WINNING STRATEGIES OF MULTIMOVE GAMES ON SOME ASIAN CHESS GAMES

BY

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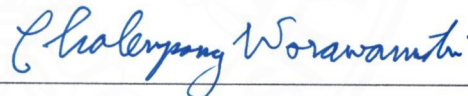
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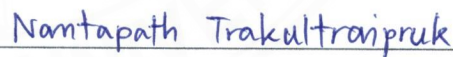the degree of Doctor of Philosophy (Mathematics)
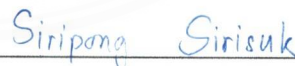
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ABSTRACT

Let i and j be positive integers. An (i, j) multimove game is a two-player game in which the first player (White/female) has i moves per turn, and the second player (Black/male) has j moves per turn. In this dissertation, we present a winning strategy for an (i, j) multimove game on Thai chess, Japanese chess, Chinese chess, and Korean chess for all (i, j) not equal to $(1, 1)$ or $(2, 2)$. Moreover, we demonstrate that Black does not have a winning strategy in a (2,2) multimove game in Thai, Japanese, Chinese, and Korean chess games.

Keywords: Multimove game, Thai chess, Japanese chess, Chinese chess, Korean chess

ACKNOWLEDGEMENTS

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CHAPTER 1

INTRODUCTION

One of the most popular board games in the world is Chess (Western international chess). This game is influenced by "Chaturanga", which was created in India before the 600s AD. Chess is similar to a war game in which two players are adversaries. Each player is given sixteen pieces to play on the squares on a board of size 8×8, which is called a chessboard. A player alternately receives a turn to perform one of the following two actions: moving a piece to an empty square or moving a piece to replace the opponent's piece (we call the second action "capturing"). The first player is called "White" and the second player is called "Black". If White can capture Black's king in her next turn, we call "Black is in check". Moreover, if White can capture Black's king in her next turn no matter how Black moves, we call "Black is in checkmate". The definitions are analogous for another player. A player who captures the opponent's king or puts the opponent in checkmate first is the winner. For more information about Chess, see [1].

Chess is widely played in many countries around the world. Moreover, some countries have their own versions of Chess depending on their culture. Cazaux [4] studied and gathered them. Some of them is similar to Chess such as Thai chess (Makruk). This chess has the same board and the same number of pieces as Chess; however, there are some differences in the rules between them. We explain about the rules of Thai chess later. There are also some other chess games with different structures from Chess. Japanese chess (Shogi) has a larger board than Chess, and there are some pieces that do not exist in Chess. Like Japanese chess, Chinese chess (Xiangqi) has a larger board and more pieces than Chess; however, it is not played on squares but on points (intersections of two lines). Korean chess (Janggi) has a similar board and pieces; however, there are some differences between them such as movement and the arrangement of the pieces.

Many researchers compared two different types of chess games. Ma [11] considered the differences in social systems, cognitive habits, and geography of Chess and Chinese chess. To develop a game's programming, Matsubara et al. [12] compared the rules and the computational aspects of Chess and Japanese chess. Yen et al. [17] compared the structures of Chess and Chinese chess. They also compared ELP and

SHIGA, which were the top Chinese chess programs during that time. Nhat and Lee [10] presented a method for detecting the Korean chessboard and pieces based on the edge and color feature to construct a Chess-playing robot.

There are many attractive problems with Chess in many areas. In graph theory, domination parameters of a rook chessboard graph and its line graph were determined by Laskar and Wallis [9]. Chia and Ong [5] considered an $m \times n$ chessboard with no closed knight's tours and gave open knight's tours for some $m \times n$ chessboards with positive integers m and n. Ionascu et al. [7] studied the maximum number of kings that can be placed on an $m \times n$ chessboard without any two kings capturing each other, where $1 \leq m, n \leq 8$. In computer science, Spoerer et al. [14] conducted research on groups of Chess programs and discovered that the winning percentage increased as the group grew in size. Wu and Beal [16] used an improved retrograde algorithm to construct Chinese chess endgame databases. In game theory, Boros et al. [3] considered Chess in term of an n-person positional game and determined existence of Nash equilibria. Wagon [15] considered Chess, which is played on a triangular hexagon board. The author studied how to place a few pieces (not necessarily identical) that can attack all hexagonal cells. Chess, played on a three-dimensional board, was considered by Laisin et al. [8], and they showed that there were non-attacking bishops on the board.

Although Chess has been played for a long time, people have not found a winning strategy. Strategies were developed in parallel with playing of the game. Reek et.al. [13] proposed short, medium, and long-term strategies for developing Chess game programming. Donovan [6] gave strategies for some situations, such as openings, endgames, quick checkmate strategies. Berger and Dubbs [2] considered an (i, j) multimove game on Chess with some additional rules, where i and j are positive integers. In the game, White must move i times per turn, and Black must move j times per turn. Berger and Dubbs discovered a winning strategy for an (i, j) multimove game on Chess except for $(i, j) \notin \{(1, 1)(2, 2)\}$, as shown in Table 1.1.

White/Black	$j = 1$	$j = 2$	$j = 3$	$j \geq 4$
$i = 1$	?	Black	Black	Black
$i = 2$	White	?	Black	Black
$i = 3$	White	White	White	Black
$i \geq 4$	White	White	White	White

Table 1.1 The player who has a winning strategy for an (i, j) multimove game on Chess

In this dissertation, we find a winning strategy for an (i, j) multimove game with (i, j) not equal to (1,1) and (2,2) on Thai chess, Japanese chess, Chinese chess, and Korean chess, and we show that Black does not have a winning strategy for a (2,2) multimove game on Thai chess, Japanese chess, Chinese chess, and Korean chess.

1.1 Thai Chess

In Thai chess, there are eight pawns (Bia), two rooks (Rua), two knights (Ma), two bishops (Khon), a queen (Med), and a king (Khun). They are arranged as shown in Figure 1.1, and each type of piece has a specific movement, as shown in Table 1.2.
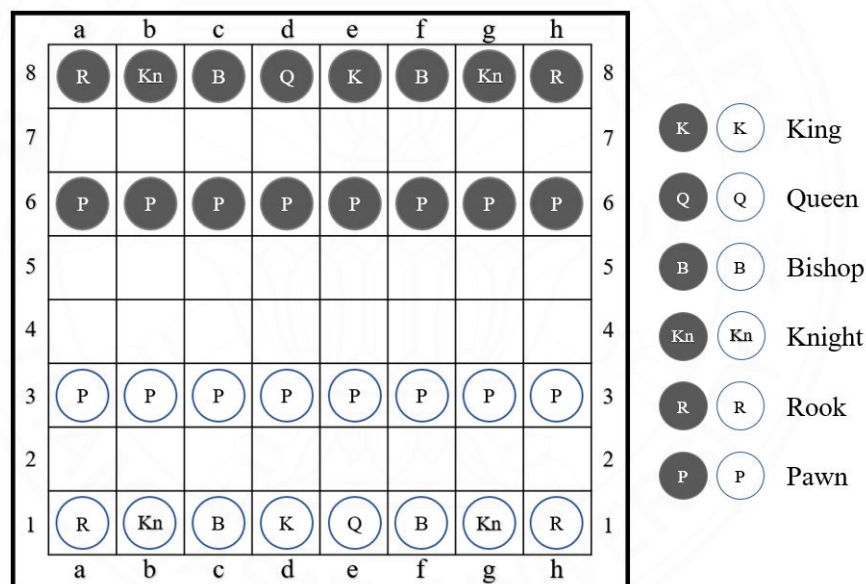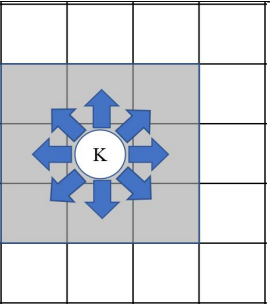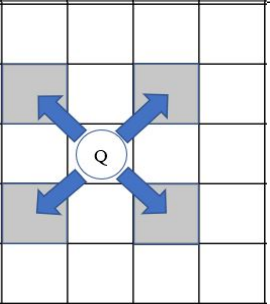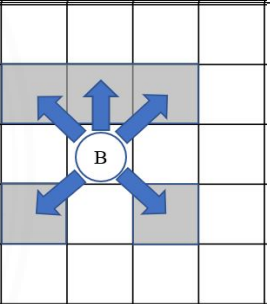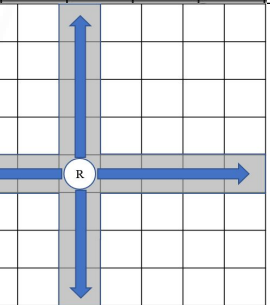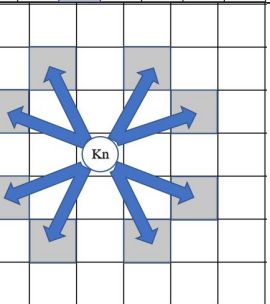
Figure 1.1 The positions of pieces in Thai chess

Pieces	Details	Movements of Pieces
King	King moves one square in any direction.	
Queen	Queen moves one square diagonally.	
Bishop	Bishop moves one square diagonally or forward.	
Rook	Rook moves vertically or horizontally to any unobstructed square without jumping over other pieces.	
Knight	Knight moves two squares orthogonally, then one square at a right angle. Moreover, Knight can jump over any piece in its path.	
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Table 1.2 – continued from previous page
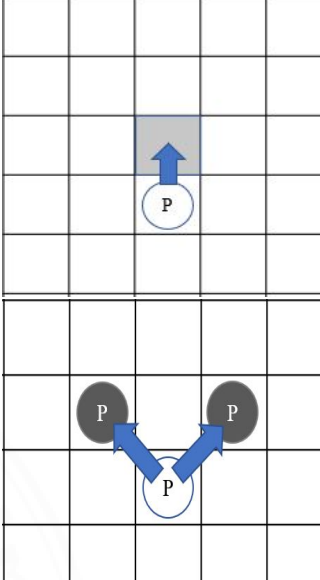
Pieces	Details	Movements of Pieces
Pawn	Pawn moves one square forward, but to capture a piece, it moves one square forward diagonally. When a pawn reaches the opponent player's third row (the sixth row for White or the third row for Black), it is flipped over, and it moves or captures like a queen.	

