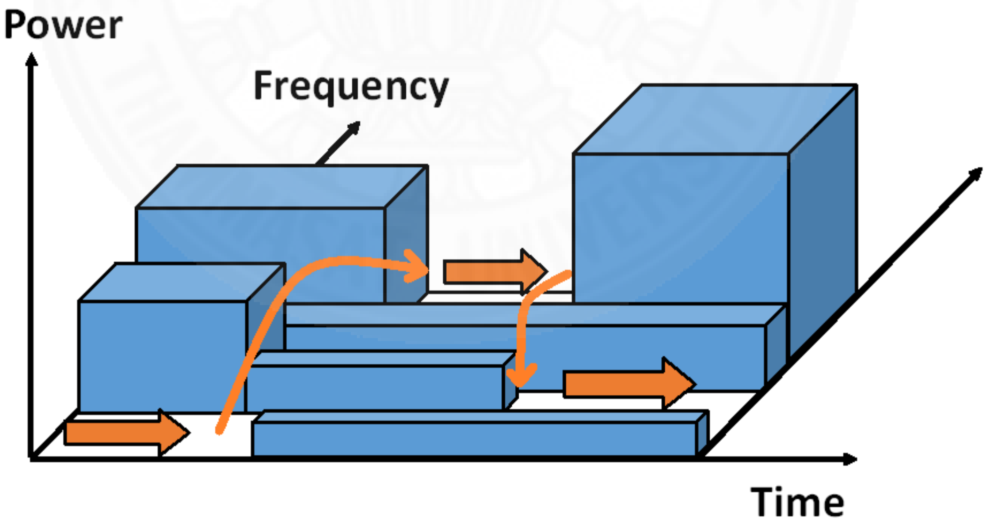

Figure 1.5 CR senses the empty spectrum and hops to different empty spectrum slots over time to use the resource enhancing the efficiency of the network [67].

Figure 1.5 shows how a CR can sense the spectrum hole and hop into empty slots to make use of the unused resource. Unlike, static resource allocation where the transmission parameters are fixed, in CRN the system adopts to changes in the network dynamically. This is the core idea of Cognitive Radio Network (CRN). The cognitive engine which is illustrated in Figure 1.6 shows how the CR senses the environment then studies the opportunities available based on its previous observations and algorithms computes; then decides and implements strategies which can optimize the transmission parameters such as frequency [18], power [15], and modulation schemes [16]. This enhances the individual and/or collective performance of the CRN [19]. In such dynamic networks, unlicensed users can also access the resources which are not being used by the licensed users; enhancing the efficiency of CRN.

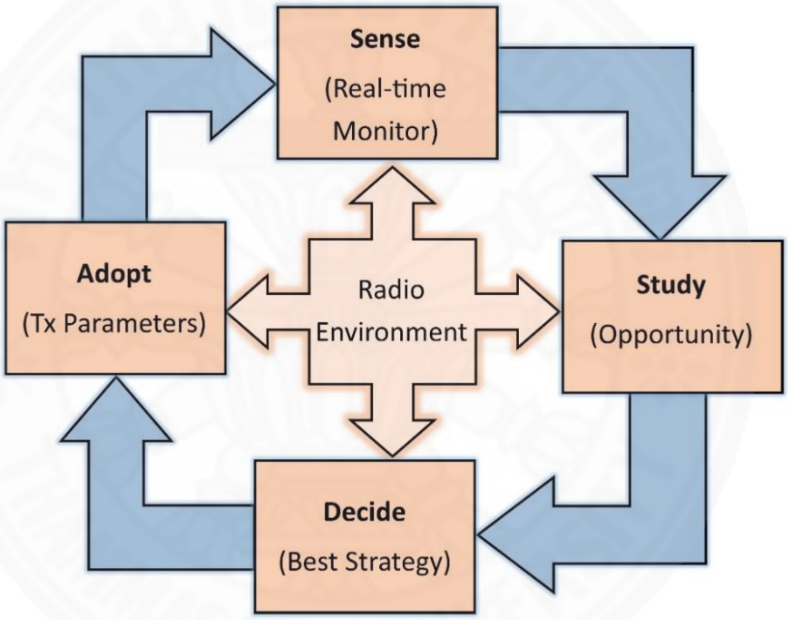

Figure 1.6 Cognitive Radio Engine shows how the intelligent radio 1. Senses the wireless environment 2. Studies the opportunities 3. Decides on the best strategy as per the opportunities available 4. Adopts its parameters like power level, spectrum hopping, etc. based on its decisions [19a].

1.3.1 Primary and Secondary Users

Although most of the frequency band is assigned to certain licensed users, which are also known as primary users (PU), they are not using them all the time [5]. In a traditional wireless network, the unused resources cannot be used by other unlicensed users, also known as secondary users (SU); because the SU are not the licensed users [21]. PUs has the right to use the resource; as illustrated in Figure 1.7.

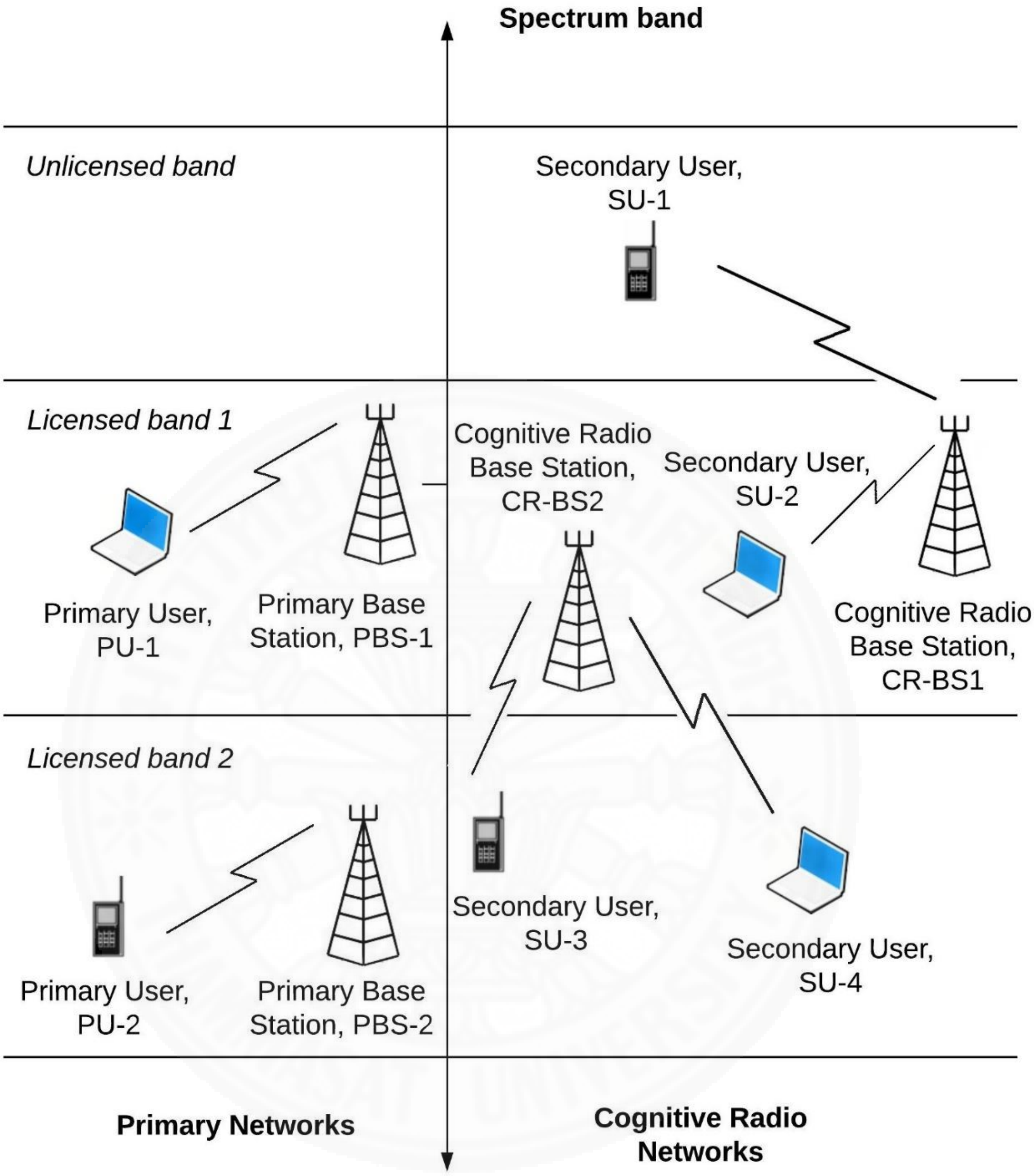

Figure 1.7 Cognitive Radio Network (CRN) are different from Primary Networks. In the Primary Network the spectrum is divided into different bands for the use of Primary Users (PU); the BS caters to the PU’s in their band only. However, for CRN the CR-BS can cater to the demands of the SU’s in different bands as per requirement [68].

However, SUs are intelligent users who make use of the unused resources allocated to PUs without interfering with or in the absence of the PUs [20]. Generally, CRNs have both PU and SU however, some CRNs only have SUs [18]. Others are working on the sensing of the PU [22] and resource allocation to PU [23]; this work focuses on the resource allocation to the SUs, after the sensing of PU and resource allocation to the PU is complete.

1.3.2 Spectrum Sensing

Spectrum sensing, that is identifying if the PU or SU is occupying specific spectrum band(s) in a network can be done in several ways. Primarily two techniques are employed, 1) Signal Processing Techniques and 2) Cooperative Sensing. These can be further elaborated as illustrated in Figure 1.8 and which is further explained below.

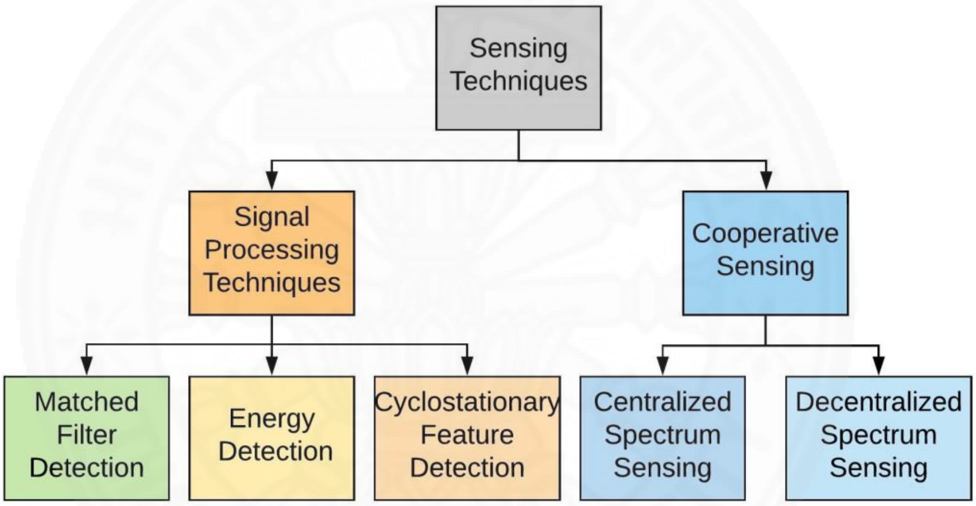

Figure 1.8 Different techniques used to sense the presence of Primary Users (PU) or Secondary User (SU) in specific spectrum band(s). Primarily segregated into Signal Processing Techniques or Cooperative Sensing [69].

Matched filter: Is an optimal way to detect the transmitter signal [70]; however, demodulation of the primary user signal has to be performed before the matched filter operation can be done. This implies the Cognitive Radio must have priori information of the transmitted signals like modulation schemes, shape of the pulse, packet format, etc. [70]. Moreover, time synchronization with the primary user has to be achieved and channel equalization as well, which makes it quite challenging. Figure 1.9 shows the signal can be detected using matched filter technique via a flow chart.

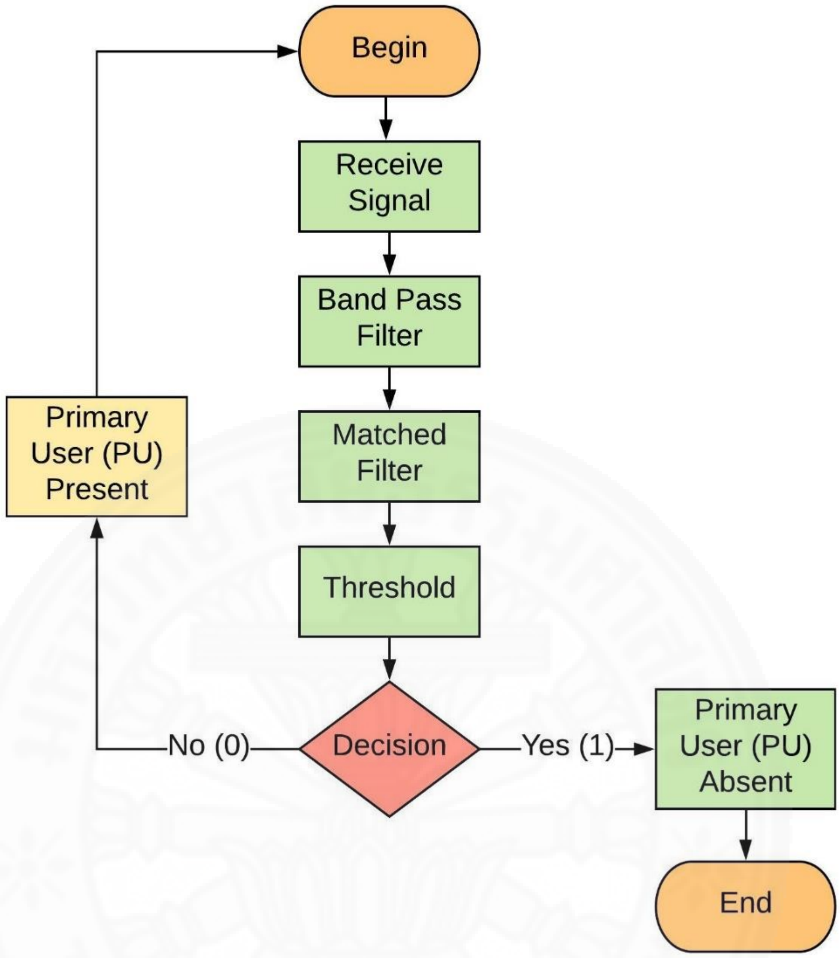

Figure 1.9 Flow Chart illustrating the basic steps taken while sensing the presence of a Primary User via Matched Filter Detection. The CR needs to know details of the PU's transmission parameters like modulation scheme and pulse shaping [71].

Energy detection: Is a comparatively easy and convenient way to ascertain the presence/absence of a signal from the PU [72]. The cognitive radio measures the received signal power at different spectrum. If the measured value is above a threshold value, then it is inferred that a primary user is present and if it below the threshold value then it is assumed the primary user is absent. However, the threshold value varies as the noise floor changes which depends upon the environment and the activity of the electromagnetic transmitters [72]. Processing gain is directly proportional to Fast Fourier Transform (FFT) of length "N" and averaging time "T". By increasing "N" the frequency resolution increases which helps to detect narrowband signals [74]. Figure 1.10 shows the flow chart representation of the Energy Detection Technique.

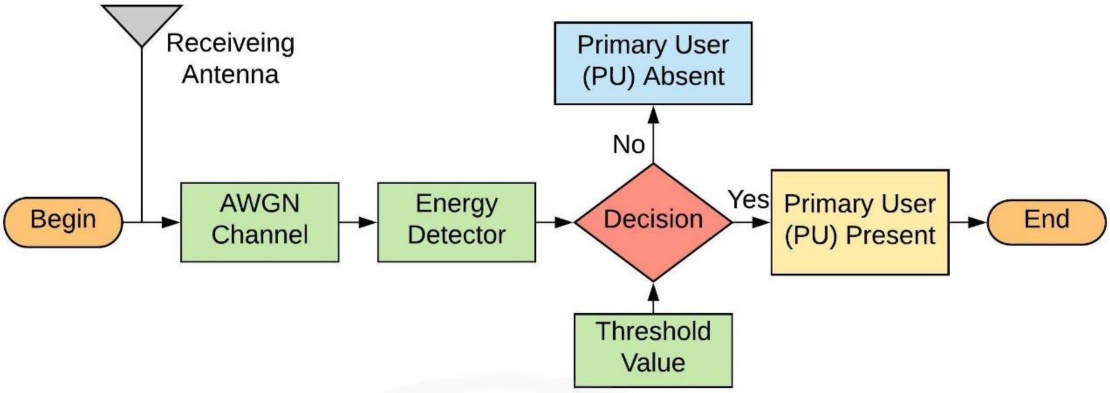

Figure 1.10 Determination of the presence/absence of a Primary User (PU) in a network using Energy Detection Technique. Energy received in the band is calculated and compared with the threshold value to determine the vacancy of the band [73].

Cyclostationary feature detection: Most of the man-made signals like AM, FSK, BPSK, QPSK, etc. display cyclostationary characteristics [75].

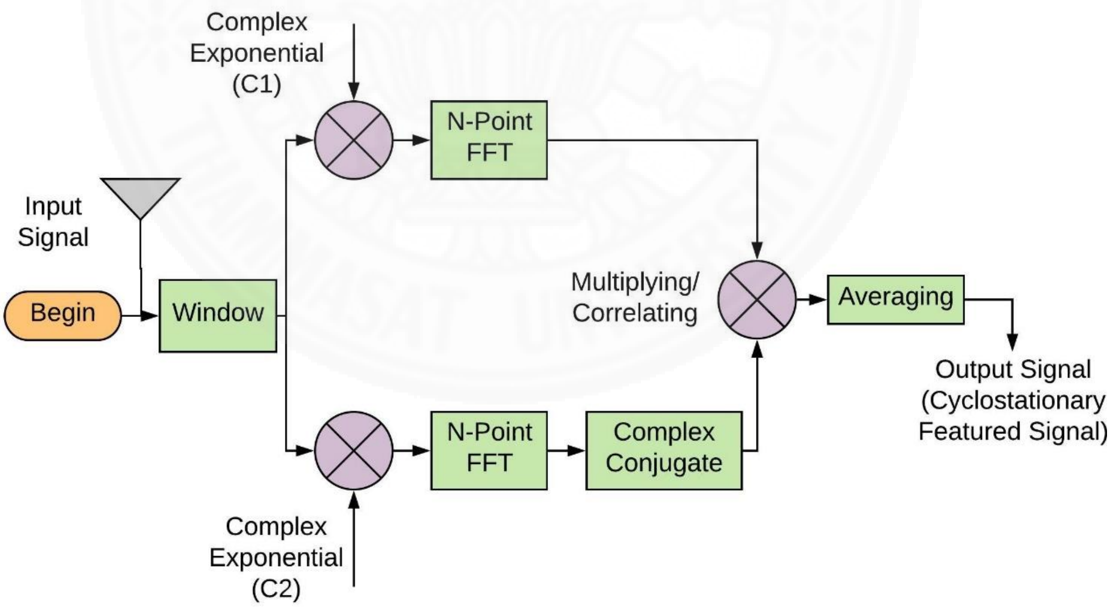

Figure 1.11 Cyclostationary Feature Detection of a signal is performed so signal hidden amidst noise can be extracted and detected [76]. Time periodic signals exhibit cyclostationary features whereas noise does not.

Cyclostationary signals have a repetitive pattern over time. Moreover, statistical information/behavior like mean, variation, spectral/auto correlation, etc. of these signals have unique signature. However, noise (especially white noise) does not exhibit cyclostationary characteristics and unique statistical behavior.

In Figure 1.11 the input signal is passed through a window to limit the length of the signal. Then it is multiplied with different complex exponentials and N-point Fast Fourier Transform (FFT) is taken respectively. In the last stage the two signals are multiplied, correlated and averaged to get the cyclic spectrum output. This property can extract and detect the signals transmitted by PU which is obscured by the noise [76].

Cooperative Detection: Spectrum sensing in an individual radio is often compromised due to the wireless channel's unpredictable characteristics: multipath fading, shadowing, etc. Cooperative spectrum sensing can alleviate this problem. When several radios which are physically located in different places cooperate to determine the presence of a primary user/s the wireless channel's unpredictable characteristics (discussed earlier) can be minimized. Although cooperative detection schemes do enhance the detection performance it does give rise to cooperation overhead.

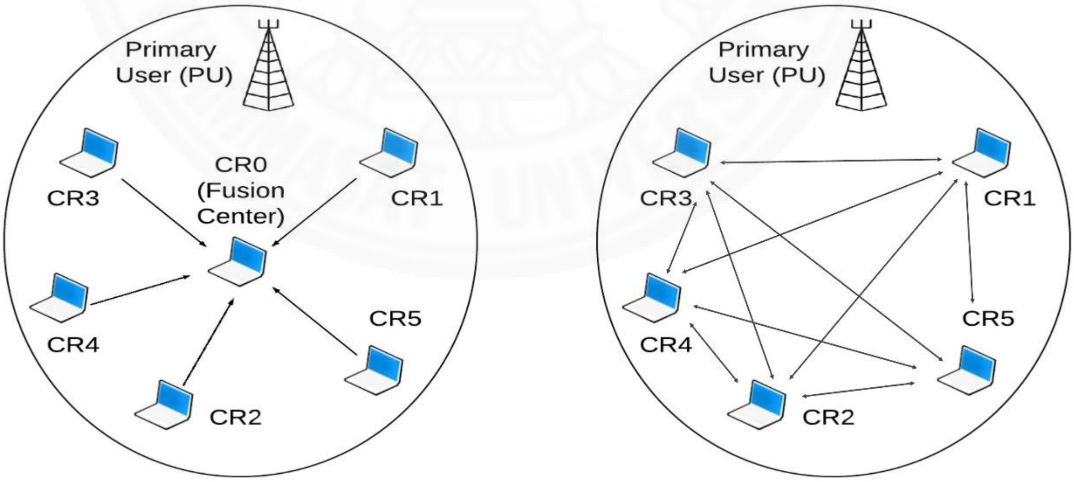

Figure 1.12 Shows the two methods for cooperative sensing of the spectrum. Left: shows a Centralized scheme wherein the Cognitive Radio (CRX) reports their observation to the Fusion Center (CR0). Right: shows a Decentralized scheme wherein there is no Fusion Center and all the CRX share their observed information with each other and perform their computation by themselves in a distributed fashion [78].

Because several radios have to relay the information they acquire from the environment and the information has to be processed to come up to a conclusion; certainly, more time, energy, computational resources are used [77]. Two prominent cooperative schemes are discussed below.

Centralized Detection: In a centralized cooperative spectrum sensing scheme a central entity generally, a Fusion Center (FC) is present as illustrated in Figure 1.12. All the SU's send their spectrum sensing information to the FC who processes to determine is the PU is present or not and relays back the decision to the SUs. In such a CRN; generally, a channel is assigned for communication between the FC and the SUs so they can relay the information regarding the spectrum sensing. In many scenarios one of the SU can act as a FC as well [77].

Decentralized: In a decentralized cooperative spectrum sensing scheme as shown in Figure 1.12; there is no central entity. Different SU's sense the spectrum and relay the information with all other radios; which are within its connectivity range. After gathering the information from different radios, each radio makes the decision by itself in a distributed way. The individual radio can make use of different algorithms to reach a decision whether a PU is present or not. The SU's might take several iterations and forward-backward communications with the other SU's before they come to a conclusive decision [78].

1.3.3 Spectrum Handoff

Spectrum handoff can dynamically make use of available yet unutilized spectrum. Most of the frequency band is designated to a specific primary users (PUs). However, the PU's are not always using the spectrum licensed by them [79]; in such scenarios the secondary users (SUs) can use these frequencies which increases the overall efficiency of the CRN and mitigates the problem bandwidth scarcity. Figure 1.13 shows the SU is assigned a new channel (as per availability) once the PU enters its licensed channel. Two prominent spectrum handoff techniques are briefly described below.

Reactive-sensing spectrum handoff: As we can see in the Figure 1.13, the SU is assigned a new channel once the PU who has the license for the channel starts to use the channel. If the target channel for the SU is selected or sensed only after the arrival of the PU then this kind of spectrum handoff is known as reactive-sensing spectrum handoff [80].

Proactive-sensing spectrum handoff: In this scheme, the vacant/prospective channels for handoff are already predetermined. So, once the PU enters the licensed channel, the SU can immediately be moved to the preassigned channel [80]. The delay for the SU to move to the next vacant channel in faster in this scheme; however more computational and memory resources are used. Because, the SU has to be always scanning and looking for prospective channels in case the PU enters its licensed channel.

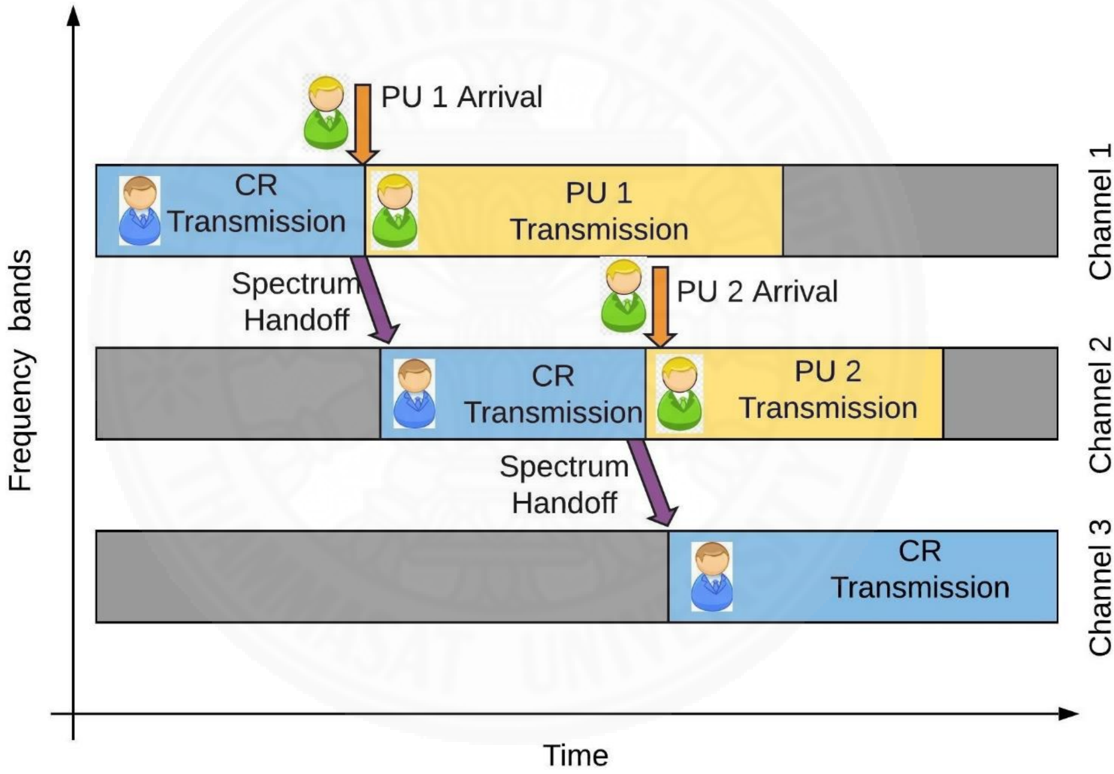

Figure 1.13 Illustrates the Spectrum Handoff in Cognitive Radio Network (CRN). At the beginning before the arrival of a Primary User (PU) CR is using Channel 1. Once PU1 arrives in Channel 1 CR hops to Channel 2. Once PU2 arrives in Channel 2 CR hops to Channel 3 [79].

1.3.4 Dynamic Spectrum Access (DSA)

In contrast to static resource (spectrum) allocation DSA allows the SU or CR to access the spectrum while sharing the spectrum with the PU or without creating interference to the PU. Two prominent ways how the spectrum can be used by the SU is illustrated in Figure 1.14 and discussed below.

Underlay: When the SU uses the same frequency band as the PU but does not cross the interference limit set by the PU or the standards in that particular network / band then it is known as underlay [103]. Because the signals transmitted by the SU is below the noise / interference floor it is perceived as noise to the PU; different transmission / modulation scheme is generally used by the SU to achieve such an effect.

Overlay: When SU senses the presence of the PU and transmits in the absence of the PU; utilizing the vacant bands/spectrum then it is known as overlay scheme. The SU needs to continually sense the absence/presence of the PU and after determining the spectrum holes, SU can transmit in full power in these bands in those time slots [103]. Generally, the SU needs to compensate the PU by direct/indirect payment/favor for using the spectrum holes.

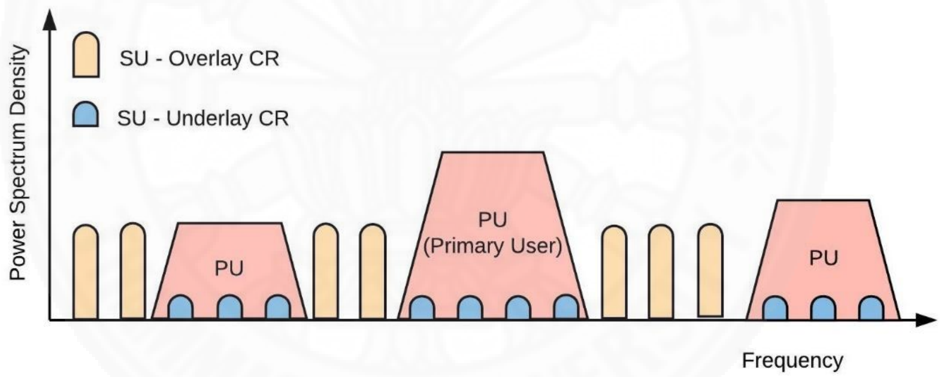

Figure 1.14 DSA allows the SU or CR to access the spectrum while sharing the spectrum with the PU or without creating interference to the PU. Two prominent ways how the spectrum can be used by the SU are Overlay and Underlay [103].

1.3.5 Classification of Cognitive Radio Networks

CRN has been categorized in three different ways. The first group "computation" refers to where the CRN runs its own algorithms. In some case, a central entity collects data from all users and does all the computation, whereas in other cases all the computation is done in a distributed way by the individual users [24]. The second group considers the "nature" of the user; users can be selfish, altruistic or partially selfish, depending upon how their payoff is defined [25]. The third group considers the "information exchange" between the users [20]. This is illustrated in Figure 1.15.

If there is no information exchange, a user has access to the information that is publicly broadcasted which is generally limited. Whereas, in a "complete-information-exchange" scheme all users have access to the network metrics of all users.