Table 1.2 The movements of pieces in Thai chess

1.2 Japanese Chess

Japanese chess is a two-player game played on a 9 x 9 chessboard. The labels of squares on the board for this game are different from the others. We used the labeling from "A world of chess: its development and variations through centuries and civilizations" by Cazaux and Knowlton [4]. This game has twenty pieces: nine pawns (Fu), two lances (Kyosha), two knights (Kei), two silvers (Gin-sho), two golds (Kin-sho), a bishop (Kaku), a rook (Hisha), and a king (Osho). The pieces are arranged and have movement as shown in Figure 1.2 and Table 1.3, respectively.
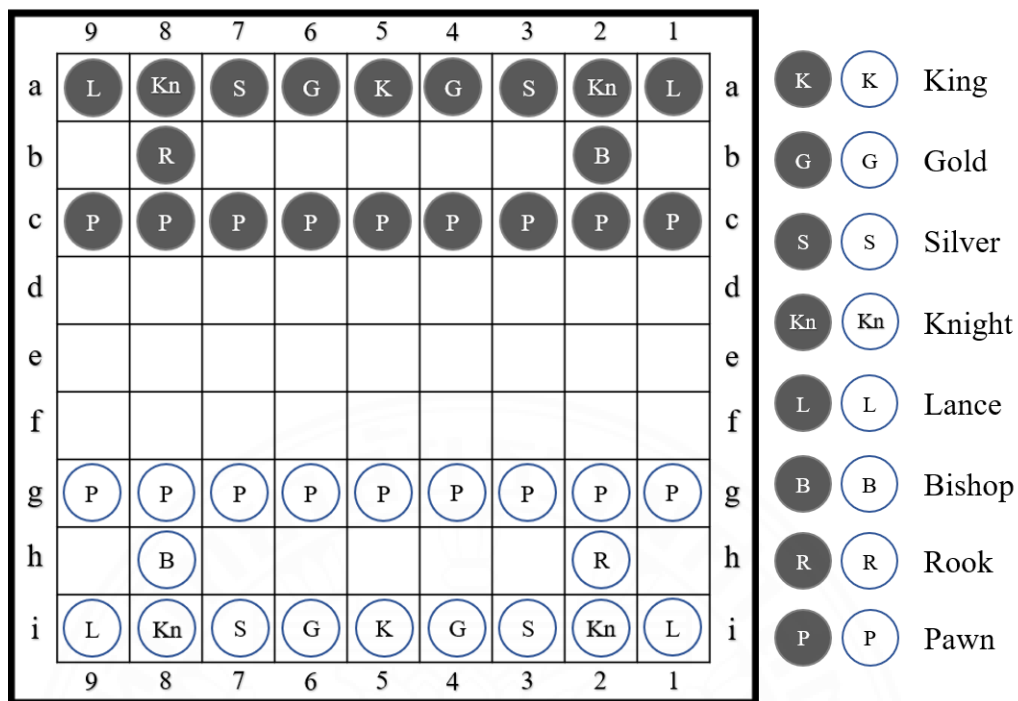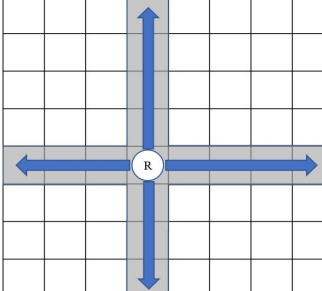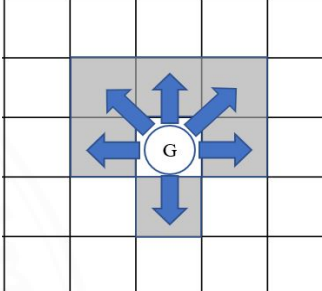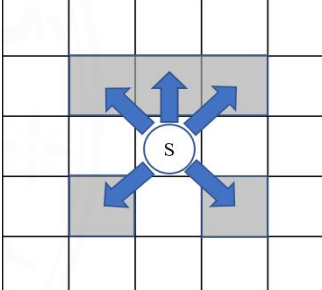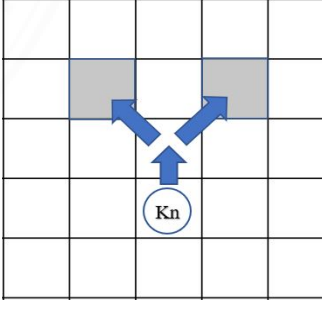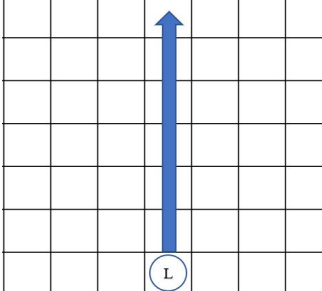
Figure 1.2 The Japanese chessboard

Pieces	Details	Movements of Pieces
King	King moves one square in any directions.	
Bishop	Bishop moves any number of squares diagonally but is not allowed to jump over any pieces.	
		Continued on next page

Table 1.3 – continued from previous page

Pieces	Details	Movements of Pieces
Rook	Rook moves vertically or horizontally to any unobstructed square without jumping over other pieces.	
Gold	Gold moves one square orthogonal or one square diagonally forward.	
Silver	Silver moves one square diagonally or forward.	
Knight	Knight moves one square forward and then one square diagonally forward. Note that a knight is the only piece that can jump over another piece.	
Lance	Lance moves any number of squares forward without jumping over another piece.	
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Table 1.3 – continued from previous page
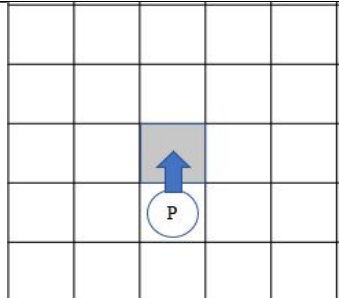
Pieces	Details	Movements of Pieces
Pawn	Pawn moves one square forward.	