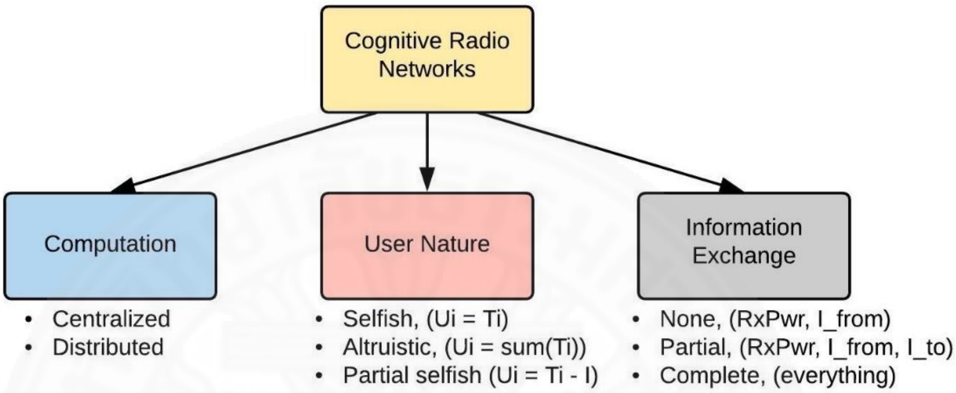

Figure 1.15 CRN is segregated based on: 1) where the Computation is performed 2) Nature of the User and 3) Information Exchange between the Users [37].

1.4 Distributed Wireless Networks

With changing time and technological advancements new sets of characteristics are visible among the wireless users. Figure 1.16 shows the difference between centralized, decentralized and distributed networks.

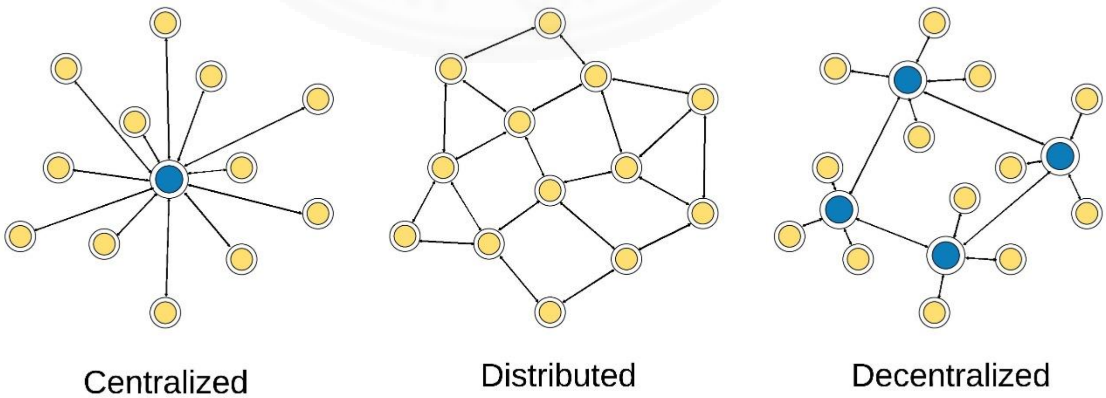

Figure 1.16 Three different networks with a Centralized, Distributed and Decentralized schemes is illustrated [81]. The amount of independence and intelligence of the nodes varies across these different schemes.

Previously, the wireless users were under the complete control of a centralized entity and they had limited computational power and independence [24]. However, as each radio has higher computational resources and artificial intelligence; decentralized and distributed networks with independent, rational, and selfish users are becoming popular [26].

1.4.1 Smart Radios - Lime SDR Mini

Smart radios/users are becoming popular. These devices have higher computational ability and have their own source of power, ability to detect/sense the environment, and intelligence (algorithms that can adopt to the changing environment and take necessary actions/strategies). Hence, these smart users make decisions rationally and act selfishly, trying to maximize their own utility or performance metrics [27]. For example, several wireless devices may connect and share information with each other via Bluetooth or WiFi in the absence of a central entity such as Access Point (AP) or Base Station (BS) [28].

Figure 1.17 Lime SDR (Software Defined Radio) Mini [82] are a set of smart radios which recently arrived in our Communication Lab. We are working with these radios to implement our algorithm/policy; so, we can verify our simulation results that show higher performance can be achieved for resource allocation in dynamic distributed wireless networks.

Figure 1.17 shows a Lime SDR (Software Defined Radio) Mini which recently arrived in our Communication Lab [82]. Lime SDR Mini is a platform for developing/prototyping high-performance and logic-intensive digital and RF designs. They use Altera's MAX 10 FPGA and Lime Microsystems' LMS7002M RF transceiver [83]. We plan to use Lime SDR Mini as a smart radio prototype and implement our research finding based on our simulations and mathematical derivations. Basically, the Lime SDR Mini will be programmed to sense the spectrum being used by different Tx-Rx pairs and optimally allocate the available channels to maximize total throughput of the network in a distributed fashion.

In a distributed network the independent and intelligent users compute and converge to resource allocation schemes themselves. A central entity who controls and manipulates the strategies of the users is not evident [29]. The users have ample computational and algorithmic resources to reach the distributed solution. And the users rely only on the information that they can measure independently without the cooperation of other users. The users are also rational; each user wants to maximize their individual payoff. The eventual equilibrium state generally reached in such a distributed network is the Nash Equilibrium (NE) [25].

1.5 Game Theory

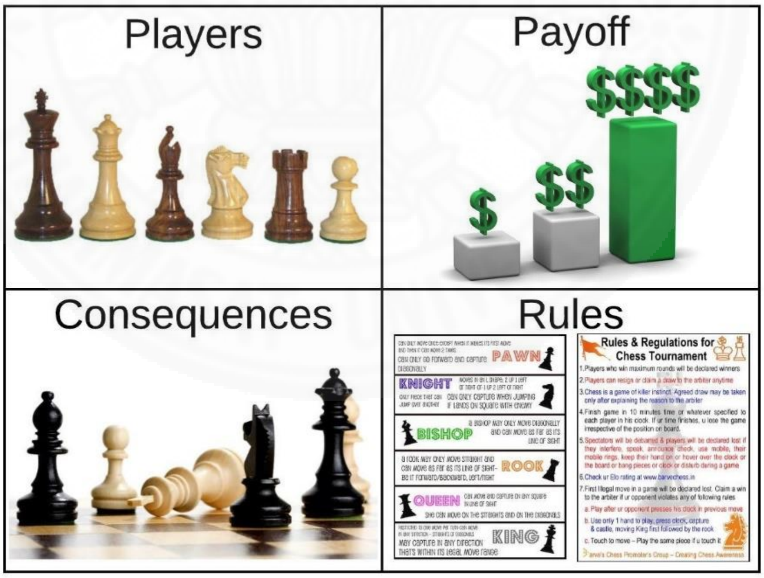

Figure 1.18 Game Theory is a mathematical tool that can be used to optimize the payoff of the players as per set rules and regulations given the consequences and action set of the users. In recent years it has been used to allocate resources in distributed wireless networks [84].

Game theory is a branch of applied mathematics which helps to solve the interactions of rational entities in conflicting situations [25]. It provided many sets of mathematical tools to model and analyze interactions among the rational entities by assigning payoff as perceived by these intelligent users. In this way the users with conflicting objectives can come to an equilibrium position [30]. Figure 1.18 shows different parameters of Game Theory, comparing with a chess game.

1.5.1 Nash Equilibrium (NE)

NE is a natural and eventual solution reached by selfish and independent users. The concept of Nash Equilibrium (NE) was initially proposed by John Nash and further refined by others in the years to follow [31]. By definition, NE none of the users will benefit by unilaterally deviating from their specific strategy.

Selfish users can reach NE in a non-cooperative manner by using game theory [32]; each user tries to maximize its own payoff irrespective of others, and after a considerable amount of time the system converges to NE.

Correlated Equilibrium: Generally, while deriving the NE of a game the players choose their strategies independently. However, in the correlated equilibrium the players follow the suggestion of a third party [85]. When the players observe and act based on the value of the public signal and if no player wants to deviate from the strategy set the result is a correlated equilibrium. The expected payoff of each player is greater than that of the pure-strategy NE or mixed-strategy NE [85]; which is further elaborated in Chapter 3.

1.5.2 Other Game Theoretic Equilibrium Concepts

Over the years, several other equilibrium concepts have been discussed and implemented to resolve resources amongst the non-cooperative users; some of the prominent ones are illustrated in Figure 1.19 and are discussed next.

Stackelberg Equilibrium: Is also known as a leader follower game. As the players of the game are segregated into a leader party and a follower party [86]. Many times, there are just two players a leader and a follower. The leader has the right to make the first move or choose the best strategy due to its power/dominance. Then the follower user has to optimize his strategy based on the strategy selected by the leader.

In a wireless network the PU (Primary User) or a BS (Base Station) or an AP (Access Point) or a SU (Secondary User) with especial power or dominance could be a leader user. Generally, users engage in Stackelberg contest if there is a first move advantage; which can be solved by finding the subgame perfect Nash equilibrium or equilibria (SPNE) [86].

Evolutionary Stable Equilibrium: This equilibrium concept was motivated by ecological biology [87]. In the classical game theory, it is assumed that each player is aware about the structure of the game and chooses strategies to maximize his own payoff. However, Evolutionary Stable Strategies (ESS) have different incentive; because it is assumed that the strategy of the user is biologically encoded or heritable [87]. And each user does not have control over their strategies and is not aware about the details of the game as well. The payoffs of the game is represented by reproductive success or biological fitness. That is, the species that choose an ideal strategy survives or reproduces more and becomes dominant. It is covered in detail in Chapter 3.

Baseyian Equilibrium: When all players do not have complete information about the payoff and other strategies of other players in the game, it is generally regarded as Baseyian equilibrium [88]. In many scenarios or games, some statistical information of a player’s previous strategies might be known but the complete payoff table and strategy list is missing. And as the game proceeds, according to the strategies played by the users and details of the probability measurements the information might be updated. The equilibrium solution of such a game is called Bayesian Nash equilibrium wherein each player tries to maximize its expected payoff given its probability confidence and strategies of other players [88].

In this work our focus is on NE, mainly due to its simplicity and prevalence and we shall delve into in deeper. Depending on the starting point, the definition of a user’s payoff and other parameters, the system might converge to NE with lower, medium or higher performance [25]. To enhance the performance of the network, we looked deeper into the network metrics at different NEs [33]. Based on the distribution of the network metrics general policies are formulated.

Several techniques have been explored to reach NE: best response (BR) technique [34], exhaustive search [37], algebraic expressions [36], etc. An exhaustive search technique for large networks is computationally overwhelming and impractical.

Developing policies for channel allocation can provide a good starting point for the best response algorithm. Although NE might not be the best possible solution, it is a converged equilibrium point; as at NE no user will benefit by unilaterally deviating from the strategy set. A global optimal allocation obtained from a centralized and co-operative CRN is impractical in a distributed CRN, as the selfish and independent SUs will deviate from such a solution if they find a way to increase their own payoff [32]. Thus, the global optimum is attained for a short time. This deviation does not occur if the system is operated at one of the equilibrium states, Nash Equilibrium [30].

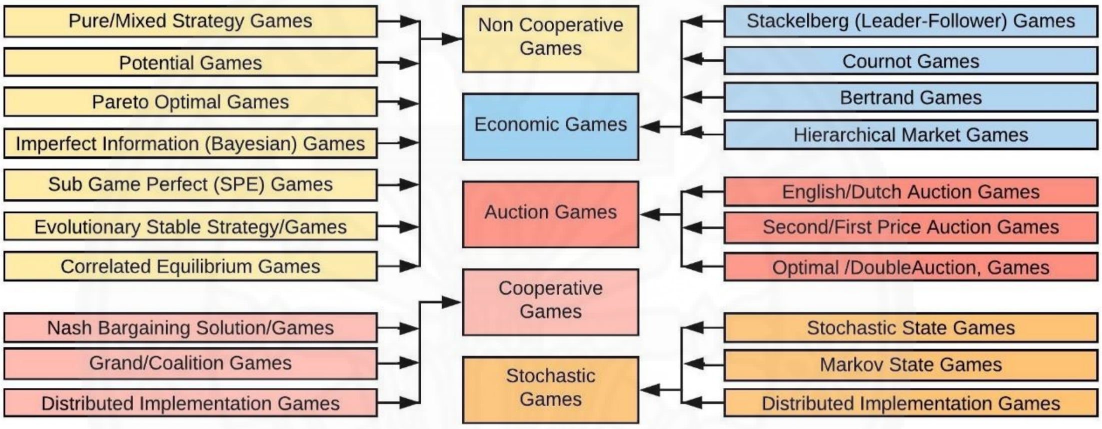

Figure 1.19 Different Game Theoretic tools and equilibrium concepts which are being used in the resource allocation in wireless networks. The main grouping are as follows: 1. Non-Cooperative Games, 2) Economic Games, 3) Auction Games, 4) Cooperative Games and 5) Stochastic Games [36]. In our work we focus on Non-Cooperative games as we our application is for Distributed Wireless Network.

1.6 Channel Allocation using Game Theory (Literature Review)

With the enlarged number of wireless devices and the information exchanged among themselves, the available bandwidth is becoming scarce [1]. Moreover, the intelligent and independent radios are competing for the limited resources- channels in a non-cooperative way to enhance their own throughput [2]. Game Theory is a mathematical tool that has been used to allocate resources among rational entities in a non-cooperative scenario [102]. In the last decade, many research groups have used game theory techniques to address the resource allocation problem in a distributed wireless network.

Figure 1.20 shows how the PUs occupy certain channels in the frequency spectrum and idle channels are sensed by the SUs and used for communication without causing interference to the licensed PUs [89]. However, the radio environment keeps changing mainly due to unpredictable nature of the wireless channels, mobility of the users, traffic variation and dynamic topology; resulting in the re-allocate of the spectrum resources [50]. In such scenarios, game theory can study, model and analyze the interaction among different users engendering efficient, self-imposing and distributed channel-sharing techniques [21]. The selfish and intelligent users compete for spectrum in a non-cooperative manner and the equilibrium solution reached can be analyzed using the different game theoretic equilibrium concepts used [32], which is discussed next.

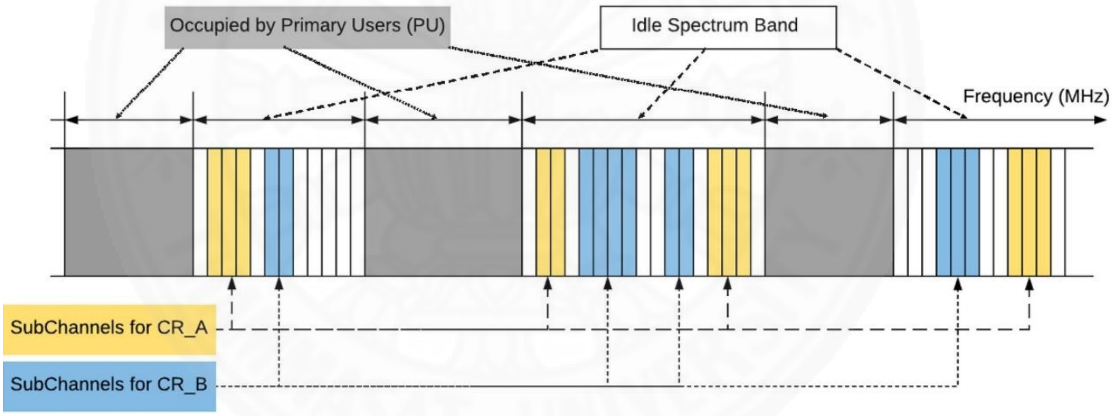

Figure 1.20 In the absence of Primary User (PU) the Cognitive Radio (CR A and B) use the spectrum sub-channels enhancing the efficiency of the network [89].

As discussed in [18], in a potential game the motive of all users to change their strategy is indicated using a global function called the potential function. Authors in [23] have defined the combined throughput of all users in the distributed network to the potential function, imitating a cooperative scheme. A potential function is a useful technique because by detecting the local optima of the potential game, set of pure NE can be established [90].

Generally, while deriving NE of a game, the users are presumed to be choose their strategies independently. However, in a correlated equilibrium the users get suggestions on the strategy set from a third party [85] which has a bigger picture of the network and can assign resources with results in minimum amount of conflict/interference among the users. In [36] the authors mix pure strategies with a certain probability distribution based on the public information (broadcast signals) relayed by other users or central entity and converge to correlated equilibrium with exhibiting better results. In [91] Evolutionary Stable Strategy/Equilibrium is used to allocate channels in CRN resulting in higher performance.

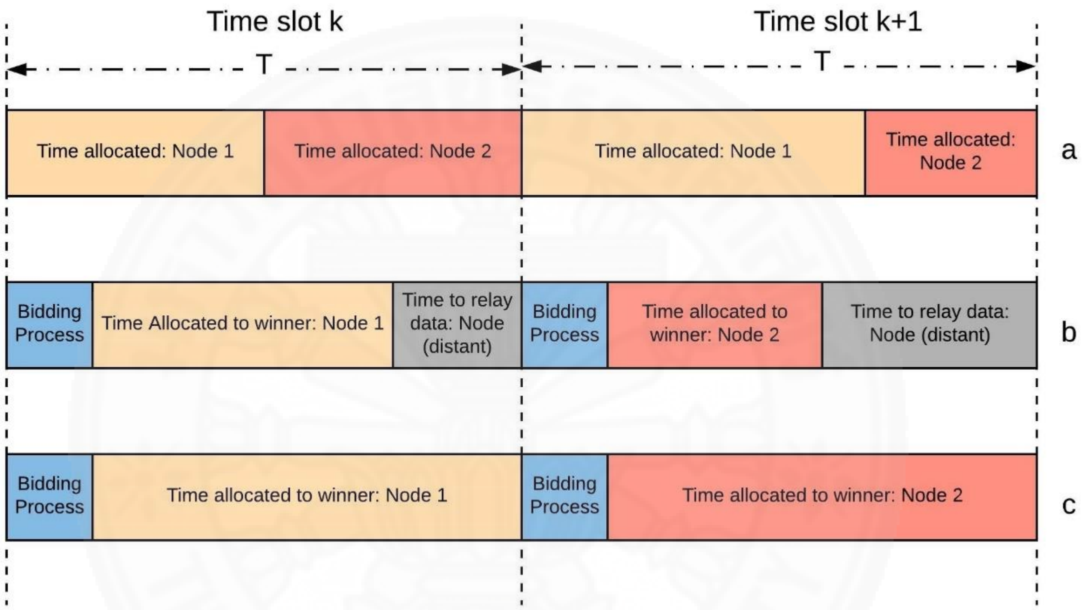

Figure 1.21 Time allocated to different nodes of CRN as per their requirement and as per the bid they submit. At the begnning to each time slot, bidding process is executed, and the winner node gets to transmit in the remaining time of the slot. At times some time is allocated for relaying data to distant nodes [25].

In [25] the users transmit data in time slots allocated to them as illustrated in Figure 1.21. This can be referred to a TDMA (Time Division Multiple Access) channel access game. Different users compete to for time slots in a channel to transmit their data. Fig 1.21.a shows two users share the time slot. The payoff of each user is a function of channel gain, transmission rate and QoS [92]. Depending upon these parameters the users converge to a NE solution and occupy a certain portion of the total time slot. In [93] auction technique is implemented to allocate the spectrum to the users. At the beginning to every time slot, players submit a bid to the central entity as is illustrated in Figure 1.21.b. The value of the bid submitted by the user is a function of the channel condition. The base station (auctioneer) assigns the channel to the user which offered the highest bid. However, the price paid by the user is the second highest bid. Some portion of the time slot is assigned for the base station to relay data to other users as well. The NE solution reached between by the users and the central entity (BS) does not require a priori knowledge of the channel distribution and results in different portion of time slots being assigned to the users [95].

In [96] repeated game-theoretic approach is implemented to allocate the spectrum among the users. In this game the user's need to convey their proper channel conditions and cooperate with each other; as, we are considering data communication over long durations of time. Hence, it is formulated as a repeated game. If the users fail to follow the agreement by not sharing the spectrum in an orderly manner; then punishment strategies are inflicted to the users.

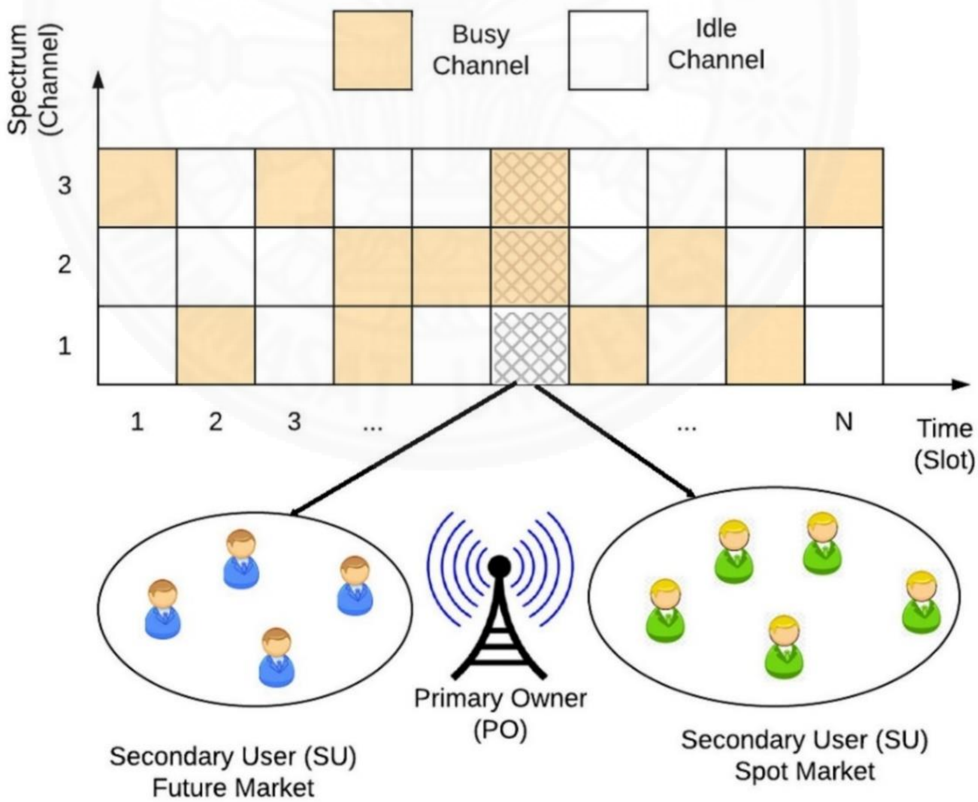

Figure 1.22 Spectrum trading between Primary Owner (PO) and Secondary Users in the Spot Market and Future Market. How the policies set by the PO can maximize the profit of the PO is discussed in [94].

Figure 1.22 shows how by proper economic stimulus successful dynamic channel sharing can be achieved. When the Primary Owner (PO) is not using the channels (y-axis) it can be outsourced with some benefit (monetary/future favor) to the SUs. The SU are classified into present (spot) market and future market [94]. Using game theoretic schemes (Nash Equilibrium concept) the PO allocates the channels to the SUs to maximize its present/future revenue using policies which properly solicit the SUs.

In a distributed network with rational and independent users, the converged equilibrium solution (NE) can fluctuate depending upon various factors; one prominent aspect is how the user's payoff is defined [30]. In [25] the authors have defined the user's payoff to be the individual throughput. Each user tried to maximize his own throughput in every iteration until they converge to a NE. This is a distributed system and there is no information exchange between the users, and the network performance is not good because when all users become selfish and try to enhance their own throughput; many times, the system converges to NE solutions with low performance. Some works also consider SINR [20] or received power [14] as their utility, which is a simply the throughput without a log function or consideration of interference received. In [36], [90] the authors have defined the user's payoff to be the total throughput of all users. Such definitions are generally made to emulate a centralized and cooperative system where each user works to maximize the total performance of the network. As each user needs to know the throughput of all other users, there needs to be exchange of complete information which is not viable for a practical distributed network that we are focusing in.

In [18], [33] the authors have defined the user's payoff to be the received power minus interference received. The authors intend to increase the received power and to minimize the received interference, endeavoring to enhance the performance of the entire network. In a similar work [35] the payoff of a user is defined as received power minus interference received from other users and interference created to other users. The objective is not just to minimize the interference a user receives rather to minimize the interference a user generates to other users. In the second case when we consider the interference to other users, there must be information exchange between the users.

Both the cases are distributed scenarios and they have a marginally better performance than their predecessors.

1.7 Problem Statement

With technological advances, distributed wireless networks with smart and independent radios are the upcoming trend [6]. These users can sense the network and based on their in-build algorithms can compute and implement optimal transmission strategies and parameters that are beneficial for themselves [13]. In such networks the number of users/links and available resources such as spectrum is dynamically varying.

Most of the time these users converge to a Nash Equilibrium solution. NE is an inevitable and natural equilibrium solution attained by independent and selfish users in a non-cooperative scenario [25]. However, NE solutions have don't have high performances, as each user focuses on his own benefits [40]. So, the main challenge is to enhance the performance (total throughput) of the dynamic distributed wireless network with independent and smart users.

1.8 Our Contribution

We studied the characteristics/ behavior of the network metrics such as individual throughput, interference from/to other users at NE with high, medium, low performance [37]. It was found that, "Adding an optimal interference received term in the user's utility term can push the distributed wireless networks towards good NE and enhance the performance."

Simulation results of thousands of random scenarios of different sized networks is used to obtain the value of α^*_{opt}. Using a software Eureqa [60] an equation for α^*_{opt} as a function of number of links (N) and channels (C) is proposed. This enhances the performance (total throughput) of the distributed dynamic wireless network by upto 20% [39].

Using definitions of NE and channel configuration constrains at NE^{Good}, we derive and propose an approximate way to mathematically express α^*_{opt} (referred to as $\hat{\alpha}_{opt}$) along with its probability density function (PDF). Then a generic equation for $\hat{\alpha}_{opt}$ is inferred for varying network size N and C. Implementing the policy $\hat{\alpha}_{opt}$ enhances the total throughput of the distributed wireless network by upto 15%. Our distributed policy can achieve upto 75% of the maximum total throughput (bench-mark value reached by centralized solution via exhaustive search) at a fraction of time and computation resources [61].

Table 1.2. Utility/Payoff is defined by different research groups in varying ways which results in different performance of the network. The operating of the network is centralized or distributed is also noted. Similarly, the amount of information shared among the users is also rated. Our research work gives highest result for distributed network with minimum information exchange scheme.

Utility/Payoff of a user/link	Computation (Algorithms)	Information Shared-Users	Performance (T_{tot})	Ref.
$U_j = T_j$ (3.8)	Distributed	Minimum	Low	[25]
$U_j = SINR_j$	Distributed	Minimum	Low	[20]
$U_j = \sum_{j=1}^{N} T_j$ (3.9)	Centralized	Maximum	High	[90]
$U_j = -\sum_{j=1}^{N} I_j^{from} - \sum_{j=1}^{N} I_j^{to}$	Centralized	Maximum	Medium	[18]

Our Contribution:

$U_j = \alpha\, p_{ij(i=j)} - \beta\, I_j^{from} - \gamma\, I_j^{to}$ (3.10)	Distributed	Medium	Medium	[33]
$U_j = T_j + \alpha_{opt}^{*}\, I_j^{from}$ (3.2)	Distributed	Minimum	Medium-High	[39]
$U_j = T_j + \hat{\alpha}_{opt}\, I_j^{from}$	Distributed	Minimum	Medium-High	[61]

Note: $\hat{\alpha}_{opt}$ is an approximation of α_{opt}^{*} obtained via mathematical derivation but can be computed in a fraction of time and computational resources required to compute α_{opt}^{*}.

CHAPTER 2
SYSTEM MODEL

This chapter discusses the main components of the model used in our work. Initially, the links (pairs of receivers and transmitters) and channels in our model are described in Chapter 2.1. Then calculation of the received power, interference and throughput of the links in the network are explained in Chapter 2.2. In Chapter 2.3 Physical / Network protocols is illustrated and the assumption made is covered in Chapter 2.4.

2.1 Links and Channels

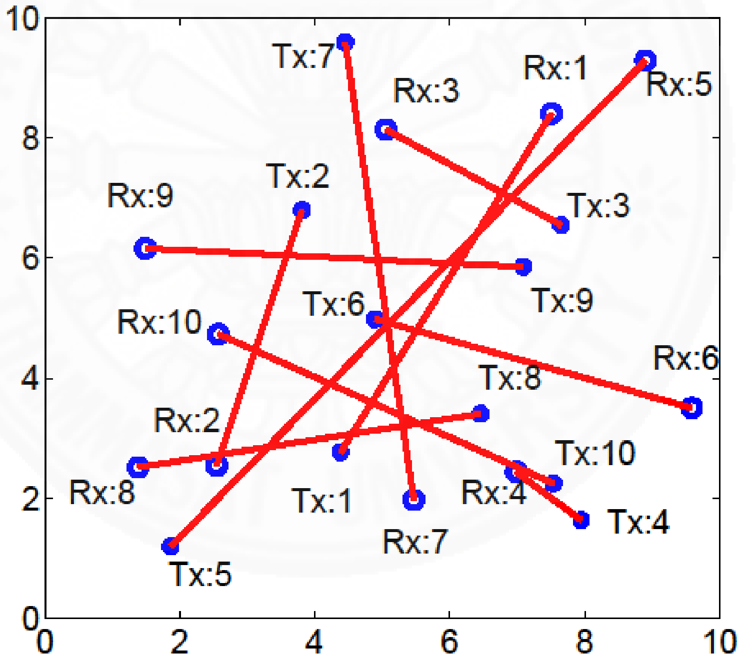

Figure 2.1 Distributed Wireless Network, random, “10 Links 4 Channels” scenario located in an area of 10 units by 10 units.

In our wireless network, there are N Links. Each link consists of a transmitter (Tx) and a receiver (Rx). There are $i = 1, 2\ldots, N$ transmitters and $j = 1, 2, \ldots, N$ receivers. Link$_j$ comprises of transmitter Tx_i connected with receiver Rx_j where $i = j$. For example, Link$_1$: Tx_1 is connected to Rx_1. Figure 2.1 shows a network with 10 links (N = 10) and

4 Channels (C=4). Symbols '*' and 'o' represent the Tx_i and Rx_j respectively. The co-ordinates of the Tx_i and Rx_j are randomly generated uniformly over an area of 10 units by 10 units. We can observe that several links overlap creating a dense network with considerable amount of interference especially for links sharing channels.

In a CRN limited resources in the form of C channels are available which needs to be shared by N links. Interesting case is when there are less channels than the number of links, $C < N$. A practical application of our work is when smart devices in a room, hall or concert arena have a direct communication with other smart devices in the area using limited bandwidth. Each link can occupy only one channel at a time.

2.2 Mathematical Formulations

Several important mathematical and equations which are used in our model are defined and formulated as follows:

2.2.1 Received Power (Friis Equation)

The power at the receiver from a transmitter, using the free space ideal propagation model, can be expressed as,

$$P_r = \frac{G_r G_t P_t}{\left(\frac{4\pi d}{\lambda}\right)^2} = \frac{A}{d^2} \tag{2.1}$$

where G_r and G_t are the antenna gain at the receiving and transmitting sides, respectively. P_r and P_t are the power at the receiver and transmitter, respectively. The distance between the transmitter and the receiver is d, and λ is the wavelength of the electromagnetic wave used in the communication system [41] as shown in Figure 2.2.

From Equation 2.1, A can be expressed as,

$$A = \frac{G_r G_t P_t}{\left(\frac{4\pi}{\lambda}\right)^2} \tag{2.2}$$

We want to explore the received power, with the variation of the link distance. Letting, the transmission power of all antennas and the gain of the antennas to be constant, unity. Similarly, as we are operating in a narrow-band spectrum, we consider the wavelength of the electromagnetic wave used in the CRN to be constant [37]. Hence, A is a constant. Without loss of generality, we normalize A to unity.

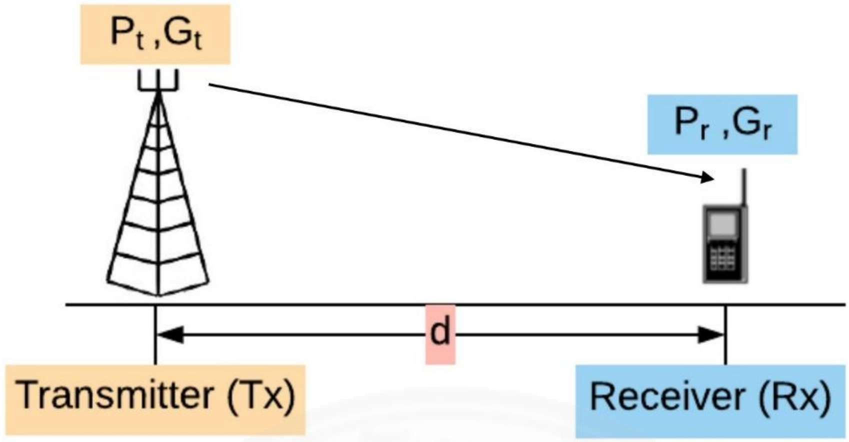

Figure 2.2 The power at the Rx can be computed using the Friis equation [11]: given the power of transmission, gain of the transmitter (Tx) and receiver (Rx) antenna and the distance between Tx and Rx and the wave-length, Equation 2.1.

The received power at Rx_j from Tx_i can be simplified using Equations 2.1 and 2.2 to,

$$p_{ij} = \frac{A}{(d_{ij})^2} = \frac{1}{(d_{ij})^2}, \tag{2.3}$$

where d_{ij} is the distance between Tx_i and Rx_j. In order to avoid very high or infinite values of p_{ij} when the distance is very small, we consider the minimum value of $d_{ij} = 1$, avoiding the near-field effects [37]. The desired power terms are those with $i = j$. The terms with $i \neq j$ is interference for link j.

2.2.2 SINR and Interference From

Then the Signal to Interference plus Noise Ratio (SINR) of Link_j is

$$\text{SINR}_j = \frac{p_{ij(i=j)}}{\sum_{j=1\neq i}^{N} p_{ij} + \sigma_n^2} = \frac{p_{ij(i=j)}}{I_j^{from} + \sigma_n^2}, \tag{2.4}$$

where $\sigma^2{}_n$ is the noise power, and $\sum_{j=1\neq i}^{N} p_{ij}$ is the received interference from other links to Link_j denoted as I_j^{from} [42]. Equation 2.4 assumes all links share the same channel; however only certain number of links share a channel and there is interference only between links sharing the same channel. In this work noise power is set at 0.001, which is 30 dB smaller than the transmit power. We have checked with 10 dB and 20 dB noise power values, which do not change our conclusion.

2.2.3 Individual and Total Throughput

The throughput (normalized over bandwidth) of Link$_j$ with AWGN channel with SINR$_j$, based on Shannon Capacity, can be expressed as [43],

$$T_j = log_2(1 + \mathrm{SINR}_j). \tag{2.5}$$

Here, the throughput is in bit/s/Hz, as the throughput is normalized over bandwidth. The total throughput of the distributed wireless network with N links is the sum of all the N link's throughput

$$T_{tot} = \sum_{j=1}^{N} T_j. \tag{2.6}$$

T_{tot} varies as different links choose different strategies and how the individual utility is defined; these are further explained in Chapter 3. T_{tot}^{max} is the maximum T_{tot} from all the different possible pure strategies obtained by exhaustive search of a random scenario, which is further explained in Chapter 4 and 5.

2.3 Physical/Network Protocol

In this section we explain how the communication takes place between the nodes. Before the links start to transmit data, they need to share certain information which is done via the beacon signal. Then based on the information shared each user can compute its payoff. Then using best response algorithm, the users can converge to NE solutions with higher performance. Figure 2.3 illustrates the protocol which is delineated as follows:

Step1: At the beginning, Link 1, 2, …, N transmits the beacon signal. The beacon signal consists of the UserID and vacant Ch.IDs that the link can observe (similar to beacon signals used in cellular networks [11] and SSID of WiFi [97]).

Step2: The beacon signal from all the links is gathered and analyzed by each user. Then the link can determine the N and C in the distributed wireless network. Each user then optimizes the parameters of its payoff which shall be covered further in the upcoming chapters.

Step3: Then using Best Response (BR) algorithm which is discussed in Chapter 3 the links converge to a NE with high performance.

Step4: The converged channel allocation scheme is adopted and the links transmit data as per the channel allocation.

As it is a dynamic network, link might be entering or exiting the network. Similarly, the number of vacant channels available also fluctuates. In such condition, the system will reset (begin from step1) after waiting for some time.

In Chapter 3, payoff for a user is defined. In Chapter 4 and 5 the network metrics are analyzed and the optimal parameters in the utility terms are determined. In Chapter 6 and 7 the results are verified via mathematical analysis and simulations and the procedure is finally implemented via a step-by-step process along with a flow-chart.

2.3 Assumptions

Five notable assumptions have been made in this work, which are as follows:

The first assumption is that the users are distributed, they compute the resource allocation schemes themselves [44]. Resource allocation in networks can be done in various ways. Central resource allocation is not practical and popular, as the individual users are independent, and hence can deviate from the global central solution. In our work we assume the users choose the strategies independently.

The second assumption is that the users are rational and selfish. Users will choose the strategies to increase their own payoff. Payoff can be defined in different ways. If the user's utility is defined as an individual throughput, a selfish behavior can be emulated; similarly, if a user's utility is defined as the total throughput of the network, a co-operative/centralized scheme can be emulated [45]. The utility used in this work will be described in detail in Chapter 3.

The third assumption is regarding the information sharing between users. Users might not share any information at all or share all possible information. Our work focuses on no information exchange. Each user measures the received power and interference it receives from other links assuming the number of links and channels available in the network are broadcasted in the beacon signal by users such as in Bluetooth and WiFi standards [46].

The fourth assumption is that the best response algorithm leads to NE (when there exists a NE in the system). NE is not an artificial imposition on the users or the distributed networks, rather it is an equilibrium solution, which the system will converge to eventually. If we let the users to independently choose the strategy that maximizes their individual payoff, after some time the system will naturally and eventually reach NE [47]. NE is further discussed in Chapter 3. In bigger networks there can be multiple NE's.

The fifth assumption is that good error control techniques are implemented in the links. System model as mentioned in Chapter 2.1 entails a dense network. Especially links sharing channels face considerable amount of interference. We assume the links have good error control capabilities which allow them to communicate with minimum bit rate even in the presence high interference (as per Shannon's formula, Equation 2.5). Over iterations, links could find other channels that have lower interference and decide to transmit at increased data rate as well.

Based on these assumptions we are going to allocate the resources available in the distributed wireless network using Game Theory, a mathematical tool which has been extensively used in similar scenarios.

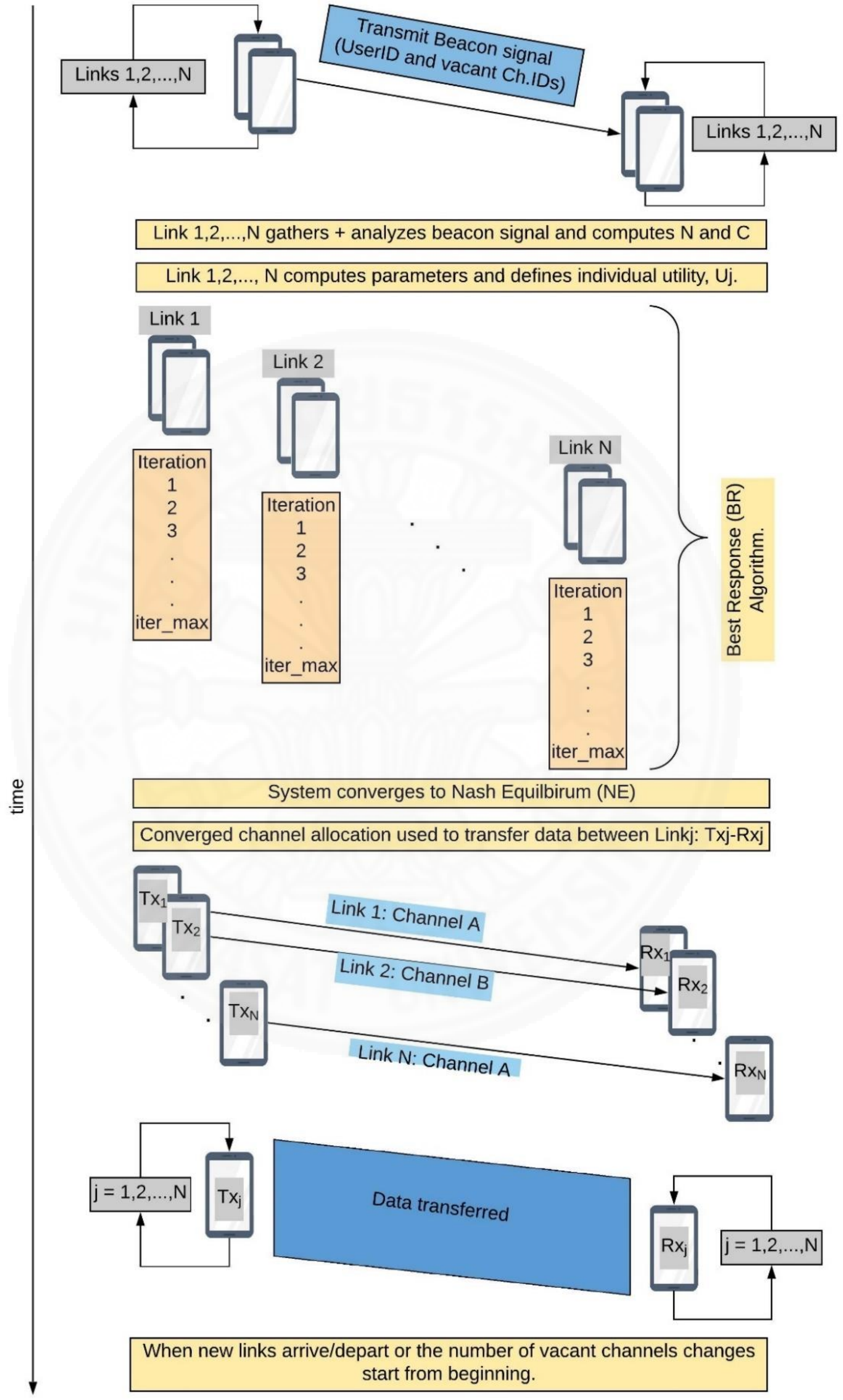

Figure 2.3 Physical/Network protocol for the links to converge to NE - transmit data.

CHAPTER 3
GAME THEORY

Game theory is a mathematical tool that is used to analyze the strategic interactions of users, especially with conflicting interests. Over the years game theory has been used in distributed wireless networks by users for resource allocation problems [25].

In this work, the players are the users or links; there are *N* links. Each link has a collection of strategies that it can adopt. S_j is the set of strategies of $Link_j$. U_j is the utility or payoff of $Link_j$, which is a function of the strategy played by the users, $< N; S_j ; U_j(S_j ; S_{-j}) >$ where $j \in N$ and S_{-j} represents the strategies played by all users apart from *j* [36],[48]. Each link chooses to transmit in a specific channel which results in different U_j.

Rational and independent users are assumed to choose a strategy that will maximize their U_j in every iteration; eventually the system converges to a Nash Equilibrium (NE) [30].

3.1 Nash Equilibrium

Nash Equilibrium (NE) is a natural and eventual equilibrium state reached by rational and independent users in a distributed network [49]. In each iteration every user chooses a strategy that will maximize its individual utility and after some time the system converges to NE. At NE, none of the user will be able to increase its utility by unilaterally changing its strategy given the strategies of other users remain unchanged [31], which is expressed mathematically as

$$U(S_j^*, S_{-j}^*) \geq U(S_j, S_{-j}^*) \; \forall j \in N, \tag{3.1}$$

where S_j^* and S_{-j}^* represent the best response strategy of user *j*, and all other users except *j*, respectively.

Nash Equilibrium is an important concept and an equilibrium solution in a distributed network with rational and independent users. A centralized cooperative global solution may result in a higher overall performance, but once the rational and selfish users are allowed to make their independent choices, they will drift away from the centralized global solution [50].

In a system, there can be multiple NEs. We want to converge to NE with higher performances which is the goal of the work. It depends on how the payoff/utility of a user is defined. In this work, the utility of a $User_j$ is defined as,

$$U_j = T_j + \alpha^*_{opt} \times I_j^{from} \tag{3.2}$$

where T_j is the individual user's throughput as defined in Equation 2.5, I_j^{from} is the interference received from other links to $Link_j$ as defined in Equation 2.4 and α^*_{opt} is the optimal weight of I_j^{from}. The optimal value α^*_{opt} is computed and mathematically derived in our work. In Chapter 3.7 other utility definitions which emulate different kinds of distributed wireless networks are presented and used as a reference for our work.

Once a system converges to a NE, none of the rational users will change their strategies, as by doing so they will get a lower U_j as defined in Equation 3.1. In every step a user chooses a strategy that gives him the highest individual payoff. A user can be playing a pure or a mixed strategy, which is discussed further in the following section.

3.2 Other Equilibriums in Game Theory

Apart from NE solution there are other game theoretic equilibrium solutions which we shall discuss as follows:

3.2.1 Correlated Equilibrium

Generally, in a NE solution we assume that the users independently select a strategy that gives them higher payoff. However, in a correlated equilibrium a third party advocates a certain mix of the strategy to the users [85]. The users follow the guideline of the third party due to faith in it from previous results. The set of mix strategy NE is a subset of the correlated equilibrium set. At the correlated equilibrium the expected payoff of the user is higher, hence none of the users wish to deviate from the equilibrium set assigned by the third party [85].

3.2.2 Bayesian Equilibrium

Many times, in a network complete information about the game is not available. We might not know the type of the user, is it a PU or SU? At times, the user might be faking its type as well. Similarly, information about the payoff and strategy set of all users might not be fully available and hidden [88]. This kind of scenarios leads us to Bayesian games where the solution is Bayesian Nash Equilibrium (BNE). However, based on historical/past observations, probabilities on the types of users and payoff of the other users can be designated [88]. As the game is played the belief might change and hence the probability distribution can be updated based on Bayes' rules.

3.2.3 Evolutionary Stable Strategy / Equilibrium

This equilibrium concept can be better understood via the natural selection process in biological evolution. A species that adopts a certain trait which is favorable for its and the species long term growth gets inherited in its genes. The species that inherit this trait then increase their population and become dominant [87]. Similarly, in the game theory context, the users that select a favorable strategy for the betterment of the population will inherit those strategies and in the long run the users using that strategy will increase its dominance. And the users choosing inappropriate strategies their population will diminish over time. Generally, in the game theoretic context we assume the users are completely rational and are familiar with their payoffs. Unlike that in Evolutionary Stable Strategy (ESS) it is assumed that the users like in biological species inherit certain strategies/traits without fully acknowledging the entire situation [87].

3.2.4 Stackelberg Equilibirum

In many situations some users in a network have dominance over others. In the Stackelberg game, one user is the leader and makes the first move and the second user or the follower chooses the strategy as per the strategy of the leader [86]. In many scenarios First Move Advantage (FMA) exists and the leader user can reap that benefit. The Stackelberg game can be solved using subgame perfect Nash equilibrium (SPNE) [86].

3.2.5 Auction Game

Wireless resources especially spectrum can be sold just like in an auction [25] where the auctioneer sells the channels to users based on the bids they submit. Generally, the central entity or the PU (Primary User) who have the license to the spectrum or channel broadcast the availability of the channels [94]. Then the SU (Secondary Users) or Cognitive Radio who need the channels for communication purposes bid a price or favor to the PU [98]. The PU considers the bid of all the SU and awards the channel to the suitable SU. Few prominent models are briefly discussed below:

English Auction (Ascending-bid): price for the channel is sequentially increased from a low initial value and at sequence all the users decide either to accept the bid or leave the game. As the price of the bid becomes greater, only one user is remaining or bidding the price; who wins and pays the amount [99].

Dutch Auction (Descending-bid): price for the channel is sequentially decreased from a high initial value. At the beginning no one bids the high initial value. As the price of the bid is sequentially lowered, whoever accepts the bid wins and pays the amount [100].

Second-price (sealed-bid) Auction: each user submits a price for the channel in a concealed way simultaneously with the other users. Whichever user made the highest bid gets the channel and needs to make a payment equal to the second highest bid [101].

First-price (sealed-bit) Auction: each user submits a price for a channel to the central entity or PU in a concealed way, simultaneously with the other users. Whichever user made the highest bid gets the channel and needs to make a payment equal to his bid [100].

Since many people prefer the second-price auction because it compels the users to bid their correct value.

However, in our work we focus on NE as it is simple and computationally less resources are used [36]. In our future work we plan to further explore and implement other game theoretic equilibrium concepts.

3.3 Pure Strategy NE

In a pure strategy game, a user selects one strategy out of the strategy pool [30], [51]. As mentioned earlier, the strategy of the user in our work is the different channels for transmission. If a user transmits in Channel 1 then the user does not transmit in Channel 2 or 3.

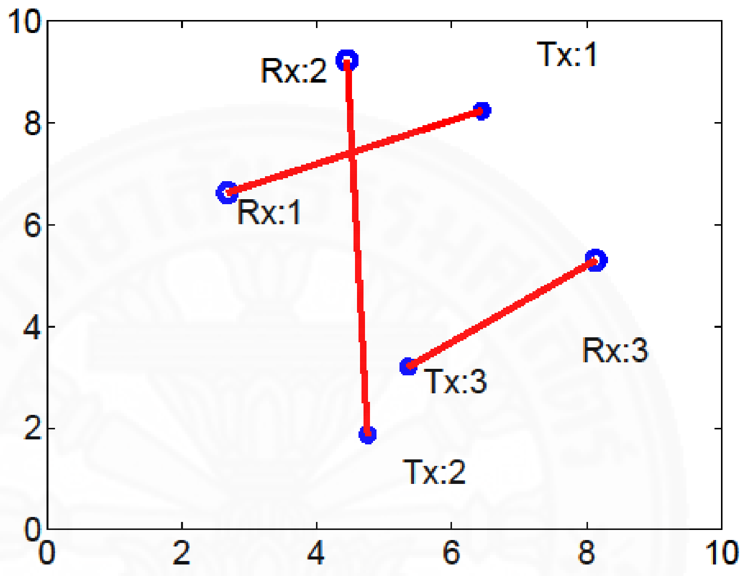

Figure 3.1 Distributed Wireless Network, random, “3 Links 2 Channels” scenario in 10 units by 10 units area.

Figure 3.1 shows an example of a random, “3 Links 2 Channels” scenario. There are three links which share two channels. Table 3.1 shows the exhaustive search analysis, all possible pure strategies for the configuration. Channel configuration ‘111’, ‘112’, ‘121’, ..., ‘222’ which denotes the channels used by $Link_1$, $Link_2$ and $Link_3$ respectively. All the eight possible configurations are listed. The second column lists the group; certain channel configurations have same grouping for example, ‘112’,‘121’,‘122’, etcetera are put into ‘122’ grouping as in all these cases one link gets a channel and the remaining two links share another channel.

As the channel configuration changes, the throughput of each link varies as the interference from links using the same channel is going to vary as shown in Table 3.1. T_{tot} for each configuration is calculated. We can see that certain configurations result in higher T_{tot} while other configurations give medium and low values of T_{tot}. In Table 3.1, two channel configurations (with bold text) are in NE with utility as in Equation 3.2.

Table 3.1 Exhaustive Search of random, "3 Links 2 Channels".

No.	Ch. Config.	Group	Throughput				NE?
			$Link_1$	$Link_2$	$Link_3$	Total	
1	111	111	0.7261	0.1129	0.7054	1.5444	0
2	112	112	1.3568	0.1271	6.3856	7.8695	0
3	121	112	1.0730	4.2816	0.9553	6.3099	0
4	**122**	**112**	**5.9186**	**0.7318**	**1.5226**	**8.1730**	**1**
5	**211**	**112**	**5.9186**	**0.7318**	**1.5226**	**8.1730**	**1**
6	212	112	1.0730	4.2816	0.9553	6.3099	0
7	221	112	1.3568	0.1271	6.3856	7.8695	0
8	222	111	0.7261	0.1129	0.7054	1.5444	0

Table 3.2 shows why the channel configuration '112' is a NE as per Equation 3.1. The third, fourth and fifth row of the table shows how the $Link_1$, $Link_2$ and $Link_3$ respectively change their strategies (bold text) while other user's strategies are fixed. As none of the users can get higher pay-off by unilaterally changing their strategies provided others do not change their strategies; the channel configuration '112' is a NE. Similarly, all other strategies in Table 3.1 were checked and there are only two NE strategies for the random, "3 Links 2 Channels" scenario. For networks with larger number of links there are more NE's and certain scenarios might not have any NE as well. In the next section we simplify the analysis by making some approximations; so generic solutions can be proposed.

3.4 Approximation using mean ($1/d^2$)

In Figure 2.1 and Figure 3.1 the co-ordinates of the transmitters and receivers are randomly selected. As we move from one random realization to another, the co-ordinates of the Tx and Rx vary greatly hence, it is difficult to make general conclusions. So, we are going to take the mean of ($1/d^2$) to approximate the random variable $1/d^2$ where d is the distance between the Tx_i and Rx_j. This helps us to approximately calculate the value of shared utility, individual/total throughput and make general conclusions. From 10^5 random scenarios the mean of $1/d^2$ is 0.11 units.

Table 3.2 Verifying Channel Configuration '112' to be NE.

Ch. Config.	Throughput			NE
	$Link_1$	$Link_2$	$Link_3$	
122	**5.9186**	**0.7318**	**1.5226**	Yes
222	**0.7261**	0.1129	0.7054	
1**1**2	1.3568	**0.1271**	6.3856	
12**1**	1.0730	4.2816	**0.9553**	

As per definition of utility in Equation 3.2, the shared utility of a link for random, "3 Links 2 Channels" scenario will be: a link gets a single channel, two links share a channel and three links share a channel as illustrated below.

$$U_{s1} = log_2\left(1 + \frac{\frac{1}{d^2}}{P_N}\right)$$

$$U_{s2} = log_2\left(1 + \frac{\frac{1}{d^2}}{\frac{1}{d^2} + P_N}\right) + \alpha\left(\frac{1}{d^2}\right) \tag{3.3}$$

$$U_{s3} = log_2\left(1 + \frac{\frac{1}{d^2}}{\frac{1}{d^2} + \frac{1}{d^2} + P_N}\right) + \alpha\left(\frac{1}{d^2} + \frac{1}{d^2}\right)$$

U_{S1}, U_{S2} and U_{S3} are the utility of user 1 or 2 or 3 when they get a single channel, share a channel with another user or share a channel with two other users. When $\alpha = 0$; $U_{S1} = 6.82$, $U_{S2} = 0.99$ and $U_{S3} = 0.58$. In Chapter 5 and 6 we are going to vary the value of α from 0, 2, 4, …, 100 for further analysis on policy formulation.

3.5 Mixed Strategy NE

In this section, we are going to explore the outcome of the system when the users play mixed strategy. In a mixed strategy game each user plays several strategies simultaneously with certain probability. In mixed strategy Nash Equilibrium (MSNE) the players expected payoff is at least as good as the expected payoff of using any other strategy [36]. For example, the expected utility of $User_1$ using Channel 1 and Channel 2 is equal at equilibrium [52].

We are going to use Table 3.3 to compute the mixed strategy probability; where, *x*, *y*, *z* is the probability of User 1, 2 and 3 respectively, while playing the strategy Channel 1. Similarly, $\bar{x} = 1 - x$, $\bar{y} = 1 - y$, and $\bar{z} = 1 - z$ are the probability of User 1, 2 and 3 respectively while playing the strategy Channel 2.

Table 3.3 Best Response table of random, "3 Links 2 Channels" scenario using approximation ($1/d^2$) = mean($1/d^2$).

$User_1$	$User_2$	$User_3$ Ch. 1 (z)	$User_3$ Ch. 2 ($1-z$)
Ch. 1 (x)	Ch. 1 (y)	U_{s3}, U_{s3}, U_{s3}	U_{s2}, U_{s2}, U_{s1}
Ch. 1 (x)	Ch. 2 ($1-y$)	U_{s2}, U_{s1}, U_{s2}	U_{s1}, U_{s2}, U_{s2}
Ch. 2 ($1-x$)	Ch. 1 (y)	U_{s1}, U_{s2}, U_{s2}	U_{s2}, U_{s1}, U_{s2}
Ch. 2 ($1-x$)	Ch. 2 ($1-y$)	U_{s2}, U_{s2}, U_{s1}	U_{s3}, U_{s3}, U_{s3}

As per the MSNE definition, $U_{1(C=1)} = U_{1(C=2)}$, where $U_{1(C=1)}$ is the utility of $User_1$ choosing strategy Channel 1 and $U_{1(C=2)}$ is the utility of $User_1$ choosing strategy Channel 2 [52]. We will use the shared utility (U_{s1}, U_{s2} and U_{s3} as defined in Equation 3.3 and presented in Table 3.3) to solve the equations for all the users:

$$
\begin{aligned}
U_{s3}xyz + U_{s2}xy\bar{z} + U_{s1}\bar{x}yz + U_{s2}\bar{x}y\bar{z} &= U_{s2}x\bar{y}z + U_{s1}x\bar{y}\bar{z} + U_{s2}\bar{x}\bar{y}z + U_{s3}\bar{x}\bar{y}\bar{z} \\
U_{s3}xyz + U_{s2}xy\bar{z} + U_{s2}\bar{x}yz + U_{s1}\bar{x}y\bar{z} &= U_{s1}x\bar{y}z + U_{s2}x\bar{y}\bar{z} + U_{s2}\bar{x}\bar{y}z + U_{s3}\bar{x}\bar{y}\bar{z} \\
U_{s3}xyz + U_{s1}xy\bar{z} + U_{s2}\bar{x}yz + U_{s2}\bar{x}y\bar{z} &= U_{s2}x\bar{y}z + U_{s2}x\bar{y}\bar{z} + U_{s1}\bar{x}\bar{y}z + U_{s3}\bar{x}\bar{y}\bar{z}
\end{aligned}
\tag{3.4}
$$

Simplifying these equations and solving for *x* and *y* and *z* results in $x = ½$, $y = ½$ and $z = ½$. Which basically means, all the three links choose Channel 1 or 2 with equal probability. Hence, the NE mixed strategy of a user is to use Channel 1 and Channel 2 with a probability of ½ and ½.

3.6 Methods to converge to (attain) NE

There are various ways we can converge to (attain) NE, Exhaustive Search [37], Method of Successive Averages (MSA) [53], Best Response (BR) [56], which are used in our work is discussed below. Other algorithms such as Lemke-Howson is used for two player bi-matrix games [55]; Simplicial Subdivision is used for n player games; and recent and more efficient algorithm Govindan-Wilson [54] also lead to NE.

3.6.1 Exhaustive Search

This technique explores all the possible configurations, and hence it gives us an exhaustive analysis of the various performance metrics of the distributed wireless network [37]. For example, there are $2^3 = 8$ different channel configurations for random, "3 Links 2 Channels" scenario which is shown in Table 3.1. The table also shows the individual and total throughput of the users. To check if a configuration is a NE or not, we need to make sure that each user cannot get a higher pay-off by deviating from the NE strategy set, as shown in Table. 3.2, and explained in Chapter 3.3.

As we move to bigger networks, exhaustive search becomes cumbersome. For random, "5 Links 3 Channels" scenario there are $3^5 = 243$ different channel configurations; whereas for random, "10 Links 4 Channels" scenario, there are $4^{10} = 1,048,576$ different channel configurations; and for bigger networks the exhaustive search analysis becomes impossible. Some of our analysis of network metrics use exhaustive search method for random, "5 Links 3 Channels" and random, "10 Links 4 Channels" scenarios which are presented in Chapter 4 and Chapter 5.

3.6.2 Method of Successive Averages (MSA)

MSA algorithm is simple, clear and requires less computer processing and memory [53]. Steps involved in computing MSA:

Step 1: Compute the expected Utility of a user at all channels.
[For the "3 Links 2 Channels" scenario; compute, E $(U_1|C = 1)$ and E $(U_1|C = 2)$].

Step 2: Compute $\bar{s}$ which gives probability of 1 to strategy that results in higher output and gives a probability of 0 to other strategies.

[For example, if $E\ (U_{1|C=1}) > E\ (U_{1|C=2})$ then $\bar{s} = [1\ 0]$ which means, $User_1$ transmits at Channel 1 with probability of 1 and transmits at Channel 2 with probability of 0].

Step 3: Compute (s^n) using,

$$s^n = (1/n)\ \bar{s} + (1 - 1/n)\ s^{n-1} \qquad (3.6)$$

where, (s^{n-1}) is the strategy from the previous iteration, $\bar{s}$ is determined in the previous step and n stands for the iteration number.

Step 4: Repeat for all users.

[For the "3 Links 2 Channel" scenario, we need to repeat steps 1, 2 and 3 for all the three users. So, at the end of this step we will be able to get a new strategy set]

Step 5: Continue the iteration until the solution converges, that is the gap between the subsequent iteration is minimal or zero.

An advantage of MSA is that it is possible to reach mixed NE as well, which is be difficult to reach via exhaustive search or algebraic equations especially in bigger scenarios.

3.6.3 Best Response (BR) Algorithms

This technique is practiced in every-day life to reach an equilibrium point where many independent and selfish users exist. A user chooses the strategy that gives the highest payoff. Then, the next user chooses a strategy that gives that particular user the highest payoff; all users keep doing this until they reach an equilibrium point [56].
Generally, best response technique converges to NE if the game is a potential function [104].

However, in Chapter 6 we start by assuming the system converges to NE at a particular channel configuration (that results in high T_{tot}). Then from the NE constrains, we derive the PDF and mean value of α^*_{opt}which is the key parameter in the utility function (Equation 3.2). So, using best response technique in the derived utility pushes the distributed network to converge to NE (mostly) with high performance. Different versions of the best-response technique are possible, which are explained in steps below:

Best Response, Simultaneous (*BR_S*)

Step 1: Users select a random channel allocation.