Table 1.3 The movements of pieces in Japanese chess

All pieces in Japanese chess can be promoted except for the kings and the golds. When a player moves a piece to one of the first three rows of the opponent's player (the a, b, or c rows for White and the g, h, or i rows for Black, as shown in Figure 1.3), the player can decide whether to promote the piece at the end of the turn. Note that the promoted pieces cannot be returned to their original forms, and the movements are shown in Table 1.4.
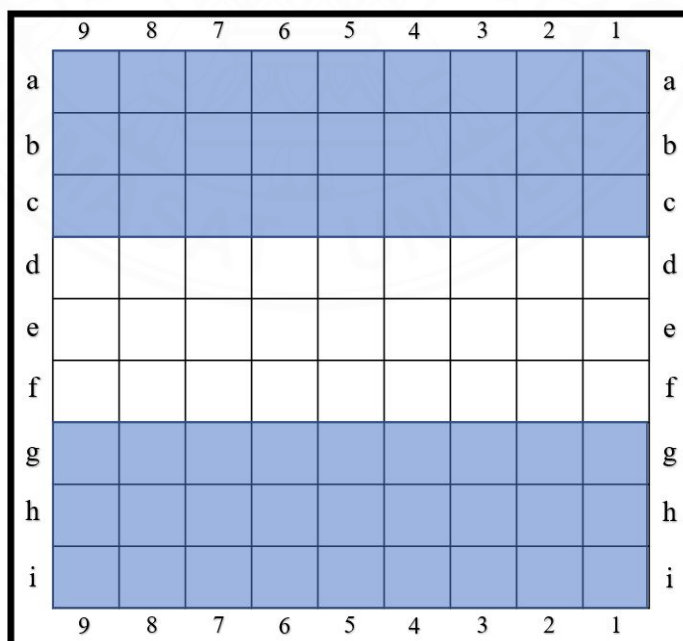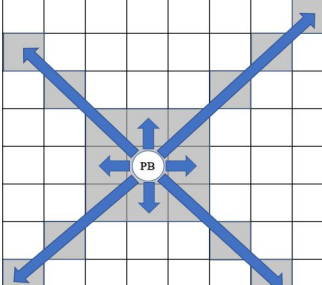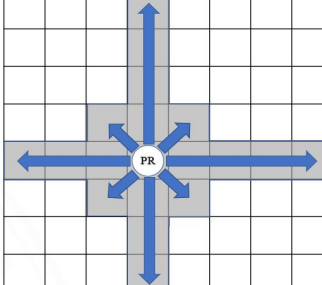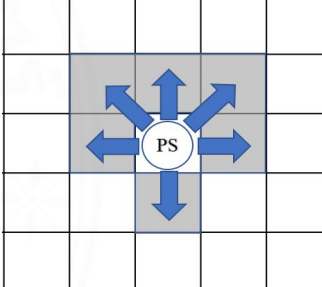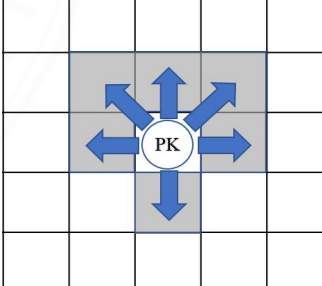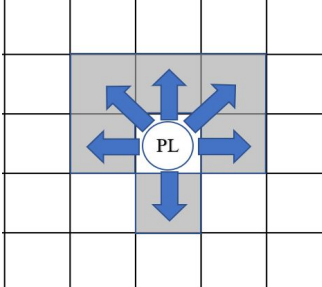
Figure 1.3 The promotion zones

Pieces	Details	Movements of Pieces
Promoted bishop	Promoted bishop moves like a normal bishop. Also, he moves one square either vertically or horizontally.	
Promoted rook	Promoted rook moves like a normal rook but acquires the power to move exactly one square diagonally.	
Promoted silver	Promoted silver moves one square orthogonal or one square diagonally forward.	
Promoted knight	Promoted knight moves one square orthogonal or one square diagonally forward.	
Promoted lance	Promoted lance moves one square orthogonal or one square diagonally forward.	
		Continued on next page

Table 1.4 – continued from previous page
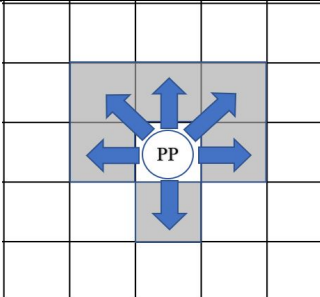
Pieces	Details	Movements of Pieces
Promoted pawn	Promoted pawn moves one square orthogonal or one square diagonally forward.	

Table 1.4 The movements of promoted pieces in Japanese chess

In Japanese chess, a player can bring a piece he/she has already captured back to the board to be his/her piece, and this action is called "dropping". There are some conditions for dropping as follows.

1. A pawn cannot be dropped in the column that has another pawn (except an opponent pawn); however, a pawn can be dropped if there is a promoted pawn in that column.

2. A pawn cannot be dropped to checkmate.

3. If a piece is captured in the promoted form, that piece can be dropped in only its original form.

4. A piece that is dropped in the promotion zone must be in its unpromoted form. It can be promoted after it is moved again and still be in the promotion zone.

5. A player cannot drop a piece on a square so that it cannot be moved. For example, a lance cannot be dropped in the last row since it can only more forward.

1.3 Chinese Chess

Chinese chess is played on a point, which is the intersection between two lines of a 10×9 board. On each side of the board, there is an X-shaped area, which is called a palace. On the center of the board, there is an area between the fourth row and the fifth row, which is called a river. Chinese chess has sixteen pieces, which are five pawns (Ping or Tsuh), two cannons (Pao), two rooks (Chuh), two knights (Ma), two elephants (Shiang), two guards (Shi), and a king (Jiang). The pieces are arranged as shown in Figure 1.4, and these movements are shown in Table 1.5.
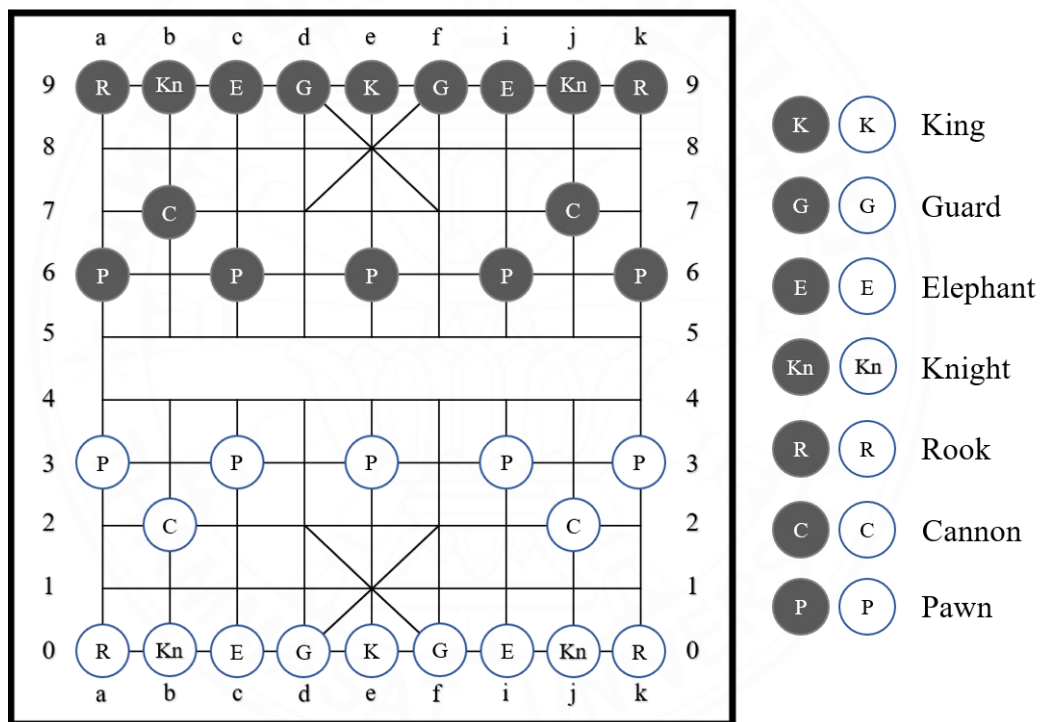
Figure 1.4 The positions of pieces in Chinese chess
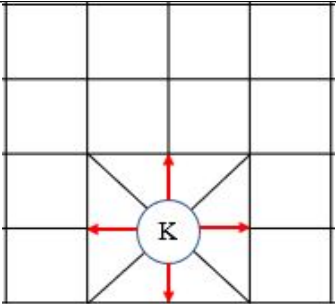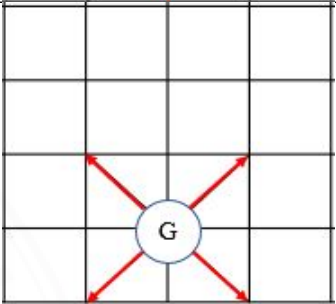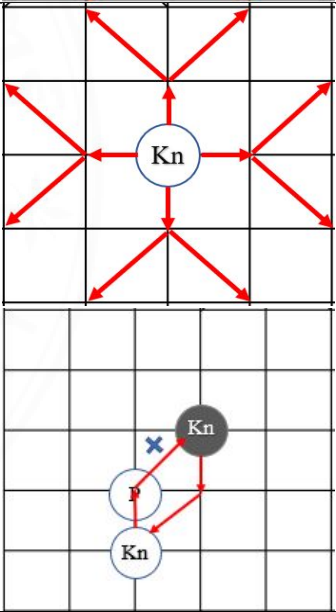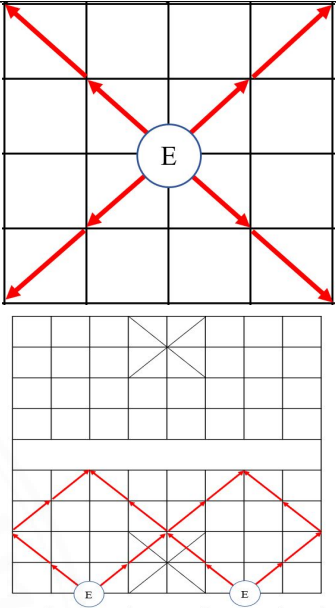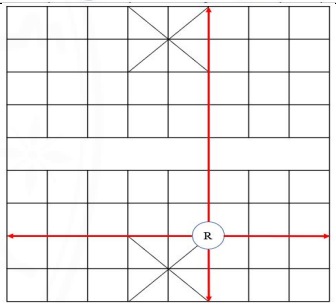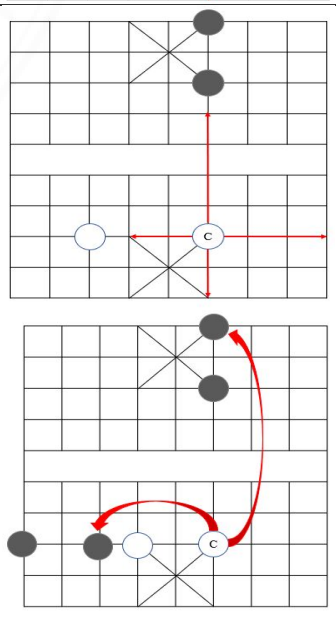
Pieces	Details	Movements of Pieces
King	King moves one position orthogonally in the palace.	
Guard	Guard moves one position diagonally in the palace.	
Knight	Knight moves one point orthogonally (forward, backward, left, or right), and then one diagonally forward point from its point of origin. If there is a piece that obstructs the movement of the knight, the knight cannot move that way (Knight cannot jump over a piece in its path).	