Step 2: $User_1$, $User_2$, ..., $User_N$ all scan the environment and then select the best-strategy simultaneously. When all the users complete their best-strategy selection and implementation, one iteration is complete.

Step 3: All the users repeat Step 2 until the $iter_{max}$, final iteration.

Best Response, Round Robin (*BR_Rr*)

Step 2: $User_1$ scans the environment and then selects the best-strategy. After $User_1$ makes the best-strategy decision and implements it, $user_2$ scans the environment and makes the best-strategy choice. This goes on until the N^{th} user. When the N^{th} user completes its best-strategy selection and implementation, one iteration is complete.

Note: Step 1 and Step 3 are the same as in algorithm BR_S.

Best Response, Random (*BR_Rn*)

Step 2: Out of the *N* links, one link randomly scans the environment, and then selects the best-strategy. One iteration is complete after one random user chooses the best-response strategy and implements it.

Note: Step 1 and Step 3 are the same as in algorithm BR_S.

Out of the three best response techniques illustrated, the round-robin algorithm converges to the distributed solution fastest, as in every iteration, $User_1$ to $User_N$ choose their best response strategy, and the other users know the strategy chosen by the previous users. In our work, the round-robin technique is used as the default technique.

In the three above-mentioned algorithms each individual user tries to maximize its own utility, U_j. Utility can be defined in different ways to emulate different scenarios which is discussed in detail in Chapter 3.7.

3.6.4 Algebraic Equations

Algebraic equations can be used to solve for NE in a game [36]. For example, in Chapter 3.5 the mixed strategy NE solution to random, “3 Links 2 Channels” scenario was obtained using a characteristic of NE. At NE each user is indifferent between any of the pure strategies played. Therefore, for the random, “3 Links 2 Channels” scenario the expected payoff of $User_1$ playing strategy Channel 1 should be equal to the expected payoff of $User_2$ playing strategy Channel 2; $U_{1(C=1)} = U_{1(C=2)}$. The solution was obtained in Chapter 3.5.

Another way of solving the NE algebraically is by solving expected utility of a user and performing a derivative with respect to the user's strategy and solving it by equating it to zero. As, at NE a user is playing its best strategy to maximize its utility. Hence,

$$\frac{d(E(U_1))}{dx} = 0; \frac{d(E(U_2))}{dy} = 0; \frac{d(E(U_3))}{dz} = 0. \quad (3.7)$$

Solving these gives values for *x, y* and *z* which is same to the value obtained in Chapter 3.5.

3.7 Varying Utility definition to portray different wireless networks

By defining the utility in different ways, it is possible to portray different wireless networks with different characteristics and performances. Some of the important ones are listed as follows:

3.7.1 Selfish and Distributed

The users are independent that is there is no central controller enforcing rules and regulations on the individual users. Each user is selfish and tries to maximize its own throughput [25]. The individual utility is defined as,

$$U_j = T_j \quad (3.8)$$

There is no information exchange between the users. A user can measure the received power and interference it receives, and hence compute individual throughput.

3.7.2 Centralized and Altruistic

This scenario portrays a centralized and co-operative wireless network. Generally, a central unit, which controls/oversees the activities of all users is present. Users report the information they sense locally to the central unit, and the central unit decides the individual user's strategy that is beneficial for the entire network [23]. In [90], individual utility is defined as the total throughput of the network,

$$U_j = \sum_{j=1}^{N} T_j \quad (3.9)$$

Each user in the network tries to maximize the total throughput of the system, and hence the users are considered altruist. The users share all information with each other. The solutions reached in such a network is also referred as global optimal solutions.

3.7.3 Distributed with Information Exchange

This represents a distributed wireless network with information exchange. There is no central entity that controls the users and hence the users compute the optimal strategies by themselves. However, the users need information from other users, so they can make proper decisions in a distributed way. In our previous work [35], we modified the utility presented in [18] to include terms I_j^{from} and I_j^{to}, which are the sum of interference user *j* receives from other users, and gives to other users, respectively. Each user tries to maximize its individual utility U_j, which results in maximizing the received power and minimizing the interference it gives and receives from other users. In this way, a higher overall performance index is achieved compared to a selfish and distributed scenario.

$$U_j = \alpha\, p_{ij(i=j)} - \beta\, I_j^{from} - \gamma\, I_j^{to}, \tag{3.10}$$

where, α, β and γ are constants which can be a discrete 0 or 1 value [35].

3.7.4 Independent and Distributed

Independent users without a centralized entity is becoming popular in wireless networks. In this scenario, the users do not share information and collaborate with each other. They rely only on the information they themselves can measure such as, SINR, Interference Received, etc. (for example from the beacon signals in cellular network and WiFi) [97]. Each user tried to maximize its own utility. In this work, we are focusing on the utility which is defined as,

$$U_j = T_j + \alpha I_j^{from} \tag{3.11}$$

where T_j is the individual user's throughput, I_j^{from} is the interference received from other links to $Link_j$ and α is the weight of the I_j^{from}. Adding the optimal interference received to the utility pushed the network towards NE with higher performance.

In Chapter 4 and 5, we are going to show how distribution of different network metrics along with mathematical analysis lead us towards such a conclusion and in Chapter 6 and 7 we are going to do mathematical analysis for α and perform simulations for different sized network and available resources; so, we can enhance the performance of the distributed wireless network.

3.7.5 Different U_j definitions (3.8-11) converges to NE with varying performance

In this section, we are going to analyze how different definitions of utility as explained in Chapter 3.7.1-4 will converge to NE solutions with varying T_{tot}; using Best Response (BR) algorithms explained in Chapter 3.6.3. These scenarios were emulated and simulated in MATLAB. Figure 3.2 shows the total throughput (normalized) on the y-axis and the iteration number on the x-axis. This shows how different algorithms with different utility definitions converge to different values over the iterations.

The results are an average of 1000 random, "5 Links 3 Channels" scenarios. The highest total throughput at the last iteration is obtained, when all users are playing a co-operative game, $U_j = \sum_{j=1}^{N} T_j$ as defined in Equation 3.9. This can be considered as a global optimal solution where a central entity tries to maximize the total throughput of the system. We label this **BR_Uj_Ttot**. Although the global optimal value is the best, which is evident in Figure 3.2; in a distributed scenario with selfish and independent users with minimum information exchange among the users (focus of our work) the users will soon deviate from the global optimal solution and the performance deteriorates drastically.

The lowest performance is obtained when as all users are selfish and try to maximize their own individual throughput, $U_j = T_j$ as defined in Equation 3.8. This is labelled as **BR_Uj_Tj**. When each user tries to maximize its own throughput and not care about other users; the total throughput of the system is lower at the converged solution.

Algorithm **BR_Uj_RpIfrIto** gives higher performance than BR_Uj_Tj but not as high as BR_Uj_Ttot. In this algorithm, $U_j = \alpha\, p_{ij(i=j)} - \beta\, I_j^{from} - \gamma\, I_j^{to}$ as defined in 3.10. In this scheme, each user tries to maximize its received power but at the same time minimizing its interference received from other users and interference generated to other users.

The users in this scenario need to share information about the interference terms with each other. This has higher performance than BR_Uj_Tj because if each user does not just become selfish and maximize its own throughput but also tries to minimize its interference to/from other users the overall throughput of the system increases.

Algorithm **BR_Uj_TiαIfr** incorporates the policy we have proposed. Adding optimal interference received in a user's utility pushes the users to select BR strategies that converges to NE^{Good} as $U_j = T_j + \alpha^*_{opt} \times I_j^{from}$ (3.11) encourages individual users to not just increase its own throughput but rather to add an optimal value of interference as well. Such behavior was observed by the analyzing the distribution of network metrics at NE^{Good} which will be discussed in the upcoming sections. This has the second-best performance, after the centralized co-operative scheme. When we consider only the distributed schemes with no information exchange, this algorithm has the highest performance.

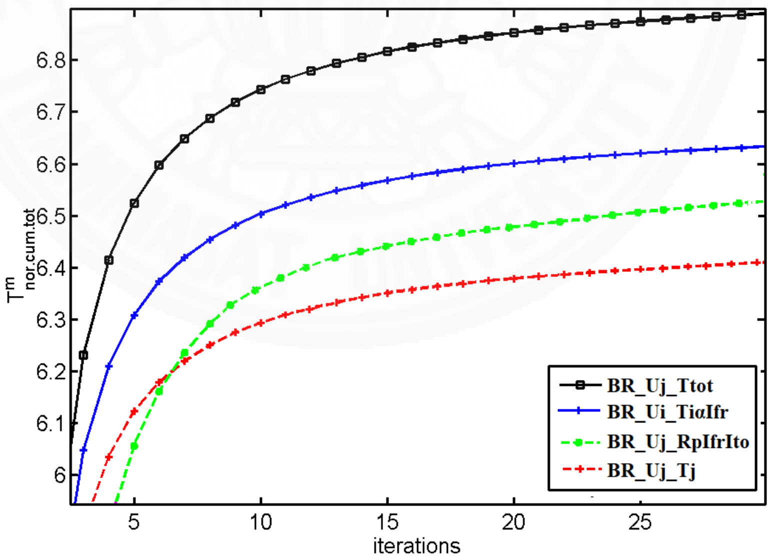

Figure 3.2 Compares the performance, T_{tot} (normalized over iterations) of different Best Response (BR) algorithms with different Utility definitions (3.8-11), based on 1000 random, "5 Links 3 Channels" scenarios [37].

Similar result is observed for random, “3 Links 2 Channels”, “10 Links 4 Channels” and “100 Links 20 Channel” scenarios which can be found in detail in our published works [37], [39], [61]. In the next section we are going to analyze network metrics so, we can formulate and finalize policies to enhance the system performance.

CHAPTER 4
DISTRIBUTION AND ANALYSIS OF NETWORK METRICS

In this chapter we are going to explore the network metric's characteristics. These results are then used to formulate channel allocation equations and policy.

The first network metric that we analyze is the link distance which is covered in Chapter 4.1. In Chapter 4.2 we explore the probability distribution function (pdf) and mean of SINR at NE for before/after channel allocation. Then in Chapter 4.3 we come up with an equation and scheme for channel allocation.

4.1 Link Distance

We start the analysis with a random, "3 Links 2 Channels" scenario like Figure 3.1. Let the distance between Tx_1 and Rx_1 be d_1, Tx_2 and Rx_2 be d_2 and Tx_3 and Rx_3 be d_3. There are total of $2^3 = 8$ channel configurations as listed in Table 3.1. Table 3.2 shows how each channel configuration can be checked for NE; there are two NE's for the random "3 Links 2 Channels" scenario. We explored four prominent cases for varying link distances and the summarized results are shown below:

Case A: $d_1 < d_2 < d_3$. The grouping at NE is '122' or '211'; $Link_1$ gets a single Channel 1; and $Link_2$ and $Link_3$ share Channel 2. The system is assigning a single channel (with no interference) to $Link_1$ which has the shortest distance; so, the throughput of $Link_1$ will be maximum and the two longer links ($Link_2$ and $Link_3$) share a channel.

Case B: $d_1 < d_2 = d_3$. When d_2 and d_3 are same, the NE reached in this case is, '122' or '211' which is like the scenario ($d_1 < d_2 < d_3$). This makes sense as a single channel is given to the link that has the shortest and the other two links share the channel.

Case C: $d_1 = d_2 = d_3$. When the three distances are equal, it makes sense to equally distribute the resources to all the links. From the exhaustive search Table 3.1, we can see there are two pure NE (apart from the '111' and '222' all the other configurations are NE); the system tried to distribute the resources evenly whenever possible.

Case D: $d_1 > d_2 > d_3$. This is similar to the initial scenario ($d_1 < d_2 < d_3$) just that the d_3 and d_1 and flipped. So, the NE solution reached is again '122' and '211'.

From this we can derive some simple policy, the shortest links gets a channel and the two longer links share another channel. However, as the distance between the links approaches and becomes equal to the other links, resources are shared as far as possible. After analyzing the NE solution, the general policy is: *"A single channel is given to a short link and the remaining two links share another channel"*.

Next, we did the exhaustive search for random, "5 Links 3 Channels" scenario. There are $3^5 = 243$ possible channel configurations as shown in Table 4.2. After analyzing the NE grouping and solutions the general policy we come up with is: *"A single channel is given to the shortest link and the remaining four links share remaining two channels. Out of the four links who share two channels; 85% of times, two links share a channel and other two links share another channel; 15% of times, three links share a channel and one link gets another channel".*

Table 4.1 Policy on Channel Allocation developed based on the link distance for different sized network and resources, channels available.

	Channel 1	Channel 2	Channel 3	Channel 4
"3 Links 2 Chs"	One Link	Two Links		
"5 Links 3 Chs"	One Link	One/Two Links	Three/Two Links	
"10 Links 4 Chs"	Two Links	Two Links	Three Links	Three Links
	Short Link ⟹			Long Link

Finally, we did the exhaustive search for random, "10 Links 4 Channel" scenario. Figure 2.1 is an example of such a scenario. Table 5.1 shows the 4^{10} = 1,048,576 different channel configurations of a random scenario. Each channel configuration is checked for NE. In total, we analyzed 1000 random scenarios with a total of $100 \times 1048576 \approx 1 \times 10^8$ different channel configurations. From this data a general policy obtained is: *"Four shorter links share two channels and six longer links share the remaining two channels"*.

Table. 4.1, combines the scenarios we analyzed and an enlists a simple policy obtained from them. (Further details can be found in our published paper [33]).

Next, we analyze the distribution of network metrics, which can give us other characteristics.

4.2 Statistical distribution of SINR, I^{from} and I^{to} at NE

In this section we are going to analyze the histogram distribution of network metrics at NE. We focus on the random, "5 Links 3 Channels" scenario. Table 4.2 shows the exhaustive search result of a random, "5 Links 3 Channels" scenario. There is a total of 3^5 = 243 different channel configurations; out of which there are mostly 6 or 12 or 24 NEs, depending upon the Tx-Rx co-ordinates and the location of the links. A total of 10,000 random scenarios were performed which was used to generate the PDF of SINR, I^{from} and I^{to} for before/after channel allocation. The flow chart in Figure 4.1 shows the steps/procedures used to obtain the data for Before/After Channel Allocation.

Which users get a single channel or sharing a channel at NE? The network metrics are segregated into the respective bins and their characteristics analyzed. The Probability Density Function (PDF) of some of the network metrics is shown in Figure 4.1.a and Figure 4.1.b.

4.2.1 Before Channel Allocation

All users are given the same channel (for example $User_1$, $User_2$, $User_3$, $User_4$, $User_5$ all communicate in Channel 1 that is channel configuration is: '11111'). The network metrics, SINR, I^{from} and I^{to} of each user, when the users use the same channel, is measured. Then, based on the exhaustive search analysis, the channel allocation at NE is determined. Figure 4.2 shows the PDF distribution of the network metrics using the Before Channel Allocation scheme.

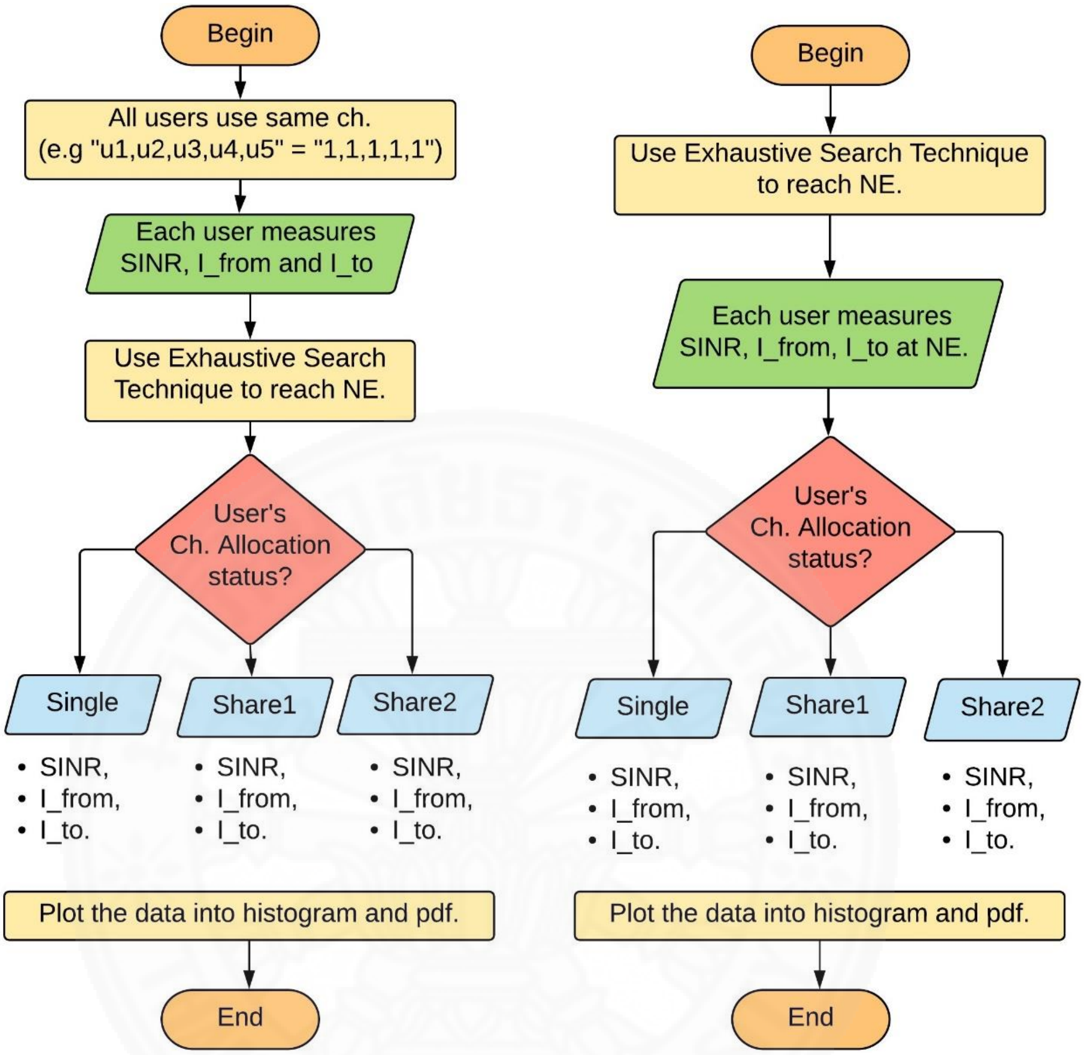

Figure 4.1 Explains using the flow-chart how the network metrics (SINR, I^{from} and I^{to}) of random "5 Link 3 Channels" scenarios is obtained and segregated into groups Single, Share1 and Share2 for a) Before Channel Allocation b) After Channel Allocation.

4.2.2 After Channel Allocation

The values of the network metrics at NE are taken and split into single and shared categories, depending upon how each of the users are allocated channels at NE. This case is labelled as "NE, after channel allocation". The PDF of some of the network metrics is shown in Figure 4.3.

The mean value of the SIINR when the link gets a single channel is much higher than when the link has to share the channel with 1 or 2 other links. Moreover, the average values of SINR at Before Channel Allocation (Figure 4.2) is lesser than the average values of SINR at After Channel Allocation (Figure 4.3). This is because at NE, the system should have converged to channel allocation solutions giving overall higher SINR to all users.

Table 4.2 Exhaustive Search of random, “5 Links 3 Channels” scenario.

No.	Ch. Config.	Throughput (T)						NE?	T_{tot}^{max}
		Link$_1$	Link$_2$	Link$_3$	Link$_4$	Link$_5$	*Total*		
1	11111	0.2492	0.1993	0.4283	0.9689	1.6524	3.4981	0	0
2	11112	0.3166	0.2179	0.6323	1.0608	9.7309	11.9585	0	0
3	11113	0.3166	0.2179	0.6323	1.0608	9.7309	11.9585	0	0
.				.			.		.
.				.			.		.
116	22132	0.5142	0.8399	5.9462	8.3674	1.9331	17.6007	0	0
117	**22133**	**0.9292**	**1.346**	**5.9462**	**3.1511**	**5.1593**	**16.5318**	**1**	**0**
118	22211	0.4714	0.2352	0.8156	3.1511	5.1593	9.8325	0	0
119	22212	0.3356	0.2137	0.5042	8.3674	1.713	11.1339	0	0
120	**22213**	**0.4714**	**0.2352**	**0.8156**	**8.3674**	**9.7309**	**19.6204**	**0**	**1**
121	22221	0.3166	0.2179	0.6323	1.0608	9.7309	11.9585	0	0
.				.			.		.
.				.			.		.
241	33331	0.3166	0.2179	0.6323	1.0608	9.7309	11.9585	0	0
242	33332	0.3166	0.2179	0.6323	1.0608	9.7309	11.9585	0	0
243	33333	0.2492	0.1993	0.4283	0.9689	1.6524	3.4981	0	0

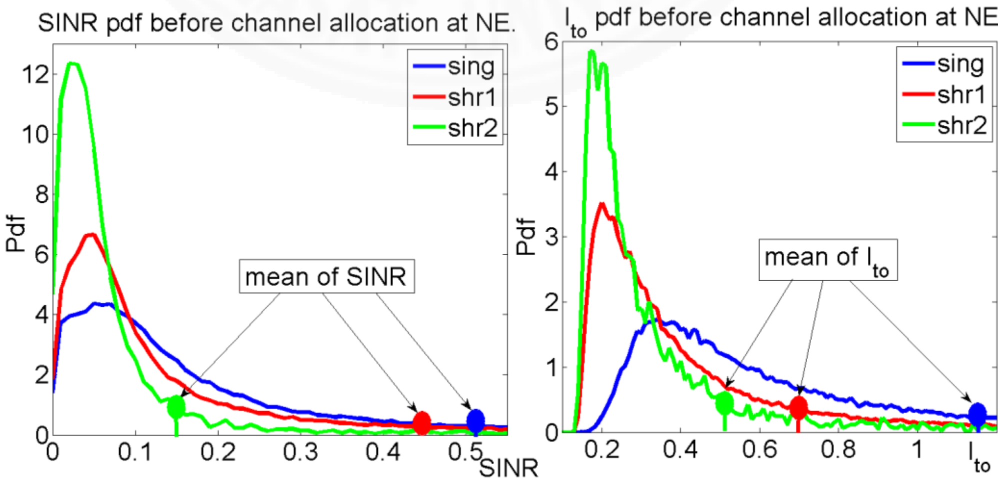

Figure 4.2 PDF of SINR (left) and I^{to} (right) for Before Channel Allocation (all users using same channel). Later the users getting single/shared channel at NE.

Random, “3 Links 2 Channels” scenario and random, “10 Links 4 Channels” scenarios were analyzed which produce similar results. The histogram plots presented in this section shows how the links with high SINR, high to/from interference tend to get good channels (less shared channels); hence, the policy is to allocate good channels (less shared channels) to links that have high SINR and high to/from interference. (Further details can be found in our published journal paper [37]).

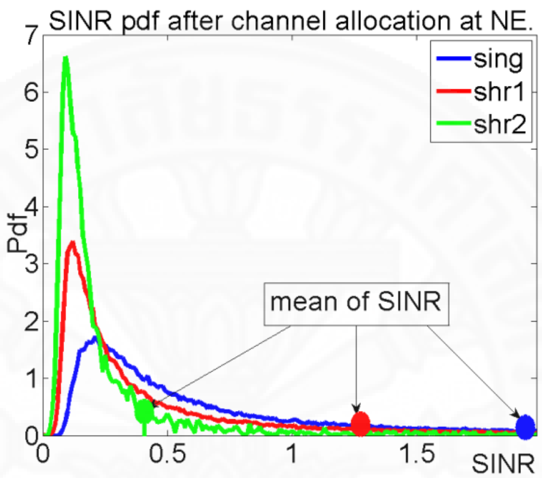

Figure 4.3 PDF of SINR at NE for After Channel Allocation. When a user gets a single or shared channel at NE.

4.3 Equation/Scheme for Channel Allocation

Based on the policy inferred in Chapter 4.1-2, we propose an equation that distributes the links into different channels, provided we know the link’s $SINR_j$, I_j^{from} and I_j^{to} before channel allocation. These PDF distributions and equations can be used to develop channel allocation schemes as illustrated in the flow chart Figure 4.4. The following steps are taken to derive the equation:

- If all channels share an equal number of links, then the number of links in Channel k can be expressed as, $c_k = N/C$, where N is the total number of links, C is the total number of channels and k is the index of the channel, $k = 1,\ldots,C$.
- We want to give preference to links that have high SINR and high interference terms. So, we sort the links based on these network metrics and allocate channels.

- c_1, c_2, have less links, which have high SINR and interference terms; whereas c_{C-1}, c_C, have more links with low SINR and interference terms. Hence, as the prefix of c, k increases, the value of ck increases as well so we have, $c_k = N/C + \theta \cdot k$. How fast c_k increases with an increment of k depends on θ, a constant which can be adjusted; in our current work, $\theta = 1$.
- To normalize c_k such that the $\sum_{k=1}^{C} c_k = N$, we need to divide the equation for c_k by the sum of c_k, and multiply by N, which results in, $c_k = \frac{\frac{N}{C}+\theta\cdot k}{\sum_{l=1}^{C}\left(\frac{N}{C}+\theta\cdot l\right)} N$
- Since the number of links in each channel can be an integer value only, the symbol ⌈::⌋ is used in the equation, which denotes nearest integer function. Hence the final equation, that represents the number of links in the kth channel, can be expressed as,

$$C_k = \left\lceil \frac{\left(\frac{N}{C} + \theta \cdot k\right) N}{\sum_{l=1}^{C}\left(\frac{N}{C} + \theta \cdot l\right)} \right\rfloor \quad and \quad \sum_{k=1}^{C} C_k = N. \tag{4.1}$$

- "3 Links 2 Channels": C_1 = 1 Link; C_2 = 2 Links,
- "5 Links 3 Channels": C_1 = 1 Link; C_2 = 2 Links; C_3 = 2 Links,
- "10 Links 4 Channels": C_1 = 2 Links; C_2 = 2 Links; C_3 = 3 Links; C_4 = 3 Links.

In this chapter complete information exchange is assumed as discussed. However, in Chapter 5 channel allocation schemes with minimum information exchange among the users will be focused. And the network metrics for distributed systems will be further analyzed. The policy (α_{opt}^{*}) formulated in Chapter 5 will be mathematically derived in Chapter 6.

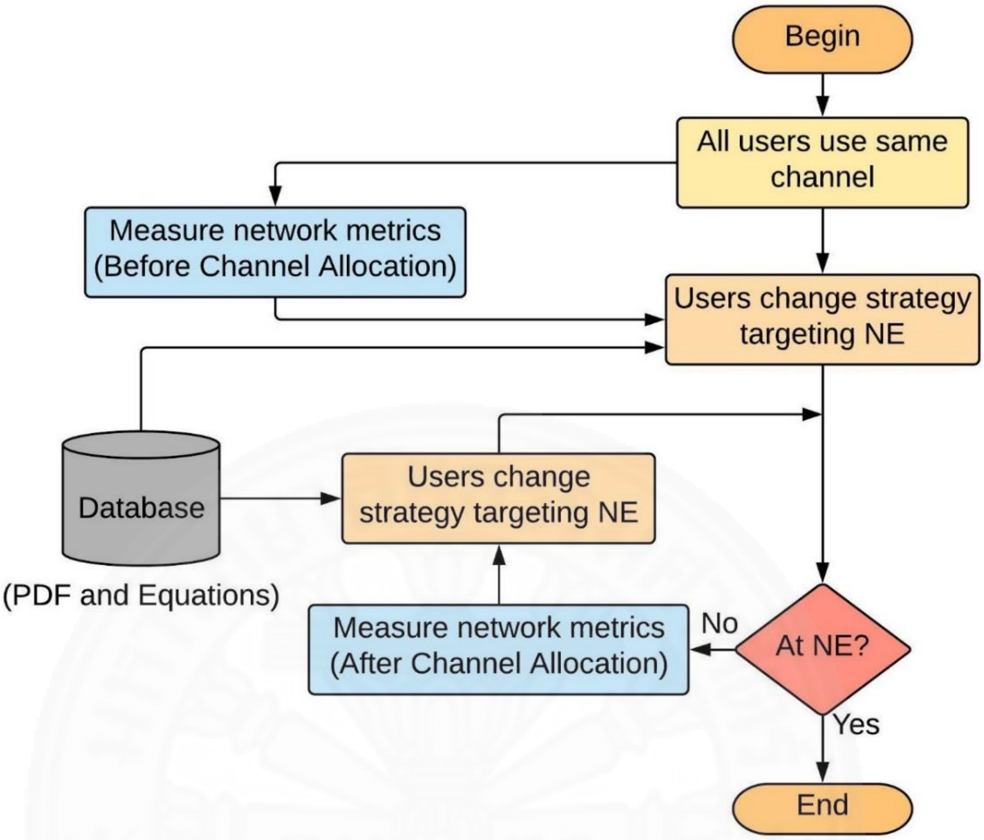

Figure 4.4 shows how network metrics distribution at NE can be used for efficiently used to allocate channel in a network to reach to NE faster and with better performance.

CHAPTER 5
POLICY FORMULATION ($\boldsymbol{\alpha}^*_{opt}$):
ANALYSIS OF NETWORK METRICS AT $NE^{GOOD\,/\,MEDIUM\,/\,LOW}$

In his chapter we are going to further analyze the network metrics features at NE with different performance. The results obtained are then used to formulate the policy which is then used to define the individual payoff of the user. So, the users in the distributed wireless network converge to a solution with higher performance.

5.1 T_j versus I_j^{from} Histogram plots at NE^{Good}, NE^{Medium}, NE^{Low}

Herein, we will focus on the network metrics (T_j and I_j^{from}) characteristics obtained from 100 random, "10 Links 4 Channels" scenarios. Table 5.1 shows a preview of one of the random scenario's Exhaustive Search results. There is a total of 4^{10} = 1,048,576 pure channel configurations in each random scenario (1st column).

Table 5.1 Exhaustive Search of random, "10 Links 4 Channels" scenario.