Continued on next page

Table 1.5 – continued from previous page

Pieces	Details	Movements of Pieces
Elephant	Elephant moves exactly two positions diagonally without jumping over another piece, and it cannot move across the river (it must be on their half of the board).	
Rook	Rook moves vertically or horizontally to any unobstructed position without jumping over other pieces.	
Cannon	Cannon moves exactly like a rook, but it must jump over exactly another piece (its own piece or an opponent's piece) to capture a piece.	

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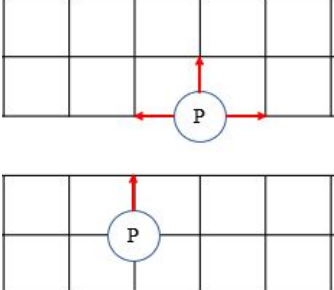
Pieces	Details	Movements of Pieces
Pawn	Pawn moves one position forward. After it has crossed the river, it may also move one step left or right.	

Table 1.5 The movements of pieces in Chinese chess

In Chinese chess, there are no promotions of pieces like Japanese chess. In addition, there is an extra rule that says both kings cannot be in the same line without other pieces between them. If there is one piece between them, that piece cannot be moved out of the line until another piece moves in between the two kings or both kings are not in the same column any more.

1.4 Korean Chess

Korean chess is played on a 10×9 chessboard, and it has pieces like ones in Chinese chess. In this game, each player is given sixteen pieces, which are five pawns (Byeong or Jol), two cannons (Po), two rooks (Cha), two knights (Ma), two elehants (Sang), two guards (Sa), and a king (Han or Cho), and they cannot be promoted. Furthermore, there are some differences between Chinese chess and Korean chess as follows.

1. The initial position of a king is in the center of the palace on the Korean chess-board, as shown in Figure 1.5.

2. The pieces, except knights and rooks, have different movements from those in Chinese chess, as shown in Table 1.6.

3. In contrast to Chinese chess, there is no river.

4. There is an extra rule "optional set-up for horses and elephants" (see below).

5. If both kings are in the same column without other pieces between them, the game is over, and both players draw.
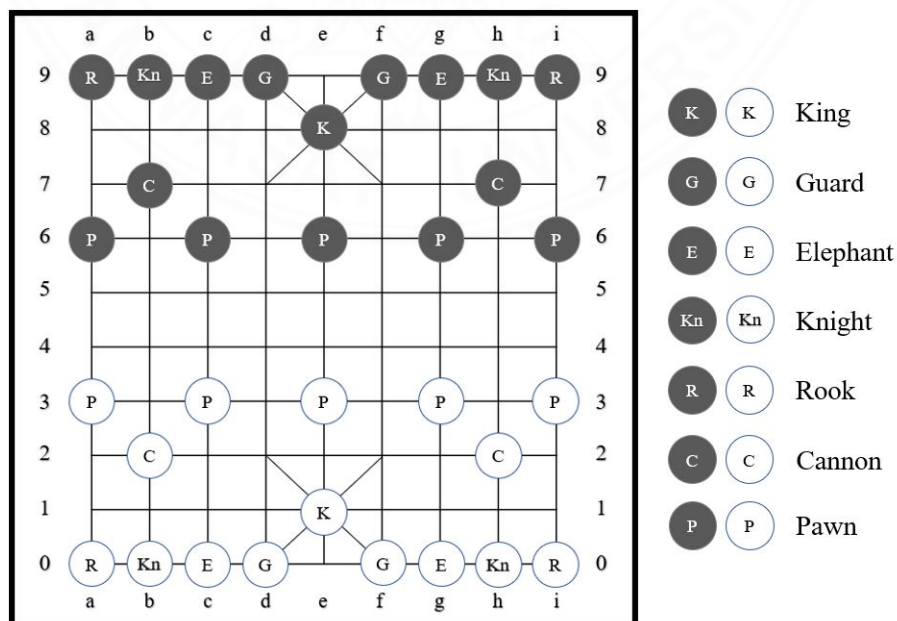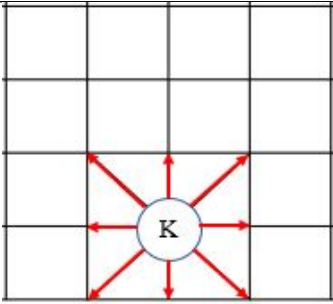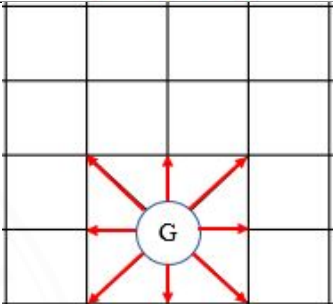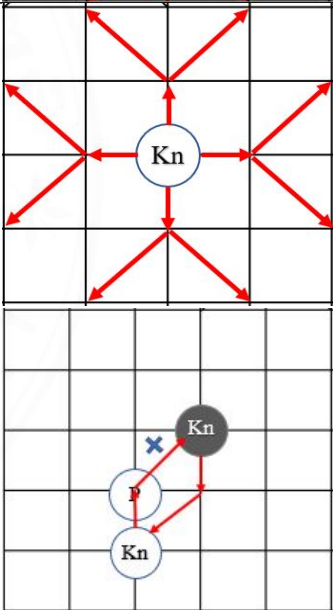
Figure 1.5 The positions of pieces in Korean chess

Pieces	Details	Movements of Pieces
King	King moves one point over a line in the palace.	
Guard	Guard moves one point over a line in the palace.	
Knight	Knight moves one point orthogonally (forward, backward, left, or right), and then one diagonally forward point from its point of origin. If there is a piece that obstructs the movement of the knight, the knight cannot move that way (Knight cannot jump over a piece in its path).	
		Continued on next page

Table 1.6 – continued from previous page

Pieces	Details	Movements of Pieces
Elephant	Elephant moves one point orthogonally (forward, backward, left, or right), and then two diagonally forward points from its point of origin. If there is a piece that obstructs the movement of the elephant, the elephant cannot move that way (Elephant cannot jump over a piece in its path).	
Rook	Rook moves vertically or horizontally to any unobstructed square without jumping over other pieces. Moreover, if a rook is in the palace, it can also move over the line in the palace.	
Cannon	Cannon moves along any straight line, including the lines within the palace. However, it must jump across a piece, except another cannon, to move or capture.	

Table 1.6 – continued from previous page

Pieces	Details	Movements of Pieces
Pawn	Pawn moves one point forward, left, or right. Moreover, if a pawn is in the palace, it can also move over the line in the palace.	

Table 1.6 The movements of pieces in Korean chess

Optional set-up for horses and elephants: before the game starts, both players can switch the positions of the knights and the elephants on either, neither, or both sides. In other words, each player can choose one of the four forms on the board at the beginning of the game, as follows:

1. Twin Ma: the positions from left to right are a knight, an elephant, an elephant, and a knight (see Figure 1.6).
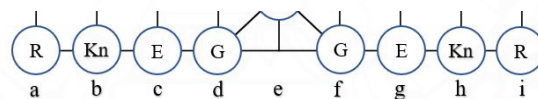
Figure 1.6 Twin Ma

2. Twin Sang: the positions from left to right are an elephant, a knight, a knight, and an elephant (see Figure 1.7).
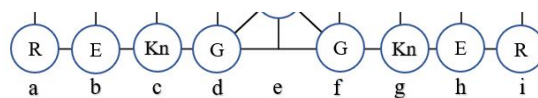
Figure 1.7 Twin Sang

3. Ma-Sang-Ma-Sang: the positions from left to right are a knight, an elephant, a knight, and an elephant (see Figure 1.8).
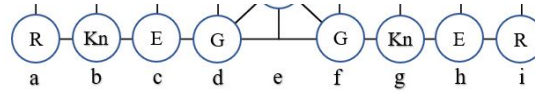
Figure 1.8 Ma-Sang-Ma-Sang

4. Sang-Ma-Sang-Ma: the positions from left to right are an elephant, a knight, an elephant, and a knight (see Figure 1.9).
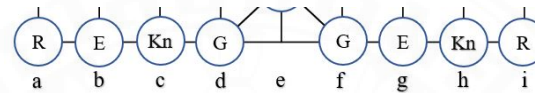
Figure 1.9 Sang-Ma-Sang-Ma

CHAPTER 2

MULTIMOVE GAMES

Let i and j be positive integers. An (i, j) multimove game is one in which the first player (White) can move i times per turn and the second player (Black) can move j times per turn. In this chapter, we consider a winning strategy of an (i, j) multimove game on Thai chess, Japanese chess, Chinese chess, and Korean chess for all $(i, j) \notin \{(1, 1), (2, 2)\}$.

2.1 Multimove Games on Thai Chess

We first present the main result for an (i, j) multimove game on Thai chess in Theorem 2.1.1 and prove it by providing winning strategies for all possible cases of i and j as appeared in the following lemmas.

Theorem 2.1.1. *For an (i, j) multimove game on Thai chess with $(i, j) \notin \{(1, 1), (2, 2)\}$, Black has a winning strategy if $i < 4$ and $i < j$; otherwise, White has a winning strategy.*

Table 2.1 shows the player who has a winning strategy, which follows Theorem 2.1.1.

White/Black	$j = 1$	$j = 2$	$j = 3$	$j \geq 4$
$i = 1$	?	Black	Black	Black
$i = 2$	White	?	Black	Black
$i = 3$	White	White	White	Black
$i \geq 4$	White	White	White	White

Table 2.1 The player who has a winning strategy for an (i, j) multimove game on Thai chess

Lemma 2.1.2. *For an (i, j) multimove game on Thai chess with $i \geq 4$, White has a winning strategy.*

Proof. To capture the Black's king, White moves the knight b1-d2-c4-d6-e8. □

Lemma 2.1.3. *For an* (i, j) *multimove game on Thai chess with* $i < 4 \le j$, *Black has a winning strategy.*

Proof. Note that White cannot capture Black's king (e8) or any knight in three moves. If White does not move the white king in the first turn, then Black moves the knight g8-e7-d5-e3-d1 to capture the White's king. Next, we consider the case that White moves the White's king in the first turn. In at most three moves, there are sixteen positions where the White's king can go to, which are a2, b1, b2, b3, c1, c2, c3, d2, d3, e1, e2, e3, f1, f2, f3 and g2. We show that Black can capture the White's king within four moves as shown in Table 2.2.

Position of the king	Moving of the knight	Position of the king	Moving of the knight
a2	g8-e7-d5-c3-a2	d3	b8-d7-c5-d3
b1	g8-e7-d5-c3-b1	e1	b8-d7-e5-d3-e1
b2	b8-d7-e5-d3-b2	e2	g8-e7-d5-c3-e2
b3	b8-d7-c5-b3	e3	g8-e7-f5-e3
c1	b8-d7-e5-d3-c1	f1	g8-e7-f5-e3-f1
c2	g8-e7-f5-e3-c2	f2	b8-d7-e5-d3-f2
c3	g8-e7-d5-c3	f3	b8-d7-e5-f3
d2	b8-d7-e5-c4-d2	g2	g8-e7-f5-e3-g2

Table 2.2 Moving of the b8 or g8 black knight to capture the white king

This completes the proof. □

Lemma 2.1.4. *For a* $(3, j)$ *multimove game on Thai chess with* $j \le 3$, *White has a winning strategy.*

Proof. White first moves the g1 knight to e2, and the a1 rook to a2. Also, White moves the b1 knight to d2 for check. Note that Black cannot capture the d1 king or any knight. If Black does not move the king, White moves the knight d2-c4-d6-e8 to capture the Black's king. Otherwise, there are sixteen positions where the Black's king can go to within j moves, which are b7, c6, c7, c8, d6, d7, d8, e6, e7, f6, f7, f8, g6, g7, g8, and h7. By symmetry of Thai Chess's board, White can easily apply the algorithm shown in Table 2.2 to capture the Black's king. □

Lemma 2.1.5. *For a* $(2, 1)$ *multimove game on Thai chess, White has a winning strategy.*

Proof. White moves the knight b1-d2-e4 for check. Note that Black cannot capture the e4 knight or the d1 king in one move. Thus, Black needs to move the e8 king to d7, e7

or f7. If Black moves the e8 king to d7 or f7, then White can capture it by the e4 knight. Otherwise, White moves the knight g1-e2-f4 for checkmate. □

Lemma 2.1.6. *For a* $(2,3)$ *multimove game on Thai chess, Black has a winning strategy.*

Proof. Note that Black can move the rook a8-a7, the knight b8-d7, and the knight g8-e7 in his first turn to make White be in check in her second turn. Furthermore, there are twenty-nine positions to which White's king can go in two turns (see Figure 2.1). If White does not capture one of the Black's knights or the Black's king in the first two turn, whenever White move the king, Black can capture it by using the moving shown in Tables 2.2 and 2.3. Then White has three options in her first turn to counter the Black's strategy.
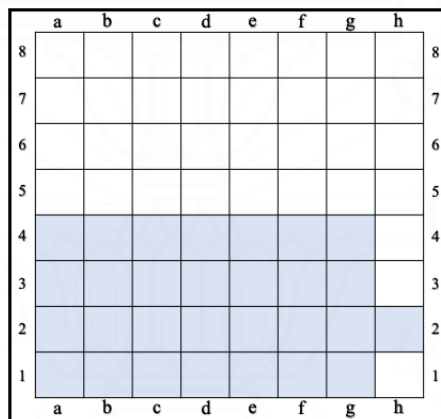
Figure 2.1 The positions where the white king can go to within four moves

Position of the king	Moving of the knight	Position of the king	Moving of the knight
a1	d7-c5-b3-a1	e4	d7-c5-e4
a3	d7-e5-c4-a3	f4	e7-d5-f4
a4	d7-c5-a4	g1	d7-e5-f3-g1
b4	e7-d5-b4	g3	e7-f5-g3
c4	d7-e5-c4	g4	d7-e5-g4
d1	e7-d5-c3-d1	h2	d7-e5-f3-h2
d4	e7-f5-d4		

Table 2.3 Moving of the d7 or e7 black knight to capture the white king

Case 1: White moves the knight b1-d2-c4 or b1-d2-e4 for check or prepares to capture the e7 knight by moving the knight g1-e2-d4 or g1-e2-f4.

Black captures the white knight by a pawn in two moves, and then moves the g8 knight to e7 for check. Note that White cannot capture or block the e7 knight. Hence, White must move the d1 king to another position. If White moves the king to b1, c2, or e2, then Black captures the king by the e7 knight; otherwise (the White's king is in b2, c1, d2, e1, or f2), Black can move the b8 knight to d7 for checkmate (see Tables 2.2 and 2.3). Even though Black has two more moves, he may move some other pieces that do not affect the game such as moving the rook a8-a7-a8.

Case 2: White prepares to capture the e7 knight by moving a pawn c3-c4-c5, e3-e4-e5, or g3-g4-g5.

Black moves a pawn to capture the white pawn, and then moves the two knights to d7 and e7 for checkmate (see Tables 2.2 and 2.3).

Case 3: White prepares to capture the e7 knight by moving the pawn c3-c4 and the knight b1-c3.

Black captures the c3 knight by moving the d6 pawn in three moves. We consider the following situations. If White moves the knight g1-e2-d4, then Black captures it by the promoted c3 pawn. Also, Black moves two knights to d7 and e7 for checkmate. If White moves the knight g1-e2-f4, then Black captures it by the e6 pawn, and Black moves the knight g8-e7 for check. Thus, White must move the king to b1. Black moves the promoted pawn c3-d4 and the knight b8-d7 for checkmate. If White moves a pawn e3-e4-e5, or g3-g4-g5, then Black captures it by a pawn, and moves the two knights to d7 and e7 for check. Thus, White must move the king to b1. Black moves the promoted pawn c3-d4 for checkmate. Otherwise, Black moves the b8 knight to d7, and moves the g8 knight to e7 for checkmate.

We are done with the proof. □

For a $(1, 3)$ multimove game on Thai chess, Black can easily apply the strategy in Lemma 2.1.6 since White is less able to block or attack. Hence, we get the following lemma.

Lemma 2.1.7. *For a $(1, 3)$ multimove game on Thai chess, Black has a winning strategy.*

Lemma 2.1.8. *For a* $(1, 2)$ *multimove game on Thai chess, Black has a winning strategy.*

Proof. We first show that White cannot check Black in the first turn. If White moves a piece on the first row in her first turn, it can be only in the second row. Moreover, if White moves a pawn in the first turn, that pawn is in the fourth row. For both cases, White cannot capture the king in the second turn. Hence, White cannot make Black in check in the first turn.

The following algorithm is Black's winning strategy, wherein whenever White moves a pawn d3-d4 or e3-e4, Black captures it in the same turn.