Number	Ch. Config.	Throughput							NE?	T_{tot}^{max}?
		$Link_1$	$Link_2$	$Link_3$	..	$Link_9$	$Link_{10}$	Total		
1	1111111111	0.172	0.139	0.122		0.014	0.260	1.667	0	0
2	1111111112	0.175	0.154	0.136	..	0.032	6.600	8.090	0	0
3	1111111113	0.175	0.154	0.136		0.032	6.600	8.090	0	0
.	.		.		.		.		.	.
.	.		.		.		.		.	.
101531	1231413233	2.437	1.941	0.358		0.024	1.149	16.372	0	0
101532	**1231413234**	**2.437**	**1.941**	**0.517**	..	**0.334**	**1.852**	**14.199**	**1**	**0**
101533	1231413241	1.921	1.941	0.920		0.196	0.383	12.619	0	0
.	.		.		.		.		.	.
.	.		.		.		.		.	.
611655	3222222123	3.058	0.164	0.169		0.034	3.006	16.692	0	0
611656	**3222222124**	**7.937**	**0.164**	**0.169**	..	**0.034**	**6.600**	**25.163**	**0**	**1**
611657	3222222131	0.448	0.171	0.197		0.787	3.034	10.126	0	0
.	.		.		.		.		.	.
.	.		.		.		.		.	.
1048574	4444444442	0.175	0.154	0.136		0.032	6.600	8.090	0	0
1048575	4444444443	0.175	0.154	0.136	..	0.032	6.600	8.090	0	0
1048576	4444444444	0.172	0.139	0.122		0.014	0.26	1.667	0	0

Second row lists the channel configuration starting from '1111111111' which denotes users- 1, 2, 3, …, 10 all using Channel 1. Channel configuration '1231413234' denotes users- 1, 4, 6 using Channel 1; users- 2, 8 using Channel 2; users- 3, 7, 9 using Channel 3; users- 5, 10 using Channel 4. This channel configuration is NE as per definition in Equation 3.1. Third column onwards lists the individual user's throughput (Equation 2.5) and ninth column lists the T_{tot} (Equation 2.6). The last two column's state if the channel configuration is NE and T_{tot}^{max} or not respectively? 1 denoting a yes and 0 denoting a no. The process of segregated the raw data is into four groups for analysis and policy derivation is described in the following steps:

***Step* 1:** Figure 5.1 shows the 100 random, "10 Links 4 Channels" scenarios. Y-axis shows the 100 random scenarios whereas the x-axis displays all the possible pure strategies (4^{10} = 1,048,576) as explained in the Exhaustive Search method and illustrated in Table 5.1. For each of the pure channel allocation the respective T_j, T_{tot} and I^{from} is computed.

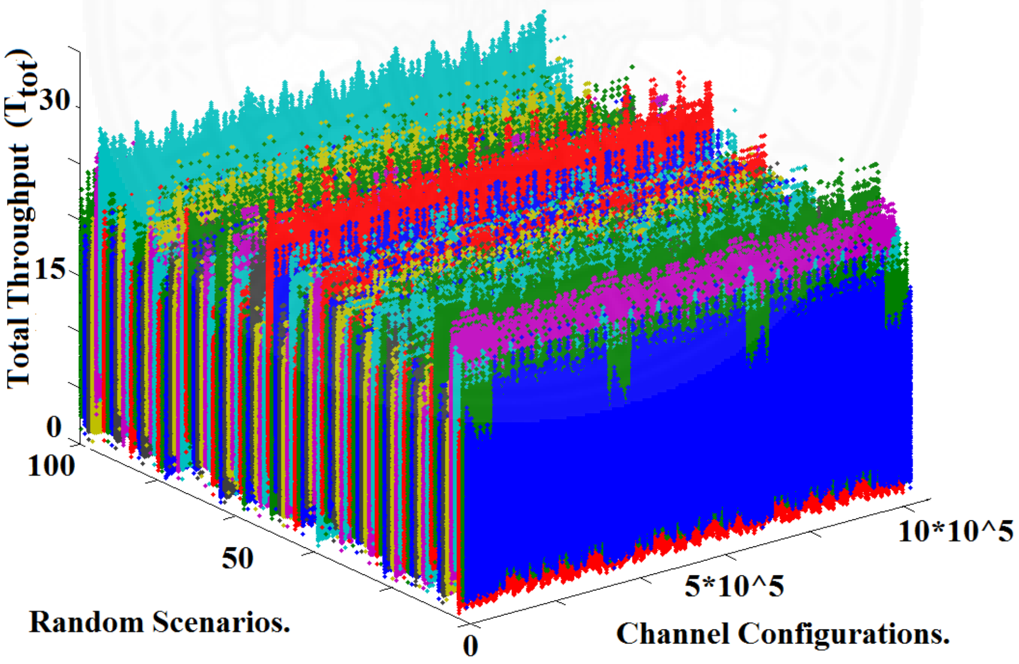

Figure 5.1. Exhaustive search of 100 random, "10 Links 4 Channels" scenarios. Each random scenario has 4^{10} = 1,048,576 pure channel allocations, which is represented in the x-axis. 100 random scenarios are plotted in the y-axis. The z-axis represents the T_{tot} for different channel configuration and random scenarios.

***Step* 2:** As the channel configurations changes for different random scenarios the value of T_{tot} (represented in z-axis of Figure 5.1) varies. When all links use the same channel, lowest T_{tot} is achieved; as all links share the same channel the interference is high and the other three channels are unused. Generally, a channel configuration that gives preference to shortest link by allocating them channels (shared with less users) and allocates longer links to channels (shared with more users) results in T_{tot}^{max}.

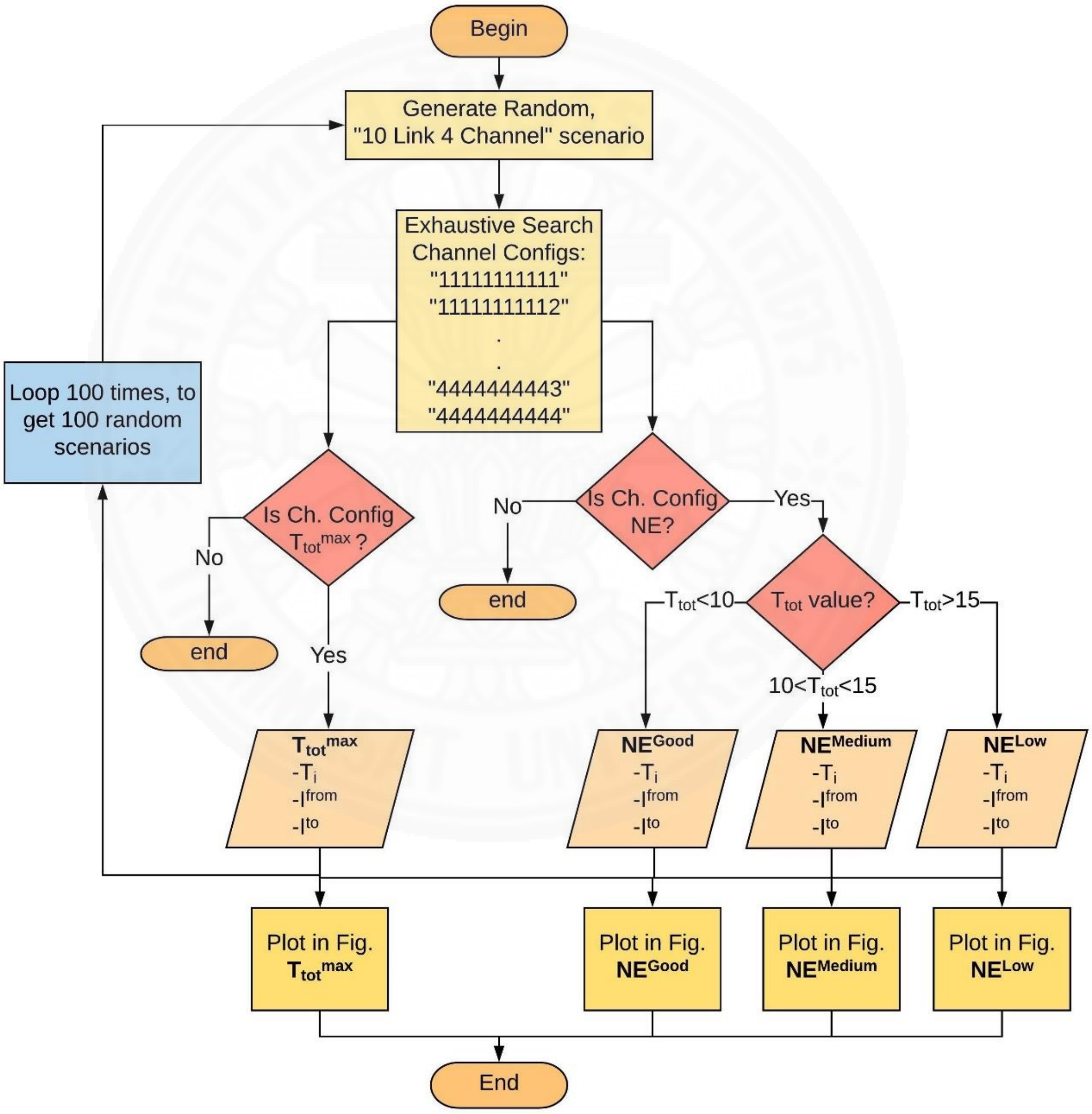

Figure 5.2. Flow chart showing how different values of *Ti*, I^{from} and I^{to} obtained from 100 exhaustive search of random "10 Links 4 Channels" are segregated into the four groups: $\boldsymbol{T_{tot}^{max}}$, $\boldsymbol{NE^{Good}}$, $\boldsymbol{NE^{Medium}}$, $\boldsymbol{NE^{Low}}$ based on their performance.

Out of the total 4^{10} = 1,048,576 channel configuration, those channel configurations that result in the maximum value of T_{tot}, i.e., T_{tot}^{max} along with the corresponding T_i, T_{tot} and I^{from} are put into the $\boldsymbol{T_{tot}^{max}}$ group.

***Step* 3:** For all pure channel configuration analyzed, check if every configuration is at NE or not? If it is, then we record the value of T_{tot} to segregate the data into groups.

-If T_{tot} is higher than 15, the channel configuration along with the corresponding network metrics (T_i, T_{tot} and I^{from}) are put into the $\boldsymbol{NE^{Good}}$ group.

-If T_{tot} is less than 15 and greater than 10, the channel configuration along with the corresponding network metrics (T_i , T_{tot} and I^{from}) are put into the $\boldsymbol{NE^{Medium}}$ group.

-If T_{tot} is less than 10, the channel configuration along with the corresponding network metrics (T_i, T_{tot} and I^{from}) are put into the $\boldsymbol{NE^{Low}}$ group.

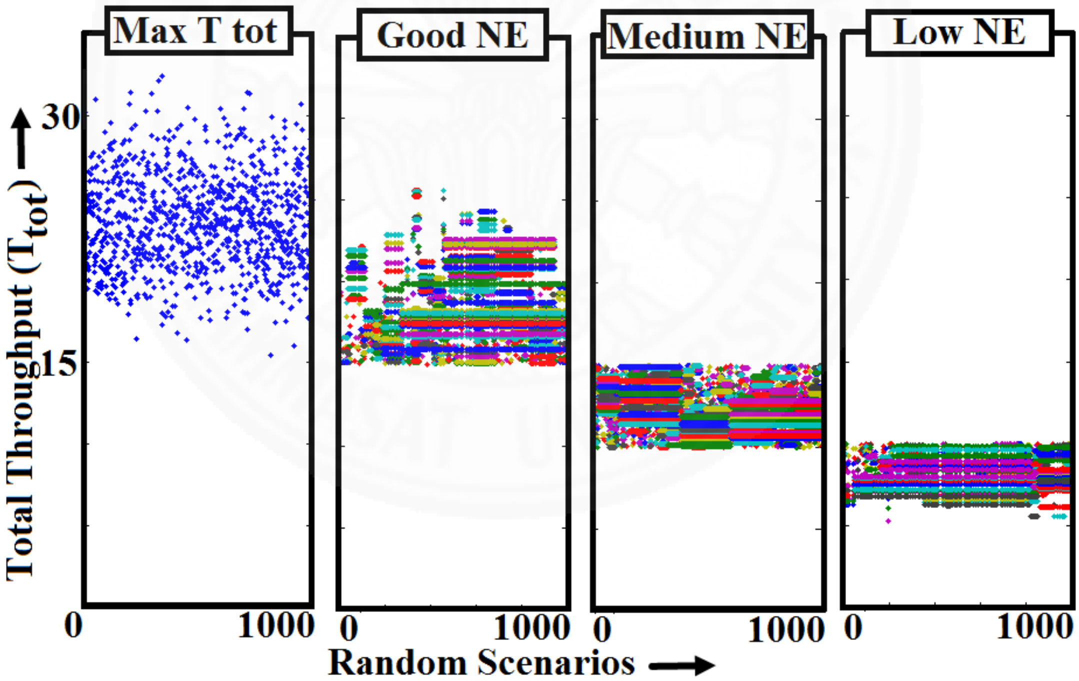

Figure 5.3 Data from 100 random scenarios are segregated into four groups: $\boldsymbol{T_{tot}^{max}}$, $\boldsymbol{NE^{Good}}$, $\boldsymbol{NE^{Medium}}$, and $\boldsymbol{NE^{Low}}$ based on T_{tot} value. The 100 random scenarios are plotted in the x-axis and T_{tot} is plotted in the y-axis. There is only one value of T_{tot} for a random scenario in the group $\boldsymbol{T_{tot}^{max}}$. However, there may be several values of T_{tot} for a random scenario in each of the groups: $\boldsymbol{T_{tot}^{max}}$, $\boldsymbol{NE^{Good}}$, $\boldsymbol{NE^{Medium}}$ and $\boldsymbol{NE^{Low}}$.

***Step* 4:** Generate another random, "10 Links 4 Channels" scenario. Repeat Step 1 to Step 3 until100 random scenarios are created.

There is only one set of channel configuration (grouping) in a random scenario that results in a T_{tot}^{max}; whereas there might be several channel configuration (grouping) that leads to NEs with different performance T_{tot} values; this is evident from Figure 5.3. The x-axis shows the 1,000 random scenarios and the y-axis shows the T_{tot}. The first sub-plot is of group $\boldsymbol{T_{tot}^{max}}$ when T_{tot} is maximum. The second sub-plot is of group $\boldsymbol{NE^{Good}}$. The third sub-plot is of group $\boldsymbol{NE^{Medium}}$. The fourth sub-plot is of group $\boldsymbol{NE_{Low}}$.

Next we examine the characteristics of T_i and I^{from} at the four groups $\boldsymbol{T_{tot}^{max}}$, $\boldsymbol{NE^{Good}}$, $\boldsymbol{NE^{Medium}}$, and $\boldsymbol{NE^{Low}}$. Figure 5.4 illustrates the 3D histogram plot of I^{from} versus T_i in the four groups. The I^{from} in group $\boldsymbol{T_{tot}^{max}}$ experiences much higher interference compared to groups $\boldsymbol{NE^{Good}}$, $\boldsymbol{NE^{Medium}}$, and $\boldsymbol{NE^{Low}}$. The T_i and I^{from} is more dispersed in group $\boldsymbol{NE^{Good}}$ than at groups $\boldsymbol{NE^{Medium}}$ and $\boldsymbol{NE^{Low}}$. T_i and I^{from} is more condensed in the lower value regions in the group $\boldsymbol{NE^{Low}}$.

Our aim is it to reach good NE and avoid low NE. Adding an optimal value of interference received term in the utility of a user pushes the independent and distributed users to select strategies such that the system converges to $\boldsymbol{NE^{Good}}$ with higher overall performance.

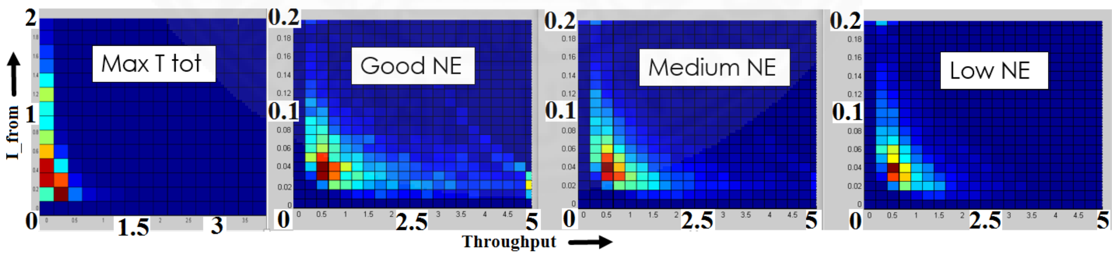

Figure 5.4 3D Histogram plot of individual user's- throughput (T_j) and interference received (I^{from}) at four groups: $\mathbf{T_{tot}^{max}}$, $\mathbf{NE^{Good}}$, $\mathbf{NE^{Medium}}$, and $\mathbf{NE^{Low}}$ are different. These attributes can be used to develop policies and implemented in the individual user's utility, to push the system towards NE^{Good}.

Hence, the payoff of a user is defined as in Equation 3.2, $U_j = T_j + \alpha_{opt}^{*} \times I_j^{from}$.

5.2 Varying α to obtain α^*_{opt} for random, "10 Links 4 Channels" scenario

In this section we are going to compute the optimal value of α which results in highest performance for random, "10 Links 4 Channels" scenario for utility, $U_j = T_j + \alpha^*_{opt} \times I_j^{from}$ (Equation 3.2). Figure 5.5 illustrates how varying the value of α from -1 to 16 results in different T_{tot} of the network. For each value of α 1,000 random "10 Links 4 Channels" scenarios are simulated and the average value of T_{tot} is displayed.

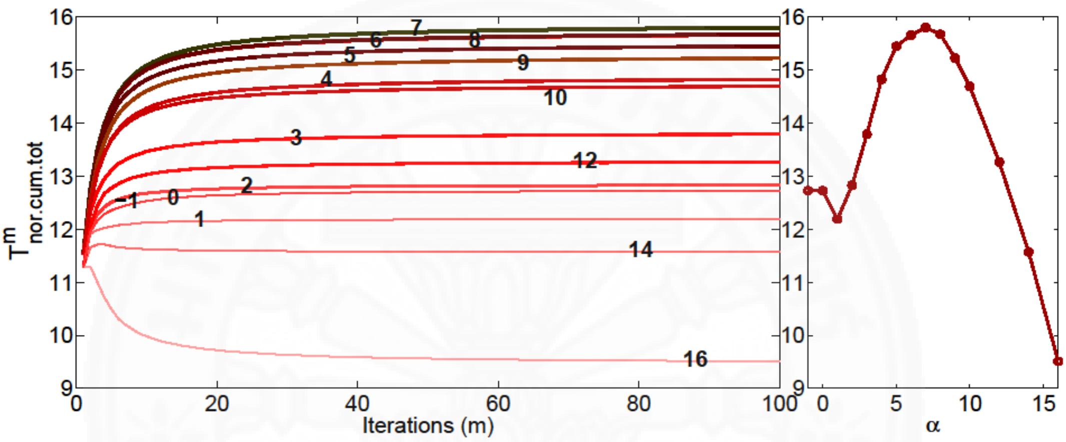

Figure 5.5 Variation of performance T_{tot} (normalized) plot as we vary the value of α from -1 to +16 in the individual utility of the user, $U_j = T_j + \alpha^*_{opt} \times I_j^{from}$ (Equation 3.2). Adding an optimal amount of interference received in the individual user's payoff increases the performance of the distributed wireless network. Results are obtained from an average of 100 random, "10 Links 4 Channels" scenarios.

Generally, the individual user's utility is defined as $U_j = T_j$, Equation 3.8. We implement the term I_j^{form} with an optimal weight α^*_{opt} in the individual utility. As we change the value of α from -1 to 16 the performance of the distributed network differs in a non-linear fashion. α^*_{opt}= 7 as highest total throughput is achieved as illustrated in Figure 5.5. As the value of α becomes larger, the performance is lower as we are giving too much emphasis to I_j^{form} term. A counter-intuitive result is noticed; adding an optimal value of I_j^{form} in the payoff gives enhanced performance. This is the characteristics of NE^{Good} as discussed earlier.

Next, we will find the value of α^*_{opt} for a dynamic network for varying *N* and *C*. Then based on these results a mathematical equation for α_{opt} is derived and proposed.

5.3 α^*_{opt} values for random, "*N* Links *C* Channels" scenario

In this section we are going to come up with the α^*_{opt} values for random, "*N* Link *C* Channels" scenarios based on the results of thousands of random simulations. We are going to run simulations with the number of links from $3 \leq N \leq 20$ and the number of channels in the network from $2 \leq C \leq 15$. For each of the value of *N* and *C*, 50 values of α from 0, 1, 2, 3, ..., 50 are used. And for each value of α, 100 to 1000 (depending on the size of the network) random scenarios are generated and let to converge to NE using the BR algorithm with utility of the individual user defined as in Equation 3.2, $U_j = T_j + \alpha^*_{opt} \times I_j^{from}$. For each value of *N* and *C* there is one optimal value of α (between 0 to 50) which results in T_{tot}^{max}; this is defined as α^*_{opt}. Figure 5.6 shows a flow chart illustrating the process of obtaining the value of α^*_{opt} for varying *N* and *C* which is then plotted in Figure 5.7.

With the help of software Eureqa [60] we propose two equations with different degrees of complexity and accuracy for,

$$\alpha^*_{opt} = \frac{104}{N} - \frac{125 \times C}{N^2} \tag{5.1}$$

$$\alpha^*_{opt} = \frac{71}{\left(N + 18 \times 8.65^{(1.69 \times C - N)} - \sin(192.38 \times C)\right)} \tag{5.2}$$

Equation 5.1 has around 10% error whereas Equation 5.2 has around 2% error and simpler. The distributed dynamic wireless network can converge to better network performance (higher values of T_{tot}) by optimally adjusting the value of α^*_{opt} as the number of links (N) and channels (C) in the network varies over time.

5.4 Performance Enhancement by Implementing policy (α^*_{opt})

From the Figure 5.8, we can observe that as we increase the value of C (number of channels), high values of T_{tot}^{max} can be achieved, this is because as the number of channels increases, resources are abundant reducing competition among users which results in higher performance.

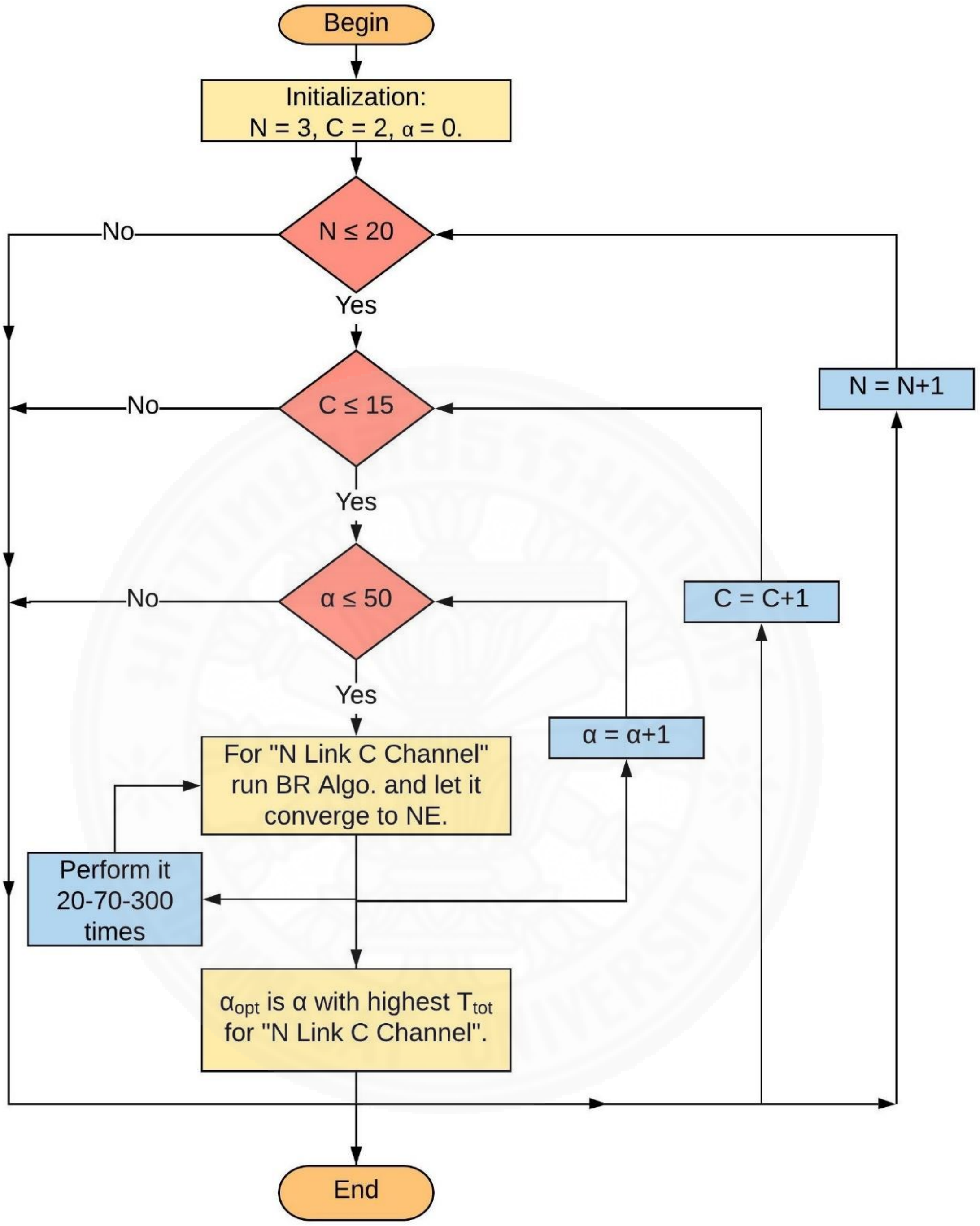

Figure 5.6 Illustrates how the values of α^*_{opt} for different sized network, (links -N) and available resource (channels - C) are obtained. Our analysis covers the range $3 \leq N \leq 20$ and $2 \leq C \leq 15$. For each scenario α is varied from 0, 1, 2, …, 50 and the value of α that results in T^{max}_{tot} is taken to be α^*_{opt}. Thousands of simulations are run for each value of α, N and C to obtain accurate results.

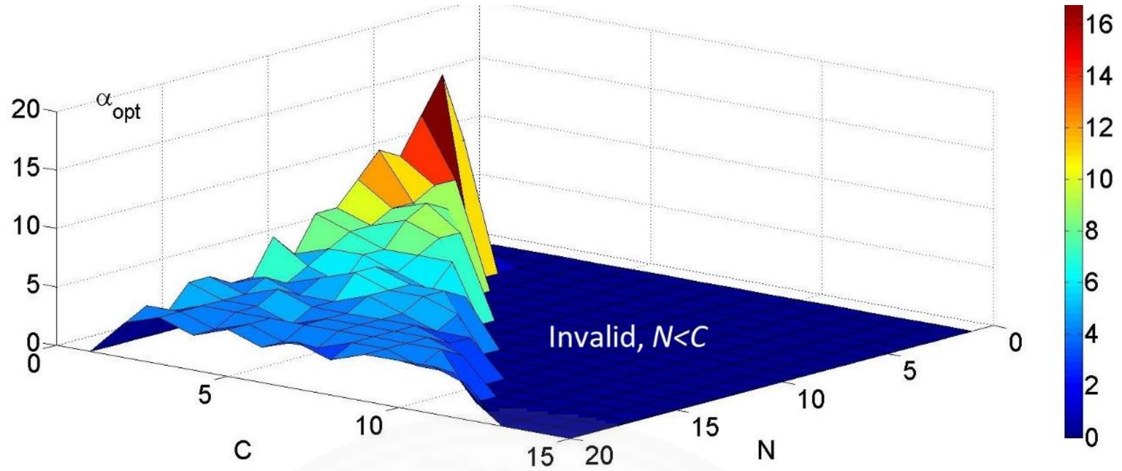

Figure 5.7 α^*_{opt} values that results in T_{tot}^{max} for varying links (N) and channels (C). Obtained from thousands of random scenarios for: $U_j = T_j + \alpha^*_{opt} \times I_j^{from}$.

For a constant *C* if we increase the value of *N*, then interference received (I_j^{from}) will increase; as more users are competing for the limited resource. As I_j^{from} increases α^*_{opt} decreases slowly to keep the utility (as defined in Equation 3.2) from increasing rapidly. This helps to create an optimal interference at the equilibrium point which is the characteristics of a NE^{High}.

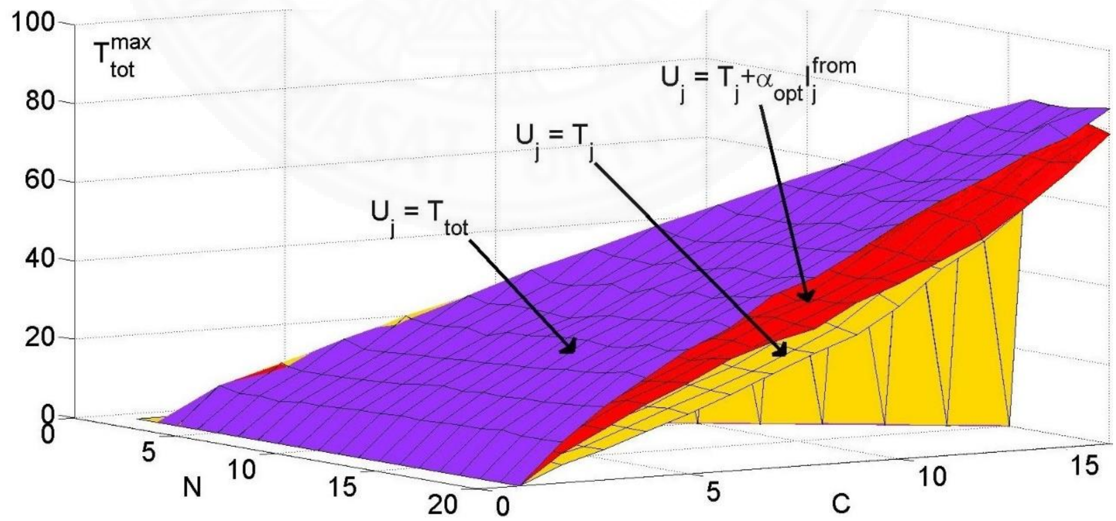

Figure 5.8 T_{tot}^{max} for varying links (N) and channels (C) obtained from thousands of random scenario simulations for: $U_j = T_{tot}$ [bench-mark, Equation 3.9]; $U_j = T_j$ [Equation 3.8]; $U_j = T_j + \alpha^*_{opt} \times I_j^{from}$[proposed policy, Equation 3.2].

The following procedure can be taken to implement our policy (α^{*}_{opt}) in a dynamic and distributed wireless network; also illustrated in a flow-chart diagram in Figure 5.9.

Step1: At the beginning (every 1-5-10 mins when the network refreshes) all the users in the network send beacon signals (like WiFi (SSID) or beacon signal of Cellular Network [97]) that includes the User ID and the vacant Channel IDs.

Step2: After gathering and analyzing the beacon signals the users infer the number of links N, and channels C in the network.

Step 3: Each user computes α^{*}_{opt} based from the values/equation presented in Chapter 5.3 and defines its utility as, $U_j = T_j + \alpha^{*}_{opt} \times I_j^{from}$.

Step 4: Using BR algorithm as explained in Chapter 3.6 each user maximizes its utility iteratively / simultaneously.

Step 5: After several iterations (20-70-300) the system converges to NE^{Good} – with better performance (high T_{tot}).