If White moves the d1 king to d2, then Black moves the knight b8-d7-e5 for check. Thus, White must move the king to c2, d1, or e2. Then Black moves the knight g8-e7-d5 for checkmate.

If White moves the c3 pawn (g3), Black moves the knight g8-e7-f5 (d5) for check. Then White must move the king to d2. After that, Black moves the knight b8-d7-c5 for checkmate.

If White moves the b3 pawn (f3), Black move the knight g8-e7-d5 for check. Thus, White must move the king to d2. Then Black moves the knight b8-d7-e5 (c5) for checkmate.

Otherwise, Black moves the knight g8-e7-d5 for check. If White moves the king to c2 or e2, then Black can capture the king by the d5 knight in the coming turn; otherwise (White moves the king to b2, c1, d2, e1, or f2), Black moves the knight b8-d7-e5 for checkmate. □

2.2 Multimove Games on Japanese Chess

For an (i, j) multimove game on Japanese chess, the main result is Theorem 2.2.1, and Table 2.4 shows the player who has a winning strategy.

Theorem 2.2.1. *For an* (i, j) *multimove game on Japanese chess with* $(i, j) \notin \{(1, 1), (2, 2)\}$, *Black has a winning strategy if* $i < 3$ *and* $i < j$; *otherwise, White has a winning strategy.*

To prove the theorem, we prove the following lemmas.

White/Black	$j = 1$	$j = 2$	$j \geq 3$
$i = 1$	?	Black	Black
$i = 2$	White	?	Black
$i \geq 3$	White	White	White

Table 2.4 The player who has a winning strategy for an (i, j) multimove game on Japanese chess

Lemma 2.2.2. *For an* (i, j) *multimove game on Japanese chess with* $i \geq 3$*, White has a winning strategy.*

Proof. White moves the pawn g7-f7 and moves the bishop h8-c3-a5. □

Lemma 2.2.3. *For a* $(1, 3)$ *multimove game on Japanese chess, Black has a winning strategy.*

Proof. Note that White is in check at the beginning of the game since Black can move the pawn c3-d3 and the bishop b2-g7-i5 to capture the i5 king in his first turn. Since White cannot capture the bishop, White must do one of the following cases.

Case 1 White moves the pawn g6-f6.

Black moves the pawn c1-d1 and the bishop b2-c1-g5 for check. Note that White cannot capture the g5 bishop or the king in one move. Moreover, no matter how Black moves, White can capture the king by using the bishop.

Case 2 White moves either the silver i7-h6, the gold i6-h6, or the rook h2-h6.

Black moves the pawn c3-d3 and the bishop b2-g7-h8 to capture the White's bishop. Also, Black promotes the h8 bishop for check. If White moves either the silver i7-h8 or the rook h6-h8 to capture the bishop, then Black drops the bishop to h4 to captures the i5 king; otherwise, he captures the king by that bishop.

Case 3 White moves either the king i5-h4 or i5-h5.

Black moves the pawn c3-d3 and the bishop b2-g7-h8. No matter how White moves in the next turn, Black can return the bishop to capture the White's king.

Thus, the proof is concluded. □

Lemma 2.2.3 implies the following lemma.

Lemma 2.2.4. *For a* $(1, j)$ *multimove game on Japanese chess with* $j \geq 3$, *Black has a winning strategy.*

Lemma 2.2.5. *For a* $(2, 3)$ *multimove game on Japanese chess, Black has a winning strategy.*

Proof. Note that White is in check at the beginning of the game since Black can move the pawn c3-d3 and the bishop b2-g7-i5 in the next turn. Thus, White must do one of the following.

Case 1 White moves the pawn g5-f5-e5.

Black moves the c1 pawn and moves the bishop b2-c1-i7 to capture the silver. White is in checkmate since Black can return the silver to capture the White's king in the next turn.

Case 2 White moves the pawn g6-f6.

If White also moves the pawn g7-f7, then Black moves the pawn c3-d3 and the bishop b2-f6-h8. White is now in checkmate since Black can return the bishop to capture the White's king in the next turn.

If White also move the pawn g8-f8, then Black moves the pawn c3-d3 and the knight a2-c3-e4. Note that Black can move the knight e4-g5, and promotes it to capture the king by the remaining two moves. Thus, White is in check. Since White cannot block or move her king to get away from the knight, White should capture the e4 knight. If White does so, the White's king is still at i5. Then Black can capture the king by moving the bishop b2-f6-g7-i5.

If White also moves the rook h2-h6, then Black moves the pawn c3-d3 and the knight a2-c3-e4. Now White is in check with the e4 knight. Note that White must use at least two moves to capture the e4 knight. If White do so, Black can move the bishop b2-f6-g5-h6 and promote it to checkmate. Otherwise, Black can capture the king by the e4 knight.

Otherwise, Black moves the pawn c3-d3 and the bishop b2-f6-d8. White is in checkmate since Black can capture the king by the d8 bishop in the next turn.

Case 3 White moves the pawn g7-f7 and the bishop from h8 to either d4, e5, or f6.

Black moves the pawn c3-d3, and the b2 bishop to captures the White's bishop and the i9 lance, so White is in checkmate.

Case 4 White moves either the i7 silver, i6 gold, or the h2 rook to h6 by using one or two moves.

If White moves the silver i7-h6 and the bishop h8-i7, Black moves the pawn c3-d3 and the bishop b2-g7-h6 for checkmate. Otherwise, Black moves the pawn c3-d3 and the bishop b2-g7-h8 for checkmate.

Case 5 White moves the king to h3.

Black moves the pawn c3-d3, and moves the bishop b2-e5-d6 for checkmate.

Case 6 White moves the king to either h4, h5, i4 or i6 by using one or two move. Black moves like Case 4 for checkmate.

In summary, the proof is now complete. □

We can easily apply Lemma 2.2.5 to get the following lemma.

Lemma 2.2.6. *For a* $(2, j)$ *multimove game on Japanese chess with* $j \geq 3$*, Black has a winning strategy.*

Lemma 2.2.7. *For a* $(2, 1)$ *multimove game on Japanese chess, White has a winning strategy.*

Proof. White moves the pawn g7-f7, and moves the bishop h8-e5 for check. Since Black cannot capture the e5 bishop or the i5 king, and he also cannot block the bishop, he must move the king a5-b5. Then White moves the knight i8-g7-e6 for checkmate. □

Lemma 2.2.8. *For a* $(1, 2)$ *multimove game on Japanese chess, Black has a winning strategy.*

Proof. Since Black can move the pawn c3-d3 and the bishop b2-e5 in his first turn (to capture the White's king in his second turn), We consider the following White's options in the first turn to counter this Black's strategy.

Case 1 White prepares to capture the e5 bishop by moving the pawn g7-f7 (to make way for the h8 bishop).

Then Black moves the pawn c3-d3 and the bishop b2-h8 to capture the White's bishop. Also, he promotes his own bishop for check. If White moves the king i5-h5, then Black can move the the promoted bishop h8-i7-i6 to capture the silver and gold for checkmate; otherwise, Black can return the bishop to capture the White's king.

Case 2 White prepares to capture the e5 bishop in the first turn by moving the pawn g5-f5.

Black moves the pawn c1-d1 and the bishop b2-c1 for check. Since White cannot capture the c1 bishop or move the king to get away from the bishop in one move, White must move the pawn g4-f4 to block the c1 bishop. Then Black moves the bishop c1-f4 to capture the pawn. He also moves the bishop f4-g5 to promote for checkmate.

Case 3 White prepares to block the e5 bishop by moving either the pawn g6-f6, or the pawn g4-f4.

Case 3.1 White moves the pawn g6-f6.

Black moves the pawn c3-d3 and the bishop b2-f6. Then White is in check. Note that White cannot capture the f6 bishop or the king in her second turn. White should not move the king in her second turn since Black can still capture it by the f6 bishop. Hence White should move either the h2 rook, the i6 gold, or the i7 silver to h6 to block the f6 bishop. Then Black moves the bishop f6-g7-h6 to checkmate.

Case 3.2 White moves the pawn g4-f4.

Black moves the pawn c3-d3 and the bishop b2-e5 to check. Note that White cannot capture the e5 bishop or the king in her second turn.

If White moves the king to get away from the e5 bishop, Black moves the knight a2-c3 and the bishop e5-f4 to checkmate.

If White moves the pawn g6-f6 to block the e5 bishop, Black moves the bishop e5-f6-g5 and promote it to checkmate.

If White moves either the h2 rook, the i6 gold, or the i7 silver to block the e5 bishop in the h6 position, Black moves the bishop e5-g7-h6 to checkmate.

Case 4 White prepares to block the e5 bishop by moving either the rook h2-h6, the silver i7-h6, or the gold i6-h6.

Black moves the pawn c3-d3 and the bishop b2-e5 for check. Now, White can block the bishop by moving either the gold i4-h4, the silver i3-h4, the rook h2-h4, or the pawn g4-f4. Also, White can move the king i5-h5 or i5-i6 (if it is possible) to get away from the bishop. Anyway, Black moves the bishop e5-g7-h6 to capture the piece at h6 (silver, gold, or rook), and promotes it for check. If White does not capture the promoted bishop, Black can move it to capture the king. Assume this is not the case. If Black captured piece is gold or rook in his second turn, then he can return it to capture the king. We next consider the case that Black captured the silver in his second turn. If White captured the promoted bishop by her king, Black can return the silver to i5 to capture the king; otherwise, Black can return the silver to h5 or i6 to capture the king.

Case 5 White prepares to block the e5 bishop by moving either the rook h2-h4, the silver i3-h4, or the gold i4-h4.

We can apply the algorithm in Case 4.

Case 6 White moves the king.

Case 6.1 White moves the king i5-h4.

Black moves the pawn c3-d3 and the bishop b2-e5 for check. Thus, White must move either the king h4-h5 or the pawn g4-f4. If White moves the king h4-h5, then Black moves the bishop e5-g3-h2 to capture the rook. No matter how White moves, Black can return the rook to capture the White's king. If White moves the pawn, then Black moves the bishop e5-d6-f4 for checkmate.

Case 6.2 White moves the king i5-h5.

Black moves the pawn c1-d1 and the bishop b2-c1 for check. Thus, White must move either the king h5-i5 or the pawn g4-f4. If White moves the king

h5-i5, then Black moves the bishop c1-f4-g5, and promotes it for checkmate. Otherwise, Black moves the bishop c1-e3-f4 for checkmate.

Case 6.3 White moves the king i5-h6.

Black moves the pawn c3-d3 and the bishop b2-e5 for check. Thus, White must move either the king h6-h5 or the pawn g6-f6. In either way, Black moves the bishop e5-g3-h2 for checkmate.

In other cases, no matter how White moves a piece in the first turn, Black moves the pawn c3-d3 and the bishop b2-e5 to check. If White moves the king, Black moves the knight a2-c3-e4 to checkmate. Otherwise, Black can capture the king by the e5 bishop. □

2.3 Multimove Games on Chinese Chess

The main result in this section is shown in Theorem 2.3.1, and Table 2.5 shows the player who has a winning strategy.

Theorem 2.3.1. *For an (i, j) multimove game on Chinese chess with $(i, j) \notin \{(1, 1), (2, 2)\}$, Black has a winning strategy if $i < 3$ and $i < j$; otherwise, White has a winning strategy.*

White/Black	$j = 1$	$j = 2$	$j \geq 3$
$i = 1$	?	Black	Black
$i = 2$	White	?	Black
$i \geq 3$	White	White	White

Table 2.5 The player who has a winning strategy for an (i, j) multimove game on Chinese chess

To prove the theorem, we prove the following lemmas.

Lemma 2.3.2. *For an (i, j) multimove game on Chinese chess with $i \geq 3$, White has a winning strategy.*

Proof. White moves the cannon b2-b4-e4-e9 to capture the Black's king. □

Lemma 2.3.3. *For a* $(2, 3)$ *multimove game on Chinese chess, Black has a winning strategy.*

Proof. White is in check at the beginning of the game since Black can move the cannon b7-b4-e4-e0, b7-b5-e5-e0, h7-h4-e4-e0, or h7-h5-e5-e0 to capture the White's king. Since White cannot capture the Black's king or any Black's cannon in two moves, White should move the king to get away from the Black's cannons or move some pieces to block them. If White moves the king to d0 or f0 in the first turn, then Black moves the cannon b7-b0-d0 or the cannon h7-h0-f0. If White moves the king to d1 or f1 in the first turn, then Black can capture the White's King by a rook in three moves. If White moves the king to e1 or e2 in the first turn, Black can move a cannon to capture the king within three moves. Hence, White should block the cannons as follows in the first turn.

Case 1 White moves the cannons to b5 and h5.

Then Black is in check. To block the White's cannons, Black moves the cannon b7-e7. Also, Black moves the rooks to a8 and i8 for check. Note that White has only one way to block the Black's rooks, that is, to move the cannons b5-d5 and h5-f5. If White does so, then Black moves the cannon h7-h4-e4-e0 to capture the king; otherwise, Black can capture the king by a rook.

Case 2 White moves either the cannon b2-b5 and the cannon h2-h6 or the cannon b2-b6 and the cannon h2-h5.

Without loss of generality, we may assume that White moves the cannon b2-b5 and the cannon h2-h6 for check. Then Black moves the cannon h7-d7-d5 to block the b5 cannon. Also, Black moves the rook a9-a8. Hence, White is in check (by the d5 cannon and the a8 rook). Note that White cannot capture both d5 cannon and a8 rook in two moves. Since the White's king cannot get away from the a8 rook, White should capture the a8 rook by the b5 cannon or block it by a rook, a guard, or a cannon. Then Black can capture the king by the d5 cannon or the b7 cannon.

Case 3 White moves the cannons to b6 and h6.

Black moves the rook a9-a8-d8 and the rook i9-i8 for checkmate.

Case 4 White moves a guard to e1, and moves an elephant or a cannon to e2.

Black moves the rooks a9-a8 and i9-i8, and moves the cannon b7-b5 for check-mate.

Case 5 White moves the pawn e3-e4-e5.

Black moves the pawn e6-e5 and the cannon b7-e7-e0.

This completes the proof. □

For an (i, j) multimove game on Chinese chess with $1 \leq i \leq 2$ and $j \geq 3$, Black can easily apply the above strategy. Hence, we get the following lemma.

Lemma 2.3.4. *For an (i, j) multimove game on Chinese chess with $1 \leq i < 3 \leq j$, Black has a winning strategy.*

Lemma 2.3.5. *For a $(2, 1)$ multimove game on Chinese chess, White has a winning strategy.*

Proof. White moves the cannon b2-b4 and the cannon h2-e2 for checkmate. □

Lemma 2.3.6. *For a $(1, 2)$ multimove game on Chinese chess, Black has a winning strategy.*

Proof. We first show that if White moves a guard or an elephant in the first turn, then Black can capture the White's king in his first turn. Without loss of generality, we may assume that White moves the d0 guard or the c0 elephant. Black then moves the cannon b7-b0-e0 to capture the White's king.

Suppose this is not the case. Note that White cannot capture the king or any cannon in the first turn. Also, White cannot move any piece for check. No matter what White does in the first turn, Black can move either the cannons b7-e7 and h7-h5, or the cannons h7-e7 and b7-b5 for check. If Black does so, then White cannot capture or get away from the cannons in one move. Clearly, White cannot capture the king or a cannon in the first turn. Also, White cannot move a piece for check, so we consider the following cases.

Case 1 White moves the king e0-e1.

Black moves the cannons b7-e7 and h7-h5 for check. Since White cannot block

the cannons in one moves, White must move the king to d1 or f1. If White moves the king to d1, then Black moves the rook a9-a8-d8 for checkmate; otherwise, Black moves the rook a9-a8-f8 for checkmate.

Case 2 White moves the pawn e3-e4.

Black moves the cannons b7-e7 and h7-h5 for check. To block the h5 cannon, White may do one of the following, which are moving the e4 pawn or moving a piece (a cannon, an elephant, a guard) to the line e. If White moves the e4 pawn, Black moves the pawn e5 and the cannon e7-e0 to capture the White's King. If White moves some piece to the line e, then Black can capture the king by the e7 cannon.

Case 3 White moves the pawn c3-c4 or g3-g4.

Without loss of generality, we assume that White moves the c3 pawn. Then Black moves the cannons b7-e7 and h7-h5 for checkmate.

Case 4 White moves the cannon b2-e2 or h2-e2.

Without loss of generality, we assume that White moves the cannon b2-e2. Black moves the cannons b7-e7 and h7-h5 for check. White must move a guard to e1 to block the e7 cannon. Then Black moves a rook to d8 for checkmate.

Case 5 White moves a cannon b2-b5, b2-b6, h2-h5, or h2-h6.

Without loss of generality, we assume that White moves the cannon b2. Then Black moves the cannons b7-e7 and h7-h5 for checkmate.

Thus, the proof is concluded □

2.4 Multimove Games on Korean Chess

For this section, Theorem 2.4.1 is the main result and is summarized in Table 2.6. To prove the theorem, we provide a strategy for each (i, j). Even though a player can choose any starting form, in this dissertation, we let the player who has a winning strategy use the Twin Ma form. We consider all possible starting forms for the other player: the Twin Ma, the Twin Sang, the Ma-Sang-Ma-Sang, and the Sang-Ma-Sang-Ma.