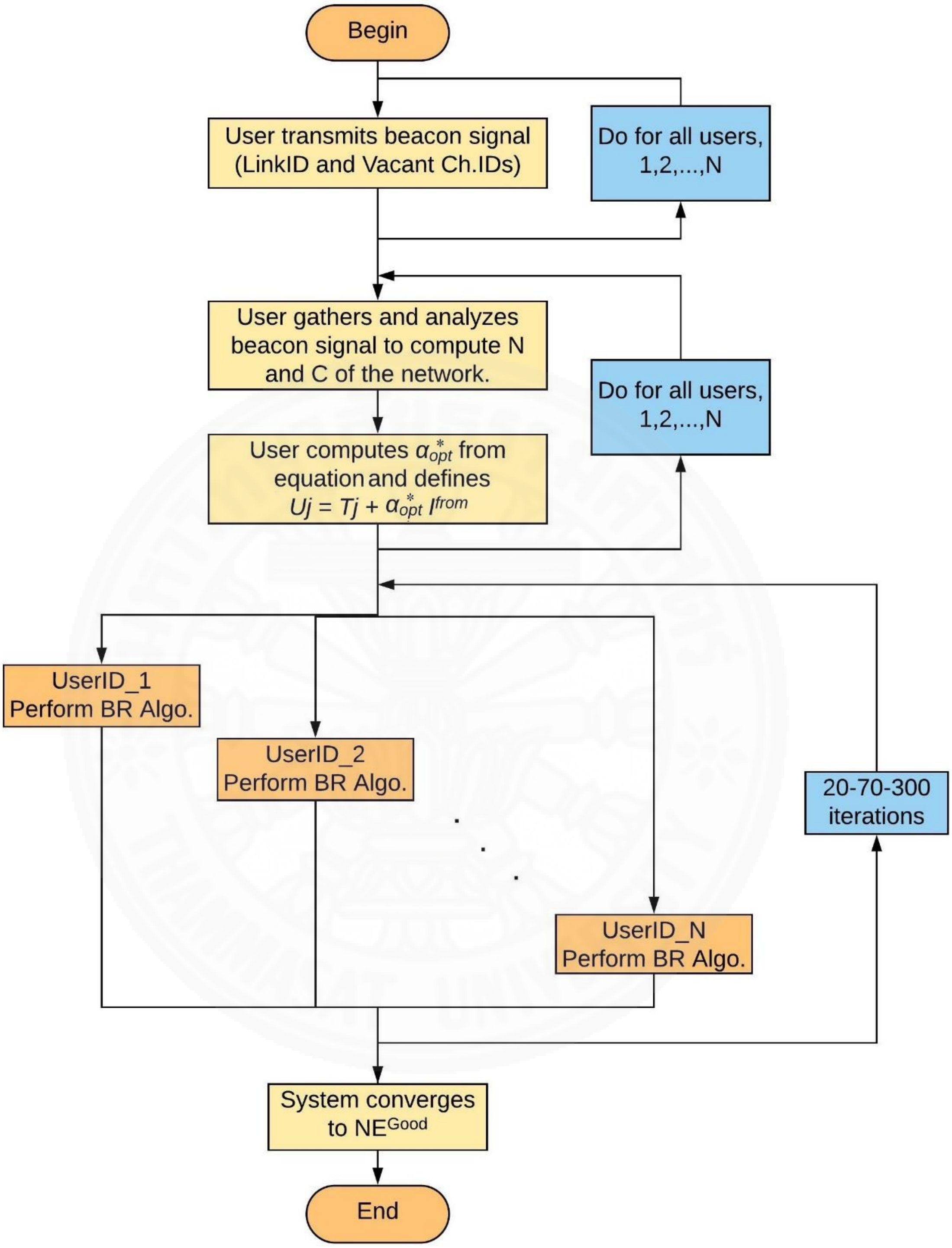

Figure 5.9 Illustrated via a flow chart diagram how the users in a dynamic distributed wireless network computes the value of α^*_{opt}, defines it payoff and using BR algorithm converges to in NE^{Good}.

CHAPTER 6
MATHEMATICAL DERIVATION AND APPROXIMATION OF α^*_{opt} (REFERRED AS $\hat{\alpha}_{opt}$)

In this chapter we are going to mathematically derive the approximate value α^*_{opt} (referred to as $\hat{\alpha}_{opt}$) along with the probability distribution function (PDF). In the previous chapters we studied different network metrics and showed the advantage of adding an optimally weighted (α^*_{opt}) interference received term in the individual link's utility; the distributed system converges to a NE solution with high T_{tot}. But to compute the value of α^*_{opt} for different networks is cumbersome (computationally and time-wise). Hence, in this chapter we mathematically derive an approximation of α^*_{opt} (referred to as $\hat{\alpha}_{opt}$) which is close to the value of α^*_{opt} but can be computed in a fraction of time and computational resources required to compute α^*_{opt}. We begin our analysis from random, "3 Links 2 Channels" scenario then move on to a medium sized scenario random, "5 Links 3 Channels", then a bigger scenario random, "10 Links 4 Channels". Then at the end we generalize the policy for random, "*N* Links *C* Channels" scenarios.

To proceed with the analysis, shared utility is defined based on the definition of a link's utility expressed in Equation 3.2: when a link gets a single channel, the utility of the link is U_{s1} which equals to $log_2\left(1+\frac{\frac{1}{d^2}}{P_N}\right)$. For U_{s1} there is no interference from other links. When L links share a channel, the utility of the link is defined as U_{sL} which is expressed as,

$$U_{sL} = log_2\left(1+\frac{\frac{1}{d_a^2}}{\sum_{m=1}^{L-1}\frac{1}{d_{Im}^2}+P_N}\right)+\alpha^*_{opt}\left(\sum_{m=1}^{L-1}\frac{1}{d_{Im}^2}\right) \quad \text{for } N \geq 2 \quad (6.1)$$

Here d_a is the distance of the signal between the transmitter and receiver of the same link and d_{Im} is the distance from the m^{th} interfering link where $m = 1, 2, \ldots, N-1$. The probability density function (PDF) of $1/d_a^2$ is illustrated in Figure 6.1. d_a is the distance between the transmitter (x_2, y_2) and the receiver (x_1, y_1) of a link and can be expressed as, $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$. As illustrated in Figure 2.1 x_1, x_2, y_1 and y_2

are random variables with uniform distribution between 0 to 10 units. To simplify our study, we assume the distribution of the interfering links d_{Im} $(m = 1, 2, \ldots, L\text{-}1)$ have the same distribution as d_a. In later sections, this distribution is used to compute the PDF of α^*_{opt}.

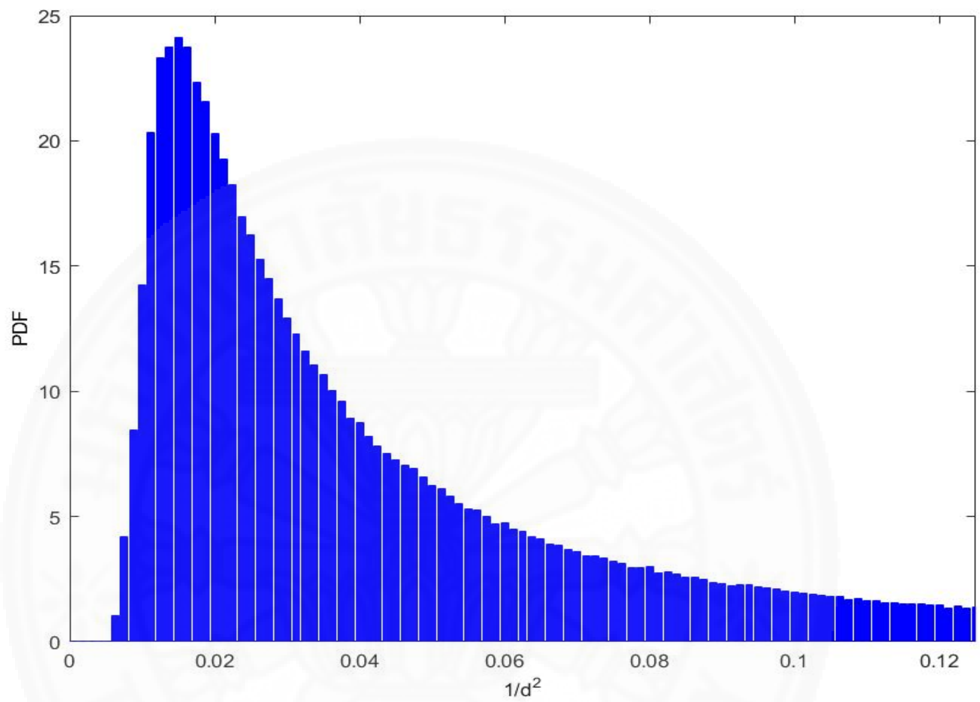

Figure 6.1 Probability Density function (PDF) of $1/d_a^2$. d_a is the distance of the signal between transmitter and receiver of the same link [mean $(1/d_a^2)$=0.11 units].

6.1 Random, "3 Links 2 Channels" and "5 Links 3 Channels" scenario

In this section we are going to vary the value of α (the weight of the interference received) and observe the change in the individual utility of the user as per definition of utility in Equation 3.2. This leads to the change in the user's best response (BR) strategy which results in different NE grouping. Different NE groups have different T_{tot}. Hence by varying the value of α, it is possible to obtain different network performance; which helps us to formulate the policy for channel allocation for a dynamic network. Mathematical analysis/derivations and verification of the results by MATLAB simulations based on average of large number of random scenarios is also presented. In this section we will cover random, "3 Links 2 Channels" scenario and random, "5 Links 3 Channels".

6.1.1 Random, "3 Links 2 Channels"

For the random, "3 Links 2 Channels" scenario, the utility of a link can be calculated under one of the following cases: when a link gets a single channel, when a link shares a channel with another link and when a link shares a channel with two other links as expressed in Equation 6.1; only U_{s1}, U_{s2} and U_{s3} are valid.

U_{s1} does not change with α however, the value of U_{s2} and U_{s3} increases at a constant rate. As illustrated in Equation 6.1 when α increases we are giving more emphasis to the interference from term as U_{s3} is receiving more interference from other links than U_{s2}. Therefore, the rate of change of U_{s3} is the highest as illustrated in Figure 6.2 (top). When the utility of the user changes, the strategy chosen by the users change which results in different channel configuration at NE. This results in different T_{tot}.

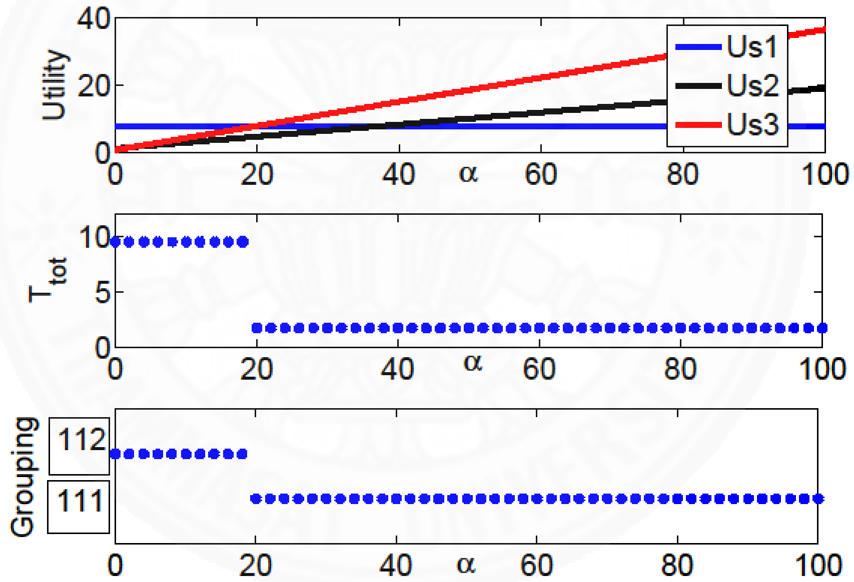

Figure 6.2 Math Analysis of how the interference received weight (α) changes shared utility, which results in different grouping at NE, which results in different T_{tot} for random, "3 Links 2 Channels" scenario; using approximation $1/d_a^2$ = mean($1/d_a^2$).

Figure 6.2 (middle) shows how the T_{tot} decreases from 9 to 2 when the α becomes higher than 28. The transition occurs when the condition changes from "U_{s1} > U_{s2}, U_{s3}" to "U_{s3} > U_{s1}, U_{s2}". When higher α value is used; users having more interference (that is links that share a channel with other links) get higher payoff, so, the BR is to choose strategies which converges to groups such as '111'; as illustrated in Figure 6.2 (bottom).

Derivation of α_{opt} from NE constrains '3L2C': Next, we analyze the behavior observed in Figure 6.2 based on mathematical analysis. For random, "3 Links 2 Channels" scenario, maximum throughput (NE^{Good}) occurs at group, '122' as illustrated in Table 3.1 *(Note: group '122' and group '112' are the same)*. From the definition of NE in Equation 3.1, for channel configuration or group '122' to be NE, the following conditions should be met.

For Link$_1$: $U_{1(c=1)} \geq U_{1(c=2)}$
'**1**22' '**2**22'
$U_{s1} \geq U_{s3}$ (Condition I)
For Link$_2$: $U_{2(c=2)} \geq U_{2(c=1)}$
'1**2**2' '1**1**2'
$U_{s2} \geq U_{s2}$
For Link$_3$: $U_{3(c=2)} \geq U_{3(c=1)}$
'12**2**' '12**1**'
$U_{s2} \geq U_{s2}$

Condition I: Solving for, $U_{s1} \geq U_{s3}$ using the definition of U_{s1} and U_{s3} from Equation 6.1 results in,

$$log_2\left(1+\frac{\frac{1}{d_a^2}}{P_N}\right) \geq log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+\frac{1}{d_{I2}^2}+P_N}\right)+\alpha\left(\frac{1}{d_{I1}^2}+\frac{1}{d_{I2}^2}\right)$$

$$\alpha_{opt}^{I} \leq \frac{\left[log_2\left(1+\frac{\frac{1}{d_a^2}}{P_N}\right)-log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+\frac{1}{d_{I2}^2}+P_N}\right)\right]}{\left(\frac{1}{d_{I1}^2}+\frac{1}{d_{I2}^2}\right)}$$

Therefore, $\alpha_{opt}^{I} \leq 28.23$ (approximating $1/d_a^2$ = mean $(1/d_a^2)$ = 0.11 units).

Verification of α_{opt} value by simulations '3L2C': As shown in Figure 6.3, U_{s1} is constant over α but U_{s2} and U_{s3} increases linearly with α; it is similar to the result obtained in Figure 6.2. However, a drastic change is not visible in Figure 6.3 regarding channel grouping and T_{tot} at NE; because we are taking an average of 1000 random scenarios. At the transition period, some random scenarios will converge to a certain grouping while others will converge to another grouping at NE, and the average value is displayed. When we increase the value of α from 0 to 100 the frequency of '111' grouping at NE is increasing whereas that of and '112' grouping at NE decreasing. Hence, the T_{tot} value is decreasing as α increases because '111' grouping has lower T_{tot} than '112' grouping.

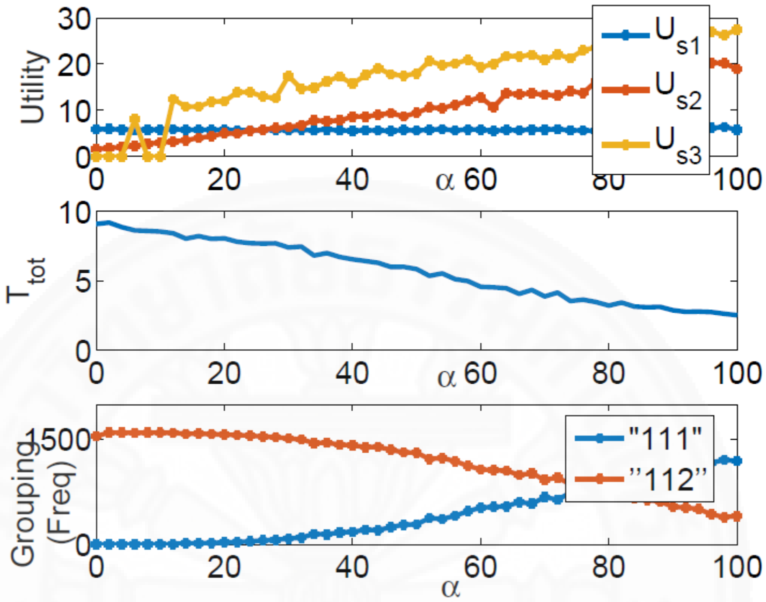

Figure 6.3 Verification of Math Analysis by random simulations to show the weight of the interference term (α), changes shared utility, which results in different NE grouping with varying T_{tot} for random, "3 Links 2 Channels" scenario.

6.1.2 Random, "5 Links 3 Channels" scenario

In the random, "5 Links 3 Channels" scenario the utility of a link can be one of the follows; when a link gets a single channel, when a link shares a channel with another one, two, three, or four links respectively. As illustrated in Figure 6.4 (top), U_{s1} is constant with α whereas U_{s2}, U_{s3}, U_{s4}, U_{s5} increases at a constant rate with α. As U_{s5} has the highest interference received value as it gets interference from four other links ($1/d_{I1}^2 + 1/d_{I2}^2 + 1/d_{I3}^2 + 1/d_{I4}^2$) its rate of change with α is the highest. We can observe in Figure 6.4 (middle) that, when α is less than 5, the T_{tot} is medium (around 11) which corresponds to '11223' grouping at NE. As the value of α increases from 5 to 15 the T_{tot} is highest (around 15.5) which corresponds to '11123' grouping at NE. When the value of α becomes larger than 20 the T_{tot} value strikingly falls to around 2. As confirmed with our previous results in Chapter 5, the NE^{Good} occurs when optimal interference is added. When α is between 5 to 15 we get highest T_{tot}; which falls in the range, "$U_{s5} > U_{s2}, U_{s3}, U_{s4}$" to "$U_{s4} > U_{s1}$".

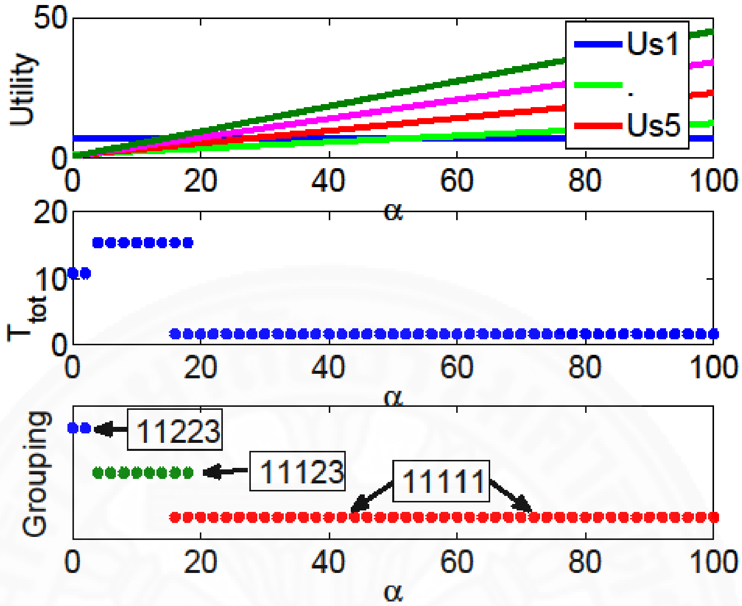

Figure 6.4 Math Analysis to show the weight of the interference received term - (α), changes shared utility, which results in different grouping at NE and different T_{tot} for random, "5 Links 3 Channels" scenario; using approximation $1/d_a^2 = \text{mean}(1/d_a^2)$.

Derivation of α_{opt} from NE constrains '5*L*3*C*': Next, we analyze the behavior observed in Figure 6.4 based on mathematical analysis. For the random, "5 Links 3 Channels" scenario, maximum throughput (NE^{Good}) occurs at group, '12333' as illustrated in Table 4.2 and Figure 6.4. *(Note: group '12333' and group '11123' are the same)*

From the definition of NE in Equation 3.1, for group '12333' to be NE, the following conditions should be met.

For Link*1*: $U_{1(c=1)} \geq U_{1(c=2)}$ *and* $U_{1(c=3)}$

'**1**2333' '**2**2333' '**3**2333'

$U_{s1} \geq U_{s2}$ *and* U_{s4} (Condition II and III)

For Link*2*: $U_{2(c=2)} \geq U_{2(c=1)}$ *and* $U_{2(c=3)}$

'1**2**333' '1**1**333' '1**3**333'

$U_{s1} \geq U_{s2}$ *and* U_{s4}

For Link3: $U_{3(c=3)} \geq U_{3(c=1)}\ and\ U_{3(c=2)}$

'12**3**33' '12**1**33' '12**2**33'

$U_{s3} \geq U_{s2}\ and\ U_{s2}$ (Condition IV)

For Link4: $U_{4(c=3)} \geq U_{4(c=1)}\ and\ U_{4(c=2)}$

'123**3**3' '123**1**3' '12323'

$U_{s3} \geq U_{s2}\ and\ U_{s2}$

For Link5: $U_{5(c=3)} \geq U_{5(c=1)}\ and\ U_{5(c=2)}$

'1233**3**' '1233**1**' '1233**2**'

$U_{s3} \geq U_{s2}\ and\ U_{s2}$

Condition II: Solving for, $U_{s1} \geq U_{s2}$ using the definition of U_{s1} and U_{s2} from Equation 6.1 results in,

$$log_2\left(1+\frac{\frac{1}{d_a^2}}{P_N}\right) \geq log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+P_N}\right)+\alpha\left(\frac{1}{d_{I1}^2}\right)$$

$$\alpha_{opt}^{II} \leq \frac{\left[log_2\left(1+\frac{\frac{1}{d_a^2}}{P_N}\right)-log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+P_N}\right)\right]}{\left(\frac{1}{d_{I1}^2}\right)}$$

Therefore, $\alpha_{opt}^{II} \leq 52.74$ (approximating $1/d_a^2$ = mean($1/d_a^2$) = 0.11 units).

Condition III: Solving for, $U_{s1} \geq U_{s4}$ using the definition of U_{s1} and U_{s4} from Equation 6.1 results in,

$$log_2\left(1+\frac{\frac{1}{d_a^2}}{P_N}\right) \geq log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+\frac{1}{d_{I2}^2}+\frac{1}{d_{I3}^2}+P_N}\right)+\alpha\left(\frac{1}{d_{I1}^2}+\frac{1}{d_{I2}^2}+\frac{1}{d_{I3}^2}\right)$$

$$\alpha_{opt}^{III} \leq \frac{\left[log_2\left(1+\frac{\frac{1}{d_a^2}}{P_N}\right)-log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+\frac{1}{d_{I2}^2}+\frac{1}{d_{I3}^2}+P_N}\right)\right]}{\left(\frac{1}{d_{I1}^2}+\frac{1}{d_{I2}^2}+\frac{1}{d_{I3}^2}\right)}$$

Therefore, $\alpha_{opt}^{III} \leq 19.33$ (approximating $1/d_a^2$ = mean ($1/d_a^2$) = 0.11 units).

Condition IV: Solving for, $U_{s3} \geq U_{s2}$ using the definition of U_{s2} and U_{s3} from Equation 6.1 results in,

$$log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+\frac{1}{d_{I2}^2}+P_N}\right)+\alpha\left(\frac{1}{d_{I1}^2}+\frac{1}{d_{I2}^2}\right) \geq log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+P_N}\right)+\alpha\left(\frac{1}{d_{I1}^2}\right)$$

$$\alpha_{opt}^{IV} \geq \frac{\left[log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+P_N}\right)-log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+\frac{1}{d_{I2}^2}+P_N}\right)\right]}{\left(\frac{1}{d_{I2}^2}\right)}$$

Therefore, $\alpha_{opt}^{IV} \geq 3.73$ (approximating $1/d_a^2$ = mean ($1/d_a^2$) = 0.11 units).

Combining Condition II, III and IV for random, "5 Links 3 Channels" scenario, the range of $\alpha_{opt}^{\wedge}$ is: $3.75 \leq \alpha_{opt}^{\wedge} \leq 19.33$. This value of $\alpha_{opt}^{\wedge}$ corresponds with the simulation results in the beginning of this section.

Verification of $\alpha_{opt}^{\wedge}$ value by simulations '5*L*3*C*': The results obtained from the simulation of random, "5 Links 3 Channels" scenario which is illustrated in Figure 6.5 is in par with the results obtained from mathematical analysis which was illustrated in Figure 6.4. The values of U_{s2} to U_{s5} fluctuate from the mean value as illustrated in Figure 6.5 (top) as we are plotting the average value from the many random simulations. However, the general trend is similar; the average T_{tot} values obtained from random scenario simulations is close to the values obtained from the mathematical analysis performed earlier in this section. A drastic change in the value of T_{tot} with α is not visible because we are taking an average of 1000 random scenarios. During the transition region ($20 < \alpha < 40$), some random scenarios might converge to higher value and others might not. Regarding the grouping at NE, three unique groups exist, '11111', '11123' and '11223'. T_{tot}^{max} occurs in '11123' grouping; when two links get a channel each and the remaining three links share the third channel. Similarly, lowest T_{tot} occurs in '11111' grouping; when all five links share the same channel. Initially when the α is zero, most of the random scenarios converges to '11223' grouping. When α is between 5 to 15 many random scenarios converge to '11123' grouping which is evident by the sharp peak in the figure and finally as value increases to more than 20 the '11111' grouping becomes prominent; this is because we are giving more emphasis to the interference received term.

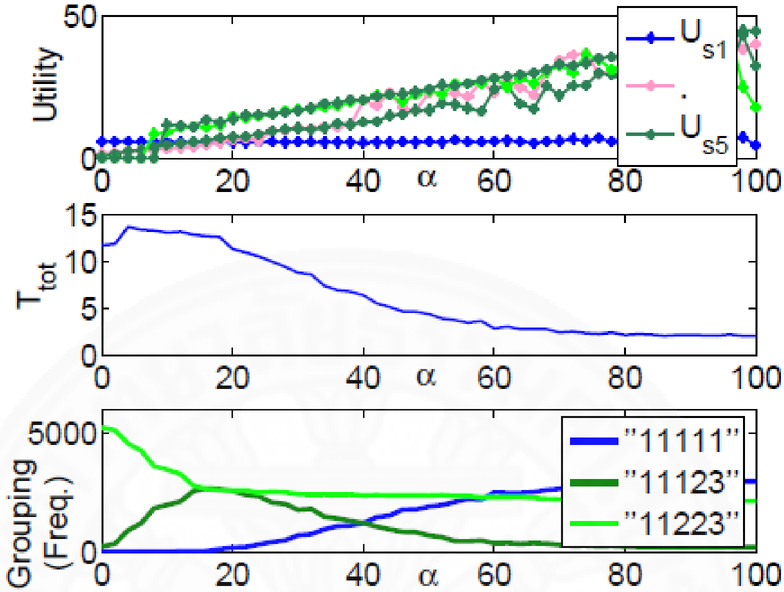

Figure 6.5 Verification of Math Analysis by random simulations to show how the weight of the interference received term - α, changes shared utility, resulting in different NE grouping with different T_{tot} for, "5 Links 3 Channels".

6.3 Random, "10 Links 4 Channels" scenario

Maximum T_{tot} occurs mostly (85%) at channel configuration, '1234444444' [39] for random "10 Links 4 Channels" scenarios [39]. Left part of the Figure 6.8 considers the first link (j = 1) which gets a 'Single Channel' and the right part of the figure considers the fourth link (j = 4) which is 'Sharing Channel' with other links. In the channel configuration '1234444444' the first, second and third links get a 'Single Channel' and have a payoff of U_{s1}, as defined in Equation 8. Once this strategy is changed, the utility of the link will be either U_{s2} if channel 2 or 3 is selected and U_{s8} if channel 4 is selected. Hence, for the NE conditions to hold true: $U_{s1} \geq U_{s2}$ and $U_{s1} \geq U_{s8}$. Similarly considering the channel configuration of '1234444444' the fourth to tenth link share channel 4 so the payoff is U_{s7}. Once the strategy is changed to channel 4 the payoff of the link will be U_{s2}. Hence, for NE condition to hold true: $U_{s7} \geq U_{s2}$.

The value of optimal α obtained from the condition $U_{s1} \geq U_{s2}$ is larger than the value obtained from the condition $U_{s1} \geq U_{s3}$, $U_{s1} \geq U_{s4}$ and so forth as illustrated in Figure 6.6. Hence, the upper bound for optimal α for the above-mentioned conditions is exponentially decreasing. As $U_{s1} \geq U_{s2}$ is a subset of $U_{s1} \geq U_{s8}$, we get one unique Upper Bound (UB) condition for “10 Links 4 Channels” scenario: $U_{s1} \geq U_{s8}$ and one Lower Bound (LB) condition: $U_{s7} \geq U_{s2}$.

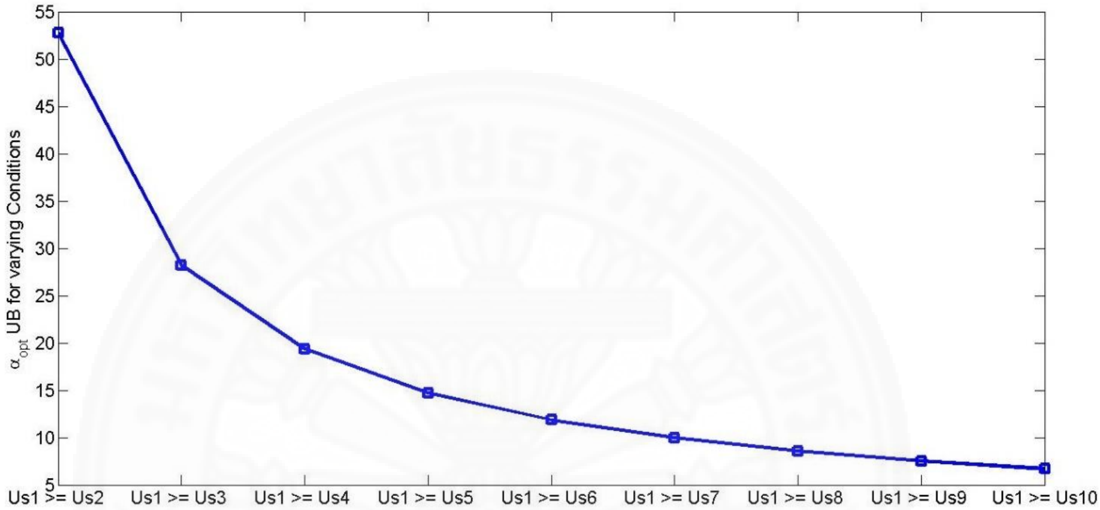

Figure 6.6 Upper Bound (UB) of optimal α for varying conditions

UB condition: Solving for $U_{s1} \geq U_{s8}$, using the definition of shared utility from Equation 6.1 when L = 1 and 8 results in,

$$log_2\left(1+\frac{\frac{1}{d_a^2}}{P_N}\right) \geq log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+\cdots+\frac{1}{d_{I7}^2}+P_N}\right)+\alpha\left(\frac{1}{d_{I1}^2}+\cdots+\frac{1}{d_{I7}^2}\right)$$

$$\alpha_{opt}^{UB} \leq \frac{\left[log_2\left(1+\frac{\frac{1}{d_a^2}}{P_N}\right)-log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+\cdots+\frac{1}{d_{I7}^2}+P_N}\right)\right]}{\left(\frac{1}{d_{I1}^2}+\cdots+\frac{1}{d_{I7}^2}\right)}$$

LB condition: Solving for $U_{s7} \geq U_{s2}$, using the definition of shared utility from Equation 6.1 when L = 7 and 2 results in,

$$log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+\cdots+\frac{1}{d_{I6}^2}+P_N}\right)+\alpha\left(\frac{1}{d_{I1}^2}+\cdots+\frac{1}{d_{I6}^2}\right) \geq log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+P_N}\right)+\alpha\left(\frac{1}{d_{I1}^2}\right)$$

$$\alpha_{opt}^{LB} \geq \frac{\left[log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+P_N}\right)-log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+\cdots+\frac{1}{d_{I6}^2}+P_N}\right)\right]}{\left(\frac{1}{d_{I2}^2}+\cdots+\frac{1}{d_{I6}^2}\right)}$$

α_{opt}^{UB} is larger than α_{opt}^{LB} because the first term of the α_{opt}^{UB} is much larger than the first term of the α_{opt}^{LB}. The first term in the numerator of α_{opt}^{UB} does not have $\frac{1}{d_{I1}^2}$ (interfering expression. Hence, $\left[log_2\left(1+\frac{\frac{1}{d_a^2}}{P_N}\right) \gg log_2\left(1+\frac{\frac{1}{d_a^2}}{\frac{1}{d_{I1}^2}+P_N}\right)\right]$ as $\frac{1}{d_{I1}^2}$ is much larger than P_N. This holds true for "*N* Links *C* Channels" scenario presented in the upcoming sections. α_{opt}^{UB} and α_{opt}^{LB} are computed using the PDF of $\frac{1}{d_a^2}$ (we have assumed the PDF of $\frac{1}{d_{Im}^2}$ to be equal to $\frac{1}{d_a^2}$). The PDF of $\hat{\alpha}_{opt}$ for random, "10 Links 4 Channels" scenario is computed by taking the average of α_{opt}^{UB} and α_{opt}^{LB}, which is shown in Figure 6.7. The mean value of $\hat{\alpha}_{opt}$ is 7.03 units.

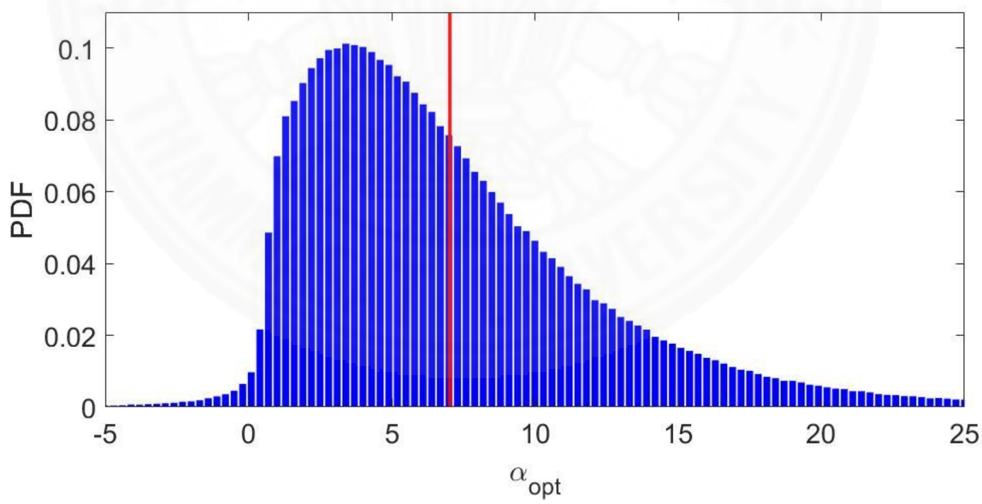

Figure 6.7 Probability Density Function (PDF) of $\hat{\alpha}_{opt}$ for "10 Links 4 Channels" scenario, the mean is 7.03 units.

We determine the value of α_{opt}^{*} in the following way: α is incremented from 0 to 30 with a step of 0.1. Many random simulations are run for each value of α and the average value of T_{tot} is stored. The α value that results in maximum T_{tot} is defined as

α^*_{opt} [39]. For random, "10 Links 4 Channels" scenario α^*_{opt} is equal to 6.8 units which is close to its mathematical approximation ($\hat{\alpha}_{opt}$) we obtained earlier. In the next section we are going to generalize the equation of $\hat{\alpha}_{opt}$ for varying networks.

6.4 Random, "*N* Links *C* Channels" scenario

In this section we are going to infer mathematical expression of $\hat{\alpha}_{opt}$ for varying number of links (*N*) and channels (*C*).

In the previous sections we analyzed the channel configurations at NE^{Good} for random, "3 Links 2 Channels", "5 Links 3 Channels" and "10 Links 4 Channels" scenarios. NE^{Good} mostly occurs when (*C-1*) links get a single channel each and the remaining channel in shared by (*N-C+1*) links. Figure 6.8 shows that $Link_j$ can be either sharing a channel or using it singlehandedly and changing its strategy will result in lower payoff. The left part of the flow chart shows that initially $Link_j$ is using the strategy 'Single Channel'. The payoff is U_{s1} as the link is singlehandedly using the channel.

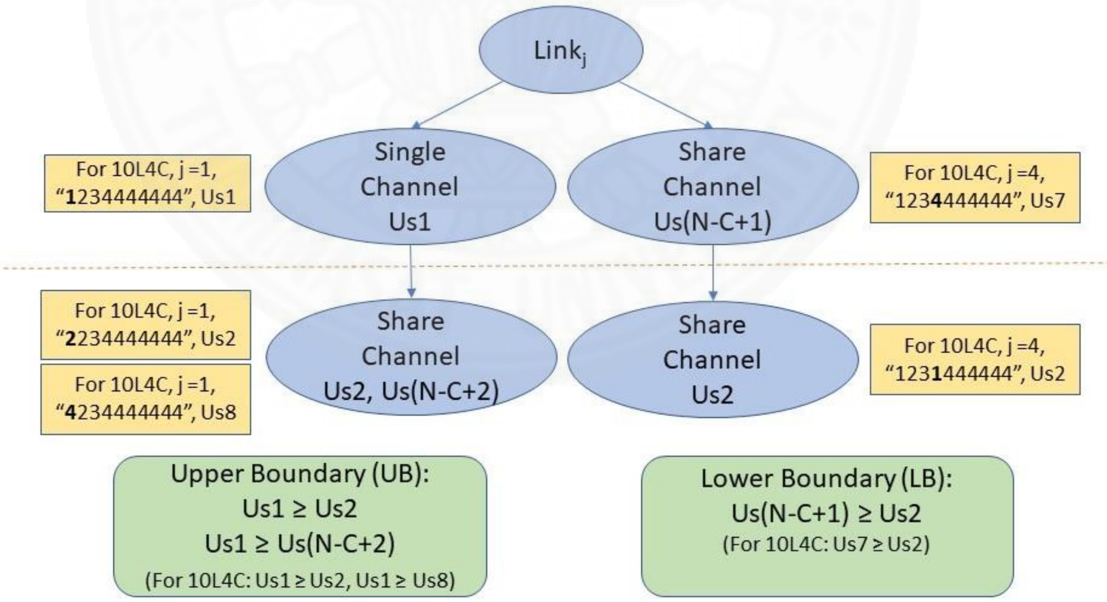

Figure 6.8 At NE^{Good}, $Link_j$ can use a 'Single Channel' or 'Share Channel' with other links. The possible outcomes can be delineated based on the channel configuration at NE^{Good}. Using the definition of NE, it is then possible to derive the Upper Bound (UB) and Lower Bound (LB) constrains of α_{opt} [61].

When the link changes the strategy to ‘Share Channel’ two different payoffs are possible. First, it shares a channel with a link which was singlehandedly using a channel before; then its payoff will be U_{s2}. Second option is if it shares a channel with links that were before sharing channels with (N-C+1) links. Then the new payoff will be $U_{s(N-C+2)}$. Therefore, the UB condition to achieve NE^{Good} are: $U_{s1} \geq U_{s2}$ and $U_{s1} \geq U_{s(N-C+2)}$. As explained earlier (Chapter 6.3), the first constrain is a subset of the second condition. The right part of the flow chart shows that initially Link$_j$ is playing the strategy ‘Share Channel’ and its payoff is $U_{s(N-C+1)}$. When the link changes the strategy, it will share a channel with a link that was previously using a ‘Single Channel’ so, its new payoff will be U_{s2}. Therefore, the LB condition is: $U_{s(N-C+1)} \geq U_{s2}$.

The expected value of $\overline{\hat{\alpha}_{opt}}$ for “N Links C Channels” scenario can be computed by taking an average of α_{opt}^{UB} and α_{opt}^{LB},

$$\overline{\hat{\alpha}_{opt}} = \frac{1}{2}\left(\frac{\left[log_2\left(1+\frac{\frac{1}{d_a^2}}{P_N}\right)-log_2\left(1+\frac{\frac{1}{d_a^2}}{\sum_{n=1}^{N-C+1}\left(\frac{1}{d_{In}^2}\right)+P_N}\right)\right]}{\sum_{n=1}^{N-C+1}\left(\frac{1}{d_{In}^2}\right)}+\frac{\left[log_2\left(1+\frac{\frac{1}{d_a^2}}{d_{I1}^2+P_N}\right)-log_2\left(1+\frac{\frac{1}{d_a^2}}{\sum_{n=1}^{N-C}\left(\frac{1}{d_{In}^2}\right)+P_N}\right)\right]}{\sum_{n=1}^{N-C-1}\left(\frac{1}{d_{In}^2}\right)}\right) \quad (6.2)$$

Equation 6.2 holds true for varying N and C, but accurate results are obtained when N/C is greater than 1.8 [61]. In the upcoming section we will compare the performance results while implementing our policy: α_{opt}^{*} and its approximate mathematical expression ($\hat{\alpha}_{opt}$) along with bench-mark schemes.

6.4 Performance Enhancement by Implementing Policy ($\hat{\alpha}_{opt}$)

In this section we are going to compare the performance of the following schemes: our policy, adding weighted optimal interference received term in the utility ($\alpha_{opt}^{*} \times I_{from}$) and approximate mathematical expression of α_{opt}^{*} (referred to as $\hat{\alpha}_{opt}$) along with standard centralized and basic distributed network. Figure 6.9 plots the T_{tot} for four different schemes in the y-axis and varying network sizes in the x-axis. Using the exhaustive search technique (centralized solution) we obtain the maximum value of T_{tot}. All possible channel configurations are listed and T_{tot} is sorted to obtain the value of T_{tot}^{max} which is denoted by the blue line. It is not possible to perform the exhaustive search method for larger networks. For example, for "13 Links 6 Channels" scenario, the total number of channel configurations is 6^{13} which is over 13 billion. Therefore, the Exhaustive Search results for '13L6C' and larger scenarios is excluded. A basic distributed wireless network is represented by $U_j = T_j$; which results in lowest T_{tot} (gray line). T_{tot} obtained by implementing our policy, $U_j = T_j + \alpha_{opt}^{*} \times I_{from}$ is shown by the red line.

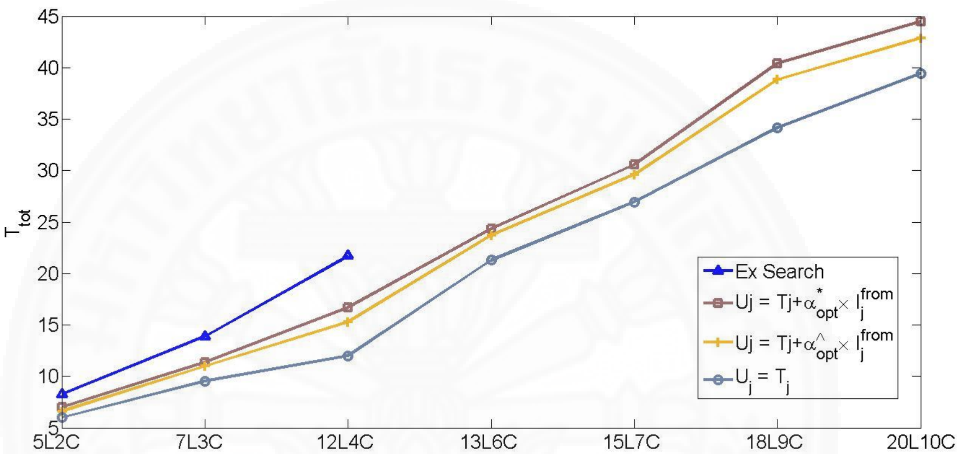

Figure 6.9 Illustrates how the distributed policy (adding $\alpha_{opt}^{*} \times I_{from}$ term in the utility) increases the T_{tot} by up to 20% for distributed wireless networks. α_{opt}^{*} is the optimal value of α that gives T_{tot}^{max}. $\overset{\wedge}{\alpha}_{opt}$ is an approximate mathematical expression of α_{opt}^{*} but can be obtained at a fraction of computational and time resources. Although, the centralized solution from exhaustive search is the ideal solution, it is not feasible to implement it (especially for larger networks) due to enormous consumption of computational resources and time.

α_{opt}^{*} is the optimal value of α and is obtained by step incrementing the value of α from 0 to 30 (0:0.1:30). For each value of α thousands of random simulations are run and the average T_{tot} is recorded. The value of α that results in maximum T_{tot} is α_{opt}^{*}. In this section we propose an approximate mathematical expression for α_{opt}^{*} (referred to as $\overset{\wedge}{\alpha}_{opt}$), represented by the orange line in Figure 6.9. Computing the optimal α via the proposed approximate mathematical expression is much faster and consumes only a fraction of time and computational resources. However, the performance is quite close to α^{*}_{opt} which is illustrated in Table 6.1. Implementing the policy ($\hat{\alpha}_{opt}$) can enhance the total throughput of a distributed network by up to 15% as shown in Figure 6.9.

Table 6.1 Comparing the essential resources (time and computation) and performance of the converged solution via centralized scheme (exhaustive search method) and our distributed policy (adding $\hat{\alpha}_{opt} \times I_{from}$ term in the utility).

Network	Centralized Solution via Exhaustive Search		Distributed Solution, Policy ($\hat{\alpha}_{opt}$)
	Total Channel Configurations	Time (Estimated) Consumed	Time Consumed
5L2C	$2^5 = 3.2\times10^1$	32 τ mseconds	39 τ mseconds
7L3C	$3^7 \cong 2.19\times10^3$	8.74 τ seconds	42 τ mseconds
12L4C	$4^{12} \cong 1.68\times10^7$	18.64 τ hours	48 τ mseconds
13L6C	$6^{13} \cong 1.31\times10^{10}$	20 τ months	50 τ mseconds
15L7C	$7^{15} \cong 4.75\times10^{12}$	602 τ years	53 τ mseconds
18L9C	$9^{18} \cong 1.50\times10^{17}$	1.90×10^7 τ years	56 τ mseconds
20L10C	$10^{20} \cong 1.00\times10^{20}$	1.27×10^{10} τ years	60 τ mseconds

Table 6.1 shows seven different networks with varying links (L) and channels (C). The second column of the table shows the total number of possible channel configurations. As N and C of a network increases, the total channel configuration exponentially increases to C^N. The third column enlists the time (estimated) consumed to run all the C^N channel configurations and sort T^{max}_{tot}. The time to run a single configuration in our desktop computer at research center is around τ = 4 milliseconds (considered as a unit time). The fourth column lists the time consumed (computing the mathematical expressions of $\hat{\alpha}_{opt}$ plus running best response technique) using the distributed policy as proposed in our work. It is evident from the table that as the link size increases it is impossible (due to time and computational resources required) to go through each channel configuration via exhaustive search method and obtain T^{max}_{tot}.

However, by implementing this policy ($\hat{\alpha}_{opt}$), the system can attain NE solution with 75% of the T_{tot}^{max} while consuming a fraction of time and computational resources in comparison to the basic distributed system ($Uj = T_j$).

CHAPTER 7
CONCLUSION

Wireless traffic is increasing in an exponential way and contemporary technologies such as fixed resource allocation is not able to meet up with the trend. Moreover, smart and independent wireless users are becoming popular. The ideal solution, based on a centralized and fully co-operative network, is not practical in distributed network with selfish and independent users as they will deviate from such a solution if they find a way to increase their benefits. The focus of this work is on distributed and decentralized wireless networks with independent and rational users. Our network has N links (with a direct communication between a transmitter and a receivers) within a limited area which share C channels. Hence, links generally overlap and when resources are limited ($C < N$) there is high interference especially to links sharing channels. If the communication distance is limited to a certain range and channels are spatially reused different solution is expected (we plan to explore it further in our future work).

The smart user in these networks try to maximize its own utility in every iteration and eventually the network converges to an equilibrium solution called Nash Equilibrium (NE). By defining the individual payoff of a user differently, various wireless network scenarios can be emulated, ranging from centralized and co-operative, to distributed and selfish. Different authors working in the field have tried to modify the utility of the users to obtain NE with good network performance. Some considered reducing interference from/to other users and maximize throughput or SINR or received power. Not all NE have high performance. From the analysis of different network metrics, it was found that communication links still have significant amount of interference values at NE with high T_{tot}. Adding weighted interference received term ($\alpha^*_{opt} \times I_{from}$) in the payoff of a link can drive the decentralized system to converge to a solution with higher T_{tot}. α^*_{opt} is a function of the number of links (N), channels (C).

Based on mathematical analysis and values obtained from thousands of random simulations we propose several equations (with different degree of complexity) for α^*_{opt}. In Chapter 5 we illustrate how this policy can be implemented in a dynamic distributed wireless network. Compared to algorithms that define utility in a selfish way our policy (α_{opt}^{*}) can enhance the network performance by upto 20%.

In Chapter 6 we derive an approximate mathematical expression along with the PDF for α_{opt}^{*}, referred as $\alpha_{opt}^{\wedge}$ for random, “10 Links 4 Channels” scenario. Then, we extract the general trend of channel configuration at NE with high T_{tot} and propose a mathematical expression for $\alpha_{opt}^{\wedge}$ for varying links (*N*) and channels (*C*). Implementing the policy ($\alpha_{opt}^{\wedge}$) in the payoff of a communication link enhances the performance of the distributed wireless network by upto 15% and the system can achieve upto 75% of the maximum total throughput (bench-mark value reached by centralized solution via exhaustive search technique) at a fraction of time and computation resources.

Some limitations of our work are: the value of noise power, area/dimension of the network and wireless path loss exponent is fixed. In reality these values can vary which may result in different channel allocation at NE with high total throughput. This might cause a shift in the value of α_{opt}^{*} and PDF of $\alpha_{opt}^{\wedge}$. When we derived a generic equation for $\alpha_{opt}^{\wedge}$, we obtained the NE conditions from the channel allocation: “each of the *C-1* links occupies its own channel and the remaining *N-C+1* links share the remaining one channel” because most of the time NE with high T_{tot} occurs at the channel configuration. However, at times especially when the ratio of N/C is less than 1.8 NE with high T_{tot} occurs at other channel allocations. We plan to address these issues in our future work.

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APPENDIX

APPENDIX A

MATLAB Codes (prominent ones) Used for Simulations

main_AlphaOpt_Ttot_ChConfig_MathPdfSimul_N3to20_C2to15

```
clear all; close all;
dir_path =
'D:\A_Amulya_B\SIIT_PhD\SIIT_PhD_MATLAB\2019_10_Matlab_NE_AlphaOpt_Tt
ot_ChConfig_MathSimul_N3to20_C2to15';
cd(dir_path);

%------------------%Initialization -----------
tic
R = 10;% length and width of the Network Size.
Pn = 0.001; %power of noise, -30 dB; Pt =1.
Gamma = 2;%wireless path exponent.
alpha = 0:0.1:60; %Ui = Ti+alpha*I_from, note: make sure step size is
1, else later when assigning the value there is discrepancy, confirm
if true?
N_max = 20; %no of links
C_max = 15; %no of channels
color_plot = ['b' 'r' 'g'];%color used for varying Pn,R,Gamma
marker_sym = ['+' 'o' '*'];%['+' 'o' '*' 'd' 'v' '^' '>'];%marker
symbols used for " " ".

f = 1*1e9; %frequency of the RF, Tx-Rx Communication
c = 3*1e8; %speed of light
lam = c/f; %wave-length

len_rs = 100;%No. of random scenario  used to average.
iter_max = 30; %No of iterations allowed for the Best Response to
converge, hopefully to NE.
simul_size_1 = 1e5; %to compute pdf of 1/d^Gamma, mean(1/d^Gamma)
simul_size_2 = 1e3;

%------------------Compute AlphaOpt for N=10 and C=4--------------
N = 10; C =4;
%[d co_tx co_rx] = fun_create_NLCC_rs(N,C,R,lam);
fun_plot_randTxRx_NLinkCCh(co_tx,co_rx,N);
figure(1); [I_d_Gamma_10L4C] =
fun_pdf_1dGamma_RandSimul(R,simul_size_1,Gamma(1),lam,'plot_on',color
_plot(1),marker_sym(1)); title('PDF of 1/d^{Gamma} for different Pn
values'); xlabel('1/d^{Gamma}'); ylabel('PDF');
figure(2); [Ch_Config_TtotCI_10L4C] =
fun_ChAlloc_TsjTtotCI(Pn,R,Gamma,lam,simul_size_2,'plot_on',color_plo
t(1),marker_sym(1)); title('T_{tot} variation with ChConfig with ');
xlabel('ChConfig'); ylabel('T_{tot}');
figure(3); [opt_alpha_PdfSimul_10L4C] =
fun_alpha_opt_PdfSimul(R,Pn,N,C,Gamma,lam,simul_size_2,'plot_on',colo
r_plot(1),marker_sym(1)); title('PDF of \alpha_{opt} for different Pn
values'); xlabel('Alpha_{Opt}'); ylabel('PDF');
figure(4); [opt_alpha_BRSimul_10L4C Ttot_AlphaOpt_10L4C
Ttot_Alpha0_10L4C] =
fun_alpha_opt_BRSimul(N,C,R,Pn,Gamma,alpha,lam,len_rs,iter_max)
```

```
figure(5);
fun_BR_algo_plot_convergence_10L4C(N,C,R,lam,Gamma,Pn,iter_max,len_rs
,alpha);

%-----------------Compute AlphaOpt for R=10, Gamma=2, Pn=0.001 and
varying N and C--------------
for N = 3:N_max
    for C = 2:C_max
        if (N > C) %N=10;C=4;
             I_d_Gamma(:,N,C) =
fun_pdf_1dGamma_RandSimul(R,simul_size_1,Gamma(1),lam,'plot_off','NA'
,'NA');
             opt_alpha_PdfSimul(N,C) =
fun_alpha_opt_PdfSimul(R,Pn,N,C,Gamma,lam,simul_size_2,'plot_off','NA
','NA');
             [opt_alpha_BRSimul(N,C) Ttot_AlphaOpt(N,C)
Ttot_Alpha0(N,C)] =
fun_alpha_opt_BRSimul(N,C,R,Pn,Gamma,alpha,lam,len_rs,iter_max);
        else
        end
    end
end
%Ttot_BRSimul_Ui_Ttot => obtained by BR Simul using Ui = Ttot;
Ttot_BRSimul_Ui_TiAlphaOptAsk = Ttot_AlphaOpt;
%Ttot_BRSimul_Ui_TiAlphaOptHat => obtained by BR Simul using
opt_alpha_PdfSimul
Ttot_BRSimul_Ui_Ti = Ttot_Alpha0
%save('data_OptAlpha_PdfSimul_BRSimul_varyNC_23rdSept2019.mat','opt_a
lpha_PdfSimul','opt_alpha_BRSimul','Ttot_BRSimul_Ui_Ttot','Ttot_BRSim
ul_Ui_TiAlphaOptAsk','Ttot_BRSimul_Ui_TiAlphaOptHat','Ttot_BRSimul_Ui
_Ti');

%load('data_OptAlpha_PdfSimul_BRSimul_varyNC_23rdSept2019.mat');
figure(6); subplot(2,1,1); imagesc(opt_alpha_PdfSimul); colorbar();
caxis([0 30]);
figure(6); subplot(2,1,2); imagesc(opt_alpha_BRSimul); colorbar();
caxis([0 30]);
%[opt_alpha_PdfSimul-opt_alpha_BRSimul];(5 2) (7,2) (8,3) (8,4)
(10,5)
%----------------------------------

N_C_good = [[5;2] [7;3] [12;4] [13;6] [15;7] [18;9] [20;10]];
for n = 1:length(N_C_good)%n=3
    N = N_C_good(1,n);
    C = N_C_good(2,n);
    xtick_name = [num2str(N) 'L' num2str(C) 'C      ' ];
    xtick_name_f(n,:) = xtick_name(1:6);
    Ttot_plot_f(n,:) = [Ttot_BRSimul_Ui_Ttot(N,C)
Ttot_BRSimul_Ui_TiAlphaOptAsk(N,C) Ttot_BRSimul_Ui_TiAlphaOptHat(N,C)
Ttot_BRSimul_Ui_Ti(N,C)];
end
figure(7);
plot(1:3,Ttot_plot_f(1:3,1)); hold on;
plot(1:length(N_C_good),Ttot_plot_f(:,2:4));
legend('Ex Search','Uj = Tj+\alpha_{opt}^{*}\times I^{from}_j','Uj =
Tj+\alpha_{opt}^{\wedge}\times I^{from}_j','U_j = T_j');
ylabel('T_{tot}');
```

```
set(gca,'xtick',[1:7],'xticklabel',xtick_name_f);
%--------------------------------

% N_C_ExSearch_Policy = [3 4 5 7 8 9 10; 2 3 3 3 4 4 4];%n=1
% for n = 1:length(N_C_ExSearch_Policy)%N_C_ExSearch_Policy(1,n)
%     N = N_C_ExSearch_Policy(1,n);
%     C = N_C_ExSearch_Policy(2,n);
%     [Ttot_ExSearch_Policy(n,:) ChConfig_ExSearch_Policy
time_ExSearch_Policy(n,:)] =
fun_ExhaustiveSearch_Comparision_AlphaOptPolicy(R,Pn,N,C,Gamma,lam,al
pha,iter_max,simul_size_2);
%     display([N C]);
% end
% %load('data_Ttot_Time_ExSearch_Policy.mat');
% stem(Ttot_ExSearch_Policy)
```

fun_pdf_1dGamma_RandSimul

```
function I_d_Gamma =
fun_pdf_1dGamma_RandSimul(R,simul_size,Gamma,lam,plot_on_off,color_pl
ot,marker_sym)

%simul_size = 1e6; R = 10; Gamma = 2; plot_on_off = 'plot_on';
x = R*rand(simul_size,2);
y = R*rand(simul_size,2);
d = sqrt( (x(:,2)-x(:,1)).^2 + (y(:,2)-y(:,1)).^2 );

%to avoid 1/d^Gamma becoming infinite when d is very small.
for n =1:length(d)
    if (d(n) < 2*lam);
        d(n) = 2*lam;
    else end
end
I_d_Gamma = (1./(d.^Gamma));
%hist(I_d_Gamma,0:0.002:1.1)

%ploting the pdf of I_d_Gamma
if (strcmp(plot_on_off,'plot_on') == 1)
    bin_w = 1/(R^Gamma)/8;
    bin_s = 0:bin_w:101*bin_w;
    axis_sn = [0 100*bin_w 0 3/(80*bin_w)];
    [a1 b1] = hist(I_d_Gamma,bin_s);
    plot(bin_s,a1/(simul_size*bin_w),color_plot,'Marker',marker_sym);
axis(axis_sn);
    %bar(bin_s,a1/(simul_size*bin_w)); axis(axis_sn);
xlabel('1/d^{Gamma}'); ylabel('PDF');
else
end
```

fun_ChAlloc_TsjTtotCI

```
function [Ch_Config_TtotCI_10L4C] =
fun_ChAlloc_TsjTtotCI(Pn,R,Gamma,lam,simul_size_pdf,plot_on_off,color
_plot,marker_sym)

%simul_size_pdf = simul_size_2; color_plot = color_plot(1);
marker_sym = marker_sym(1);
for m = 1:simul_size_pdf%m=3
    for n = 1:20 %max times Ts1,2,...,10 occurs while computing Ttot
        I_d_gamma(m,n,:) =
fun_pdf_1dGamma_RandSimul(R,55,Gamma,lam,'plot_off','NA');
        %Shared Throughput
        Ts1(m,n)  = log2(1+I_d_gamma(m,n,1)/(Pn));
        Ts2(m,n)  = log2(1+I_d_gamma(m,n,2)/(I_d_gamma(m,n,3) + Pn));
        Ts3(m,n)  = log2(1+I_d_gamma(m,n,4)/(I_d_gamma(m,n,5) +
I_d_gamma(m,n,6) + Pn));
        Ts4(m,n)  = log2(1+I_d_gamma(m,n,7)/(I_d_gamma(m,n,8) +
I_d_gamma(m,n,9) + I_d_gamma(m,n,10) + Pn));
        Ts5(m,n)  = log2(1+I_d_gamma(m,n,11)/(I_d_gamma(m,n,12) +
I_d_gamma(m,n,13) + I_d_gamma(m,n,14) + I_d_gamma(m,n,15) + Pn));
        Ts6(m,n)  = log2(1+I_d_gamma(m,n,16)/(I_d_gamma(m,n,17) +
I_d_gamma(m,n,18) + I_d_gamma(m,n,19) + I_d_gamma(m,n,20) +
I_d_gamma(m,n,21) + Pn));
        Ts7(m,n)  = log2(1+I_d_gamma(m,n,22)/(I_d_gamma(m,n,23) +
I_d_gamma(m,n,24) + I_d_gamma(m,n,25) + I_d_gamma(m,n,26) +
I_d_gamma(m,n,27) + I_d_gamma(m,n,28) + Pn));
        Ts8(m,n)  = log2(1+I_d_gamma(m,n,29)/(I_d_gamma(m,n,30) +
I_d_gamma(m,n,31) + I_d_gamma(m,n,32) + I_d_gamma(m,n,33) +
I_d_gamma(m,n,34) + I_d_gamma(m,n,35) + I_d_gamma(m,n,36) + Pn));
        Ts9(m,n)  = log2(1+I_d_gamma(m,n,37)/(I_d_gamma(m,n,38) +
I_d_gamma(m,n,39) + I_d_gamma(m,n,40) + I_d_gamma(m,n,41) +
I_d_gamma(m,n,42) + I_d_gamma(m,n,43) + I_d_gamma(m,n,44) +
I_d_gamma(m,n,45) + Pn));
        Ts10(m,n) = log2(1+I_d_gamma(m,n,46)/(I_d_gamma(m,n,47) +
I_d_gamma(m,n,48) + I_d_gamma(m,n,49) + I_d_gamma(m,n,50) +
I_d_gamma(m,n,51) + I_d_gamma(m,n,52) + I_d_gamma(m,n,53) +
I_d_gamma(m,n,54) + I_d_gamma(m,n,55) + Pn));
    end
    Ttot_pdf_ChConfig(m,:) =    [sum([Ts10(m,1) Ts10(m,2) Ts10(m,3)
Ts10(m,4) Ts10(m,5) Ts10(m,6) Ts10(m,7) Ts10(m,8) Ts10(m,9)
Ts10(m,10)])...
                                 sum([Ts2(m,1) Ts2(m,2) Ts2(m,3)
Ts2(m,4) Ts3(m,1) Ts3(m,2) Ts3(m,3) Ts3(m,4) Ts3(m,5) Ts3(m,6)])...
                                 sum([Ts2(m,5) Ts2(m,6) Ts2(m,7)
Ts2(m,8) Ts2(m,9) Ts2(m,10) Ts4(m,1) Ts4(m,2) Ts4(m,3) Ts4(m,4)])...
                                 sum([Ts1(m,1) Ts2(m,11) Ts2(m,12)
Ts3(m,7) Ts3(m,8) Ts3(m,9) Ts4(m,5) Ts4(m,6) Ts4(m,7) Ts4(m,8)])...
                                 sum([Ts1(m,2) Ts2(m,13) Ts2(m,14)
Ts2(m,15) Ts2(m,16) Ts5(m,1) Ts5(m,2) Ts5(m,3) Ts5(m,4) Ts5(m,5)])...
                                 sum([Ts1(m,3) Ts1(m,4) Ts2(m,17)
Ts2(m,18) Ts6(m,1) Ts6(m,2) Ts6(m,3) Ts6(m,4) Ts6(m,5) Ts6(m,6)])...
                                 sum([Ts1(m,5) Ts1(m,6) Ts1(m,7)
Ts7(m,1) Ts7(m,2) Ts7(m,3) Ts7(m,4) Ts7(m,5) Ts7(m,6) Ts7(m,7)])];
end
```

```
%save('data_Ch_Config_Ttot_10L4C','Ts1','Ts2','Ts3','Ts4','Ts5','Ts6'
,'Ts7','Ts8','Ts9','Ts10','Ttot_pdf_ChConfig');
%load('data_Ch_Config_Ttot_10L4C');
Ch_Config_TtotCI_10L4C =  [ 1 1 1 1 1 1 1 1 1 1
quantile(sort(Ttot_pdf_ChConfig(:,1)),0.25)
quantile(sort(Ttot_pdf_ChConfig(:,1)),0.5)
quantile(sort(Ttot_pdf_ChConfig(:,1)),0.75);
                            1 1 2 2 3 3 3 4 4 4
quantile(sort(Ttot_pdf_ChConfig(:,2)),0.25)
quantile(sort(Ttot_pdf_ChConfig(:,2)),0.5)
quantile(sort(Ttot_pdf_ChConfig(:,2)),0.75);
                            1 1 2 2 3 3 4 4 4 4
quantile(sort(Ttot_pdf_ChConfig(:,3)),0.25)
quantile(sort(Ttot_pdf_ChConfig(:,3)),0.5)
quantile(sort(Ttot_pdf_ChConfig(:,3)),0.75);
                            1 2 2 3 3 3 4 4 4 4
quantile(sort(Ttot_pdf_ChConfig(:,4)),0.25)
quantile(sort(Ttot_pdf_ChConfig(:,4)),0.5)
quantile(sort(Ttot_pdf_ChConfig(:,4)),0.75);
                            1 2 2 3 3 4 4 4 4 4
quantile(sort(Ttot_pdf_ChConfig(:,5)),0.25)
quantile(sort(Ttot_pdf_ChConfig(:,5)),0.5)
quantile(sort(Ttot_pdf_ChConfig(:,5)),0.75);
                            1 2 3 3 4 4 4 4 4 4
quantile(sort(Ttot_pdf_ChConfig(:,6)),0.25)
quantile(sort(Ttot_pdf_ChConfig(:,6)),0.5)
quantile(sort(Ttot_pdf_ChConfig(:,6)),0.75);
                            1 2 3 4 4 4 4 4 4 4
quantile(sort(Ttot_pdf_ChConfig(:,7)),0.25)
quantile(sort(Ttot_pdf_ChConfig(:,7)),0.5)
quantile(sort(Ttot_pdf_ChConfig(:,7)),0.75)];

if strcmp(plot_on_off,'plot_on') == 1
    size_Ch_Config_TtotCI_10L4C = size(Ch_Config_TtotCI_10L4C);
    for n = 1:size_Ch_Config_TtotCI_10L4C(1)
        xtick_name(n,:) = 
strrep(num2str(Ch_Config_TtotCI_10L4C(n,1:10)),' ','');
    end

    bar([Ch_Config_TtotCI_10L4C(:,11) Ch_Config_TtotCI_10L4C(:,13)-
Ch_Config_TtotCI_10L4C(:,11)],'stacked'); hold on;
    bar(Ch_Config_TtotCI_10L4C(:,12),0.1,'g');
    set(gca,'xtick',[1:7],'xticklabel',xtick_name);
    %xticklangle(90); works in recent Matlab but not the 2014 one.
else
end
```

fun_alpha_opt_PdfSimul

```
%if a single value is needed
function [mean_opt_alpha_PdfSimul] =
fun_alpha_opt_PdfSimul(R,Pn,N,C,Gamma,lam,simul_size,plot_on_off,colo
r_plot,marker_sym);

%if three different values are needed, mean of Cond I, II and III.
%function [mean_alpha_cn1_cn2_cn3] =
fun_alpha_opt_PdfSimul(R,Pn,N,C,Gamma,lam,simul_size,plot_on_off,colo
r_plot,marker_sym);

%simul_size = 1e6; N=10; C=4; plot_on_off='plot_on';
i1 = 10; i2 = i1+(N-C+1); i3 = i2+(N-C+1); i4 = i3+(N-C); i5 = i4+(N-
C-1); %[i1+1:i2 i2+1:i3 i3+1:i4 i4+1:i5]
for n = 1:simul_size%n=1
    I_d_Gamma_sim(n,:) =
fun_pdf_1dGamma_RandSimul(R,i5+1,Gamma,lam,'plot_off','NA');
    alpha_cn1(n) = [ log2(1 + I_d_Gamma_sim(n,1)/Pn) - log2(1 +
I_d_Gamma_sim(n,2)/(I_d_Gamma_sim(n,3)+Pn)) ] / (I_d_Gamma_sim(n,4));
    alpha_cn2(n) = [ log2(1 + I_d_Gamma_sim(n,5)/Pn) - log2(1 +
I_d_Gamma_sim(n,6)/( sum(I_d_Gamma_sim(n,i1+1:i2)) + Pn)) ] / (
sum(I_d_Gamma_sim(n,i2+1:i3)) );
    alpha_cn3(n) = [ log2(1 +
I_d_Gamma_sim(n,7)/(I_d_Gamma_sim(n,8)+Pn)) - log2(1 +
I_d_Gamma_sim(n,9)/( sum(I_d_Gamma_sim(n,i3+1:i4)) + Pn)) ] / (
sum(I_d_Gamma_sim(n,i4+1:i5)) );
    %using alpha_opt: average of condition II and III. Condition I is
much larger and hence a subset of the condition II and III.
    if (N == C+1)
        opt_alpha_PdfSimul(n) = alpha_cn2(n)/2;%coz, when N = C+1,
alpha_c3 becomes infinite-NaN, as its divied by (N-C-1).
    else
        opt_alpha_PdfSimul(n) = mean([alpha_cn2(n) alpha_cn3(n)]);
    end
end
mean_opt_alpha_PdfSimul = mean(opt_alpha_PdfSimul);
%save('opt_alpha_PdfSimul_10L4C.mat','opt_alpha_PdfSimul')
%load('opt_alpha_PdfSimul_10L4C.mat');

%hist((alpha_cn2 + alpha_cn2)/2,-5:0.1:35)
%mean_alpha_cn1_cn2_cn3 = [mean(alpha_cn1) mean(alpha_cn2)
mean(alpha_cn3)];

%plotting alpha_opt: average of condition II and III. Condition I is
much larger and hence a subset of the condition II and III.
if strcmp(plot_on_off,'plot_on') == 1
    bin_w = 3*0.1;
    [a1 b1] = hist(opt_alpha_PdfSimul,-20:bin_w:30);
    %plot(b1,a1/(sum(a1)*bin_w))%,color_plot,'Marker',marker_sym);
    bar(b1,a1/(sum(a1)*bin_w)); axis([-5 25 0 0.11]);
xlabel('\alpha_{opt}'); ylabel('PDF');
    line([mean_opt_alpha_PdfSimul mean_opt_alpha_PdfSimul],[0 0.11]);
else
end
%mean_opt_alpha_PdfSimul => 7.0335
```

fun_alpha_opt_BRSimul

```
function [opt_alpha_BR Ttot_AlphaOpt Ttot_Alpha0] =
fun_alpha_opt_BRSimul(N,C,R,Pn,Gamma,alpha,lam,len_rs,iter_max)

%N=10; C=4; R=10; iter_max = 30; len_rs = 100; alpha = 0:0.1:30;

%run len_rs number of random scenarios.
for o = 1:len_rs
    clear d Pr co_tx co_rx ch_ini_rand T_1toN_tot U_1toN group ch_ff;
    [d co_tx co_rx] =
fun_create_NLCC_rs(N,C,R,lam);%fun_plot_randTxRx_NLinkCCh(co_tx,co_rx
,N);
    %compute Pr based on different channel/wireless models!
    [Pr] = fun_PathLoss(d,lam,Gamma,'Friss');
    %randomly create a channel.
    ch_ini_rand = randi(C,N,1);
    %----------BR algorithm, utility_def = T+alpha_opt*I_from;
    for p = 1:length(alpha)
        [T_1toN_tot(:,:,p) U_1toN(:,:,p) group(:,:,p) ch_ff(:,:,p)] =
fun_BR_algo(ch_ini_rand,Pr,Pn,N,C,iter_max,alpha(p),'Ti+alpha_opt*Ifr
om');
        %just taking the converged-last of the iter_max values.
        T_1toN_tot_conv(p,:,o) = T_1toN_tot(iter_max,:,p);
        U_1toN_conv(p,:,o) = U_1toN(iter_max,:,p);
        group_conv(p,:,o) = group(iter_max,:,p);
        ch_ff_conv(p,:,o) = ch_ff(iter_max,:,p);
    end
end
%computing the average Ttot (corresponding to alpha_opt value) from
len_rs random scenarios.
for p = 1:length(alpha)
    Ttot_varyAlpha(p) = mean(T_1toN_tot_conv(p,N+1,:));
end

%-----------------
[Ttot_AlphaOpt AlphaOpt_index] = max(Ttot_varyAlpha);
opt_alpha = alpha(AlphaOpt_index);
Ttot_Alpha0 = Ttot_varyAlpha(1);

plot(alpha,Ttot_varyAlpha); xlabel('\alpha'); ylabel('T_{tot}'); grid
on;
%alpha->6.7 => Ttot-> from 13.18 to 15.60 increment of 18.36% (approx
upto 20%).
```

fun_BR_algo

```
function [T_1toN_tot_f U_1toN_f group_f ch_ff]  =
fun_BR_algo(ch_rand,Pr,Pn,N,C,iter_max,alpha,utility_def)
%-------------------------------------------
%BR algorithm, (u1->u2->u3... ->u5)till iter_max. choose the B.R ch
based on the previous iteration... until iter_max.
%{
clear d Pr co_tx co_rx ch_ini_rand T_1toN_tot U_1toN group ch_ff;
%fun_plot_randTxRx_NLinkCCh(co_tx,co_rx,N);
[d co_tx co_rx] = fun_create_NLCC_rs(N,C,R,lam);
[Pr] = fun_PathLoss(d,lam,Gamma,'Friss');
ch_ini_rand = randi(C,N,1);
alpha = alpha(1); ch_rand = ch_ini_rand; utility_def = 'Ttot';
%}

ch_ff(1,:) = ch_rand;
[T_1toN_tot_f(1,:) U_1toN_f(1,:) group_f(1,:)] =
fun_T_U_NLCC_rs(ch_ff(1,:),Pr,Pn,N,C,alpha);

for m = 2:iter_max
    ch = ch_ff(m-1,:);
    ch_un_f = ch;

    for n = 1:N%n=2
        ch_f = [repmat(ch_un_f(1:n-1),C,1) (1:C)'
repmat(ch_un_f(n+1:end),C,1)];
        for (o = 1:C)
            [T_1toN_tot(o,:) U_1toN(o,:) group(o,:)] =
fun_T_U_NLCC_rs(ch_f(o,:),Pr,Pn,N,C,alpha);
        end
        if strcmp(utility_def,'Ti+alpha_opt*Ifrom')==1
            %maximize Ui in every iteration
            [a1 b1] = sort(U_1toN(:,n),'descend'); ch_un = ch_f(:,n);
ch_un_f(n) = ch_un(b1(1));
        elseif strcmp(utility_def,'Ttot')==1
            %maximize Ui = Ttot in every iteration.
            [a1 b1] = sort(T_1toN_tot(:,N+1),'descend'); ch_un =
ch_f(:,n); ch_un_f(n) = ch_un(b1(1));
        else
            display('Error: utility_def not as used in our work');
        end
    end
    ch_ff(m,:) = ch_un_f;
    [T_1toN_tot_f(m,:) U_1toN_f(m,:) group_f(m,:)] =
fun_T_U_NLCC_rs(ch_ff(m,:),Pr,Pn,N,C,alpha);
end
%plot(T_1toN_tot_f(:,N+1))
%[Is_ch_set_NE_f] =
fun_verify_NE_chset_10L4C(ch_ff(m,:),Pr,Pn,N,C,alpha)
```

fun_BR_algo_plot_convergence_10L4C

```
function [] =
fun_BR_algo_plot_convergence_10L4C(N,C,R,lam,Gamma,Pn,iter_max,len_rs
,alpha)
%to check the convergence to NE via BR algorithm.

for o = 1:len_rs
    [d co_tx co_rx] =
fun_create_NLCC_rs(N,C,R,lam);%fun_plot_randTxRx_NLinkCCh(co_tx,co_rx
,N);
    %compute Pr based on different channel/wireless models!
    [Pr] = fun_PathLoss(d,lam,Gamma,'Friss');
    %randomly create a channel.
    ch_ini_rand = randi(C,N,1);
    %----------algorithm1c,
    %Ui = Ti + 0*Ifrom
    [T_1toN_tot U_1toN group ch_ff] =
fun_BR_algo(ch_ini_rand,Pr,Pn,N,C,iter_max,alpha(1));
    T_tot_alpha0(:,o) = T_1toN_tot(:,11);
    T_tot_norm_cum_alpha0(:,o) =
cumsum(T_tot_alpha0(:,o))./[1:iter_max]';

    %Ui = Ti + (alpha_opt*)*Ifrom. alpha_opt* = 6.8
    [T_1toN_tot U_1toN group ch_ff] =
fun_BR_algo(ch_ini_rand,Pr,Pn,N,C,iter_max,alpha(69));
    T_tot_alphaOpt(:,o) = T_1toN_tot(:,11);
    T_tot_norm_cum_alphaOpt(:,o) =
cumsum(T_tot_alphaOpt(:,o))./[1:iter_max]';

    %Ui = Ti + (alpha_opt^)*Ifrom. alpha_opt^ = 7.03
    [T_1toN_tot U_1toN group ch_ff] =
fun_BR_algo(ch_ini_rand,Pr,Pn,N,C,iter_max,alpha(73));
    T_tot_alphaOpt_hat(:,o) = T_1toN_tot(:,11);
    T_tot_norm_cum_alphaOpt_hat(:,o) =
cumsum(T_tot_alphaOpt_hat(:,o))./[1:iter_max]';
end

plot(sum(T_tot_norm_cum_alphaOpt')/len_rs,'r'); hold on;
plot(sum(T_tot_norm_cum_alphaOpt_hat')/len_rs,'g');
plot(sum(T_tot_norm_cum_alpha0')/len_rs);
legend('U_j = T_j','U_j = T_j+\alpha^{*}_{opt}*I^{from}_j','U_j =
T_j+\alpha^{\wedge}_{opt}*I^{from}_j')
xlabel('Iteration (time)'); ylabel('T^{tot}_{nor.cum}')
```

fun_create_NLCC_rs

```
function [d co_tx co_rx] = fun_create_NLCC_rs(N,C,R,lam)

%----------------------------
%creating random N links (Rx-Tx).
co_tx = R*rand(N,2); %transmitter co-ordinates, 1st col= x, 2nd col
=y.
co_rx = R*rand(N,2); %receiver co-ordinates, 1st col= x, 2nd col =y.

%----------------------------
%computing the distance and path loss betw all tx and rx.
for i = 1:N
    for j = 1:N
        d(i,j) = sqrt( (co_tx(i,1)-co_rx(j,1))^2 + (co_tx(i,2)-
co_rx(j,2))^2 );
        if  (d(i,j) < 2*lam)
            d(i,j) = 2*lam;
        else
        end
    end
end
```

fun_plot_randTxRx_NLinkCCh

```
function [] = fun_plot_randTxRx_NLinkCCh(co_tx,co_rx,N)

%---------------------------------------------------
%plot the Tx and Rx; Random Scenario
figure(1)
for n = 1:N
        str1 = [' Tx:' num2str(n)];
        plot(co_tx(n,1),co_tx(n,2),'*'); %tx

text(co_tx(n,1),co_tx(n,2),str1,'HorizontalAlignment','left');
        hold on;
        str2 = [' Rx:' num2str(n)];
        plot(co_rx(n,1),co_rx(n,2),'o'); %rx

text(co_rx(n,1),co_rx(n,2),str2,'HorizontalAlignment','left','Vertica
lAlignment','top');
        plot([co_tx(n,1) co_rx(n,1)],[co_tx(n,2) co_rx(n,2)],'r');
end

figure(2)
plot(co_tx(1,1),co_tx(1,2),'*'); %tx1
text(co_tx(1,1),co_tx(1,2),' Tx:1','HorizontalAlignment','left');
hold on;
plot(co_rx(1,1),co_rx(1,2),'o'); %rx1
text(co_rx(1,1),co_rx(1,2),'
Rx:1','HorizontalAlignment','left','VerticalAlignment','top');
plot([co_tx(1,1) co_rx(1,1)],[co_tx(1,2) co_rx(1,2)],'r'); %axis([0
10 0 10]);
for n = 1:N
        str1 = [' Tx:' num2str(n)];
        str2 = ['          d_{' num2str(n) '1}'];
        plot(co_tx(n,1),co_tx(n,2),'*'); %tx

text(co_tx(n,1),co_tx(n,2),str1,'HorizontalAlignment','left');

text(co_tx(n,1),co_tx(n,2),str2,'HorizontalAlignment','left');
        plot([co_tx(n,1) co_rx(1,1)],[co_tx(n,2) co_rx(1,2)],'-.r');
end
```

fun_T_U_NLCC_rs

```
function [T_1toN_tot U group] = fun_T_U_NLCC_rs(ch,Pr,Pn,N,C,alpha)
%compute interference matrix, row-users, column-users; 1 if same
ch,else 0.
%N=10; ch = [1 2 2 4 3 3 2 1 1 4]; ch = ch_ini_rand; alpha =
alpha(1);

inter = zeros(N,N);
for n = 1:N
    inter(n,:) = (ch(n)==ch);
end
%multiply path-loss with interference; exists whenever users have
same ch.
inter_f = inter.*Pr;

%computer SNR
for n = 1:N
    %p_signal/(p_interference + p_Pn); assume p_tx for all is 1.
    Pr_f(n) = Pr(n,n);
    I_from(n) = (sum(inter_f(:,n)) - inter_f(n,n) + Pn);
    I_to(n) = (sum(inter_f(n,:)) - inter_f(n,n) + Pn);
    SINR(n) = Pr_f(n)/I_from(n);%h(n,n)/( sum(inter_f(:,n)) -
inter_f(n,n) + Pn);
    T(n) = log2(1+SINR(n));
    U(n) = T(n) + alpha*I_from(n);
end
T_1toN_tot = [T sum(T)];
%U = T_1toN_tot(N+1)*ones(1,N);%when Ui = Ttot

inter_g = zeros(C,N+1);%1st row=> ch1; 2nd row=> ch2; 3rd row=> ch3;
for n = 1:N
    for m = 1:C
        if (ch(n) == m); inter_g(m,n) = 1; else end
    end
end
inter_g(:,N+1) = sum(inter_g(:,1:N)');
[group_A group_B] = sort(inter_g(:,N+1),'descend');
group = ones(1,group_A(1));
for n = 2:C%clear group
      group = [group n*ones(1,group_A(n))];
%     group_i(n) = strrep(num2str(n*ones(1,group_A(n))),' ','');
group = strcat(group,group_i(n));
end
```

fun_PathLoss

```
function [Pr] = fun_PathLoss(d,lam,Gamma,model)
%model = 'Friss'; d = 100; f=1e9;
% Pt = 1; %Tx power
% Gr = 1; %gain of Rx antenna
% Gt = 1; %gain of Tx antenna
% c = 3*1e8; %speed of light
% hb = 50; %height of base station
% hm = 1.65; %height of MS

for m = 1:length(d)%m=1;n=1;d(m,n)=1
    for n = 1:length(d)
%         if (strcmp(model,'Friss')==1)
            %Pr(m,n) = Gr*Gt*Pt/(4*pi*d(m,n)/(lam^Gamma));
            Pr(m,n) = 1/(d(m,n)^Gamma);
%          elseif (strcmp(model,'Okumura')==1)
%              alpha_PL(m,n) = 3.2*(log10(11.75*hm))^2 - 4.97; %for f>
300 MHz
%              L_PU(m,n) = 69.55 + 26.16*log10(f/1e6) -
13.82*log10(hb) - alpha_PL(m,n) + [44.9 -
6.55*log10(hb)]*log10(d(m,n)/1000);
%              Pr(m,n) = Pt/(10^(L_PU(m,n)/10));
%          elseif (strcmp(model,'Hata')==1)
%              Pr(m,n) = 5;
%          elseif (strcmp(model,'Urban')==1)
%              Pr(m,n) = 10;
%         end
    end
end
```

fun_verify_NE_chset_10L4C

```
function [Is_ch_set_NE_f] =
fun_verify_NE_chset_10L4C(ch,Pr,Pn,N,C,alpha)
%alpha = alpha(1)
%ch = [2     3     4     3     1     2     1     1     4];
clear ch_f Is_ch_set_NE_f Is_ch_set_NE;
ch_f = [ch; ch; ch; ch; ch; ch; ch; ch; ch; ch; ch; ch; ch; ch; ch;
ch; ch; ch; ch; ch; ch; ch; ch; ch; ch; ch; ch; ch; ch; ch; ch];
%size(ch_f)

%---------------------------------------------------
%if u1=ch1; 2,3row= ch2,ch3: if u1=ch2; 2,3row=ch1,ch3; if
u1=ch3;2,3row=ch1,ch2.
if (ch(1)==1) ch_f(2,1) = 2; ch_f(3,1) = 3; ch_f(4,1) = 4; else end;
if (ch(1)==2) ch_f(2,1) = 1; ch_f(3,1) = 3; ch_f(4,1) = 4; else end;
if (ch(1)==3) ch_f(2,1) = 1; ch_f(3,1) = 2; ch_f(4,1) = 4; else end;
if (ch(1)==4) ch_f(2,1) = 1; ch_f(3,1) = 2; ch_f(4,1) = 3; else end;

if (ch(2)==1) ch_f(5,2) = 2; ch_f(6,2) = 3; ch_f(7,2) = 4; else end;
if (ch(2)==2) ch_f(5,2) = 1; ch_f(6,2) = 3; ch_f(7,2) = 4; else end;
if (ch(2)==3) ch_f(5,2) = 1; ch_f(6,2) = 2; ch_f(7,2) = 4; else end;
if (ch(2)==4) ch_f(5,2) = 1; ch_f(6,2) = 2; ch_f(7,2) = 3; else end;

if (ch(3)==1) ch_f(8,3) = 2; ch_f(9,3) = 3; ch_f(10,3) = 4; else end;
if (ch(3)==2) ch_f(8,3) = 1; ch_f(9,3) = 3; ch_f(10,3) = 4; else end;
if (ch(3)==3) ch_f(8,3) = 1; ch_f(9,3) = 2; ch_f(10,3) = 4; else end;
if (ch(3)==4) ch_f(8,3) = 1; ch_f(9,3) = 2; ch_f(10,3) = 3; else end;

if (ch(4)==1) ch_f(11,4) = 2; ch_f(12,4) = 3; ch_f(13,4) = 4; else
end;
if (ch(4)==2) ch_f(11,4) = 1; ch_f(12,4) = 3; ch_f(13,4) = 4; else
end;
if (ch(4)==3) ch_f(11,4) = 1; ch_f(12,4) = 2; ch_f(13,4) = 4; else
end;
if (ch(4)==4) ch_f(11,4) = 1; ch_f(12,4) = 2; ch_f(13,4) = 3; else
end;

if (ch(5)==1) ch_f(14,5) = 2; ch_f(15,5) = 3; ch_f(16,5) = 4; else
end;
if (ch(5)==2) ch_f(14,5) = 1; ch_f(15,5) = 3; ch_f(16,5) = 4; else
end;
if (ch(5)==3) ch_f(14,5) = 1; ch_f(15,5) = 2; ch_f(16,5) = 4; else
end;
if (ch(5)==4) ch_f(14,5) = 1; ch_f(15,5) = 2; ch_f(16,5) = 3; else
end;

if (ch(6)==1) ch_f(17,6) = 2; ch_f(18,6) = 3; ch_f(19,6) = 4; else
end;
if (ch(6)==2) ch_f(17,6) = 1; ch_f(18,6) = 3; ch_f(19,6) = 4; else
end;
if (ch(6)==3) ch_f(17,6) = 1; ch_f(18,6) = 2; ch_f(19,6) = 4; else
end;
if (ch(6)==4) ch_f(17,6) = 1; ch_f(18,6) = 2; ch_f(19,6) = 3; else
end;
```

```
if (ch(7)==1) ch_f(20,7) = 2; ch_f(21,7) = 3; ch_f(22,7) = 4; else
end;
if (ch(7)==2) ch_f(20,7) = 1; ch_f(21,7) = 3; ch_f(22,7) = 4; else
end;
if (ch(7)==3) ch_f(20,7) = 1; ch_f(21,7) = 2; ch_f(22,7) = 4; else
end;
if (ch(7)==4) ch_f(20,7) = 1; ch_f(21,7) = 2; ch_f(22,7) = 3; else
end;

if (ch(8)==1) ch_f(23,8) = 2; ch_f(24,8) = 3; ch_f(25,8) = 4; else
end;
if (ch(8)==2) ch_f(23,8) = 1; ch_f(24,8) = 3; ch_f(25,8) = 4; else
end;
if (ch(8)==3) ch_f(23,8) = 1; ch_f(24,8) = 2; ch_f(25,8) = 4; else
end;
if (ch(8)==4) ch_f(23,8) = 1; ch_f(24,8) = 2; ch_f(25,8) = 3; else
end;

if (ch(9)==1) ch_f(26,9) = 2; ch_f(27,9) = 3; ch_f(28,9) = 4; else
end;
if (ch(9)==2) ch_f(26,9) = 1; ch_f(27,9) = 3; ch_f(28,9) = 4; else
end;
if (ch(9)==3) ch_f(26,9) = 1; ch_f(27,9) = 2; ch_f(28,9) = 4; else
end;
if (ch(9)==4) ch_f(26,9) = 1; ch_f(27,9) = 2; ch_f(28,9) = 3; else
end;

if (ch(10)==1) ch_f(29,10) = 2; ch_f(30,10) = 3; ch_f(31,10) = 4;
else end;
if (ch(10)==2) ch_f(29,10) = 1; ch_f(30,10) = 3; ch_f(31,10) = 4;
else end;
if (ch(10)==3) ch_f(29,10) = 1; ch_f(30,10) = 2; ch_f(31,10) = 4;
else end;
if (ch(10)==4) ch_f(29,10) = 1; ch_f(30,10) = 2; ch_f(31,10) = 3;
else end;

%------------------------------------------------------
%find the SINR,T,etc. for each of the 11 configs
for n = 1:length(ch_f)
    [T_1toN_tot(n,:) U(n,:) group(n,:)] =
fun_T_U_NLCC_rs(ch_f(n,:),Pr,Pn,N,C,alpha);
end

%check if the original Ch->Utility is the highest while each user
changes its strategies? If not its not NE.
[a1 b1] =    sort(U(1:4,1),'descend');                  if (U(1,1) >=
a1(1))       Is_ch_set_NE(1) = 1;    else Is_ch_set_NE(1) = 0; end;
[a2 b2] =    sort([U(1,2);U(5:7,2)],'descend');         if (U(1,2) >=
a2(1))       Is_ch_set_NE(2) = 1;    else Is_ch_set_NE(2) = 0; end;
[a3 b3] =    sort([U(1,3);U(8:10,3)],'descend');        if (U(1,3) >=
a3(1))       Is_ch_set_NE(3) = 1;    else Is_ch_set_NE(3) = 0; end;
[a4 b4] =    sort([U(1,4);U(11:13,4)],'descend');       if (U(1,4) >=
a4(1))       Is_ch_set_NE(4) = 1;    else Is_ch_set_NE(4) = 0; end;
[a5 b5] =    sort([U(1,5);U(14:16,5)],'descend');       if (U(1,4) >=
a5(1))       Is_ch_set_NE(5) = 1;    else Is_ch_set_NE(5) = 0; end;
[a6 b6] =    sort([U(1,6);U(17:19,6)],'descend');       if (U(1,6) >=
a6(1))       Is_ch_set_NE(6) = 1;    else Is_ch_set_NE(6) = 0; end;
```

```
[a7 b7] =   sort([U(1,7);U(20:22,7)],'descend');      if (U(1,7) >=
a7(1))      Is_ch_set_NE(7) = 1;    else Is_ch_set_NE(7) = 0; end;
[a8 b8] =   sort([U(1,8);U(23:25,8)],'descend');      if (U(1,8) >=
a8(1))      Is_ch_set_NE(8) = 1;    else Is_ch_set_NE(8) = 0; end;
[a9 b9] =   sort([U(1,9);U(26:28,9)],'descend');      if (U(1,9) >=
a9(1))      Is_ch_set_NE(9) = 1;    else Is_ch_set_NE(9) = 0; end;
[a10 b10] = sort([U(1,10);U(29:31,10)],'descend');    if (U(1,10) >=
a10(1))    Is_ch_set_NE(10) = 1;   else Is_ch_set_NE(10) = 0; end;

if (sum(Is_ch_set_NE)==10) Is_ch_set_NE_f = 1; else Is_ch_set_NE_f =
0; end;
```

BIOGRAPHY

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