Theorem 2.4.1. *For an (i, j) multimove game on Korean chess with $(i, j) \notin \{(1, 1), (2, 2)\}$, Black has a winning strategy if $i < 4$ and $i < j$; otherwise, White has a winning strategy.*

White/Black	$j = 1$	$j = 2$	$j = 3$	$j \geq 4$
$i = 1$	?	Black	Black	Black
$i = 2$	White	?	Black	Black
$i = 3$	White	White	White	Black
$i \geq 4$	White	White	White	White

Table 2.6 The player who has a winning strategy for an (i, j) multimove game on Korean chess

2.4.1 Twin Ma

In this subsection, both players use the Twin Ma form, and the corresponding board is shown in Figure 2.2.
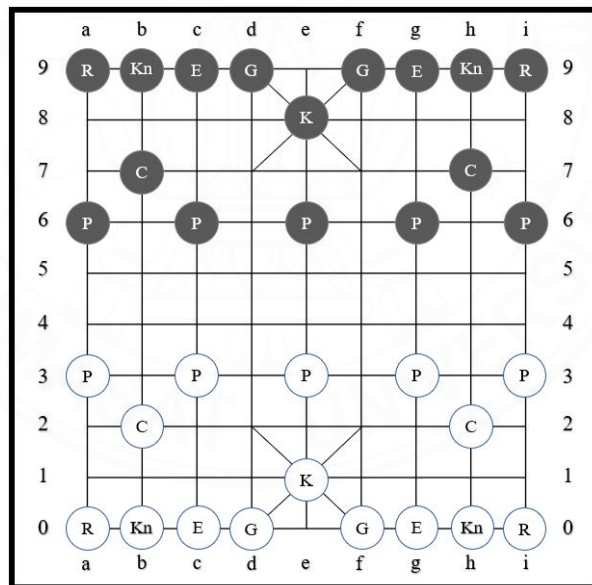
Figure 2.2 The board where both players use the Twin Ma form

Lemma 2.4.2. *For an (i, j) multimove game on Korean chess with $i \geq 4$, if Black uses the Twin Ma form, White has a winning strategy.*

Proof. White moves the rook a0-a1-d1-d7-e8 to capture the Black's king. □

Lemma 2.4.3. *For a* $(3, 4)$ *multimove game on Korean chess, if White uses the Twin Ma form, Black has a winning strategy.*

Proof. At the beginning of the game, White is in check by both rooks and both cannons since Black can move either the rook a9-a8-d8-d2-e1, the rook i9-i8-f8-f2-e1, the pawn a6-b6 and the rook a9-a3-a1-e1, the pawn i6-h6 and the rook i9-i3-i1-e1, the knight b9-c7 and the cannon b7-e7-e5-e1, or the knight h9-g7 and the cannon h7-e7-e5-e1. Also, White cannot capture the Black's king in his first turn. Consequently, in the first turn, White must do one of the following: capture the Black's rooks and cannons; move some pieces to block them; move the White's king to get away from them; or make a game end with a draw. Note that White cannot capture all of these pieces in his first turn, and White does not have enough moves to block them. Since the White's king can be only in the palace, Black can always apply the above strategies to capture the White's king. To make a game end with a draw, White needs to make that there is no piece between both kings. Then White must capture the e6 pawn with some piece, and move the piece and the e3 pawn to the other column, as shown in Figure 2.3.
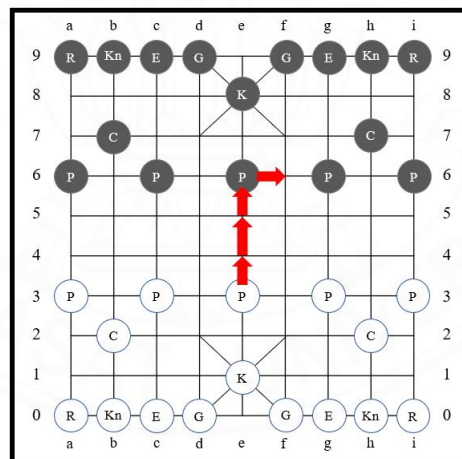
Figure 2.3 The drawing strategy of White in the first turn

However, White does not have enough move to do that. Therefore, White is in checkmate at the beginning of the game. □

For an (i, j) multimove game on Korean chess with $1 \leq i \leq 3$ and $j \geq 4$, Black can easily apply the above strategy. Hence, we get the following lemma.

Lemma 2.4.4. *For an* (i, j) *multimove game on Korean chess with* $1 \leq i \leq 3$ *and* $j \geq 4$, *if White uses the Twin Ma form, Black has a winning strategy.*

Lemma 2.4.5. *For a* $(3, 3)$ *multimove game on Korean chess, if Black uses the Twin Ma form, White has a winning strategy.*

Proof. White first moves the rook a0-a1-d1 and the knight b0-c2. Since White can move the rook d1-d7-e8 in her next turn, Black is now in check. Note that Black cannot capture White's king in his first turn. Then Black must do some of the following: capture the d1 rook; move some pieces to block it; move Black's king to get away from the rook; or make the game end in a draw. To make the game ends with a draw, Black needs to move some pieces to capture the e3 pawn, and then move that piece to another column. Since Black must use at least three moves to capture the pawn and one to move that pawn to another column, the game cannot end in a draw. Note that, in the first turn, Black has only one way to capture the d1 rook, which is moving the rook a9-a8-d8-d1. If Black does so, White moves the cannon b2-e2-e4-e8. If Black moves the king only, White can still capture the king by the rook d1. Then we consider the following cases.

Case 1 Black moves the king to the column d.

> No matter how Black moves some pieces to block the d1 rook, White can capture the king with the d1 rook.

Case 2 Black moves the king to e7 or e8.

> If Black moves only one piece to the column d, White can capture the king with the d1 rook.

> Next, we assume Black moves two pieces to d5 and d6 to block the rook (for example, Black moves the pawn c6-d6 and the knight b9-c7-d5). Since Black must use at least three moves to do that, the Black's king must be at e8. If there is a piece between the Black's king and the e3 pawn, White moves the cannon b2-e2-e4-e8 to capture the king; otherwise, White moves the cannon b2-e2-e8 to capture the king.

Case 3 Black moves the king to e9.

> Then Black has at most two remaining moves. If Black moves two pieces to d6 and d8, then White can capture the king by the b2 cannon; otherwise, White capture the king by the d1 rook.

Case 4 Black moves the king to f7 or f8.

Then he has at most two remaining moves. Thus, there is at most one blocking piece in d2, d3, d4, d5, d6, and d7. Hence, White can capture the king with the d1 rook.

Case 5 Black moves the king to f9.

Then he has at most one remaining move. Since Black needs at least two moves to move a cannon to the column f, Black cannot do that. Then White moves the cannon b2-f2. If there are some pieces between the f2 cannon and the Black's king, White moves the cannon to capture the king; otherwise, he moves the pawn g3-f3 and the cannon f2-f9 to capture the king.

In summary, the proof is now complete. □

For a $(3, j)$ multimove game on Korean chess with $1 \leq j \leq 3$, Black can easily apply the above strategy. Hence, we get the following lemma.

Lemma 2.4.6. *For a $(3, j)$ multimove game on Korean chess with $1 \leq j \leq 3$, if Black uses the Twin Ma form, White has a winning strategy.*

Lemma 2.4.7. *For a $(2, 3)$ multimove game on Korean chess, if White uses the Twin Ma form, Black has a winning strategy.*

Proof. First, we show that White does not have a drawing strategy. Note that White needs at least four moves to make the game end in a draw (e.g., the pawn e3-e4-e5-e6-f6). If White tries to do so, Black moves the knight b9-c7, the cannon b7-e7, and the rook a9-a8 in his first turn to make White in checkmate. Next, we give a Black's winning strategy by determining whether White makes Black in check in the first turn.

Case 1 White makes Black in check in the first turn.

Since the board is symmetric, we can consider the following five cases.

Case 1.1 White moves the pawn a3-b3, and then moves the rook a0-a6 to capture the pawn.

Since White can move the rook a6-a8-e8 in the next turn, Black is in check. However, Black can move the rook a9-a6-a1-e1 to capture the king in his first turn.

Case 1.2 White moves the rook a0-a1-d1.

Since White can move the rook d1-d7-e8 in the next turn, Black is in check. Black moves the pawn c6-d6 to block the rook, and then moves the rook i9-i8-f8 for check. Since White cannot capture the king or the f8 rook in this turn, White should do one of the following.

> **Case 1.2.1** White moves some pieces (the b2 cannons, the h2 cannons, the d1 rook, the e3 pawn, and the g3 pawn) to block the f8 rook, so White has at most one remaining move.
> No matter how White moves, Black can use the f8 rook to capture the king.

> **Case 1.2.2** White moves the king to get away from the rook.
> It is easy to check that no matter where the White's king go to in the palace, Black can capture the king by the f8 rook.

Case 1.3 White moves the e3 pawn to another column, and also moves either the knight b0-c2 or the king e1-d2 to make Black in check by the b2 cannon.

If White moves the knight b0-c2, then Black moves the knight b9-c7 and the cannon b7-e7-e1. If White moves the pawn e3-d3 and the king e1-d2, then Black moves the knight b9-c7 and the cannon b7-d7-d2. Otherwise, Black moves the rook a9-a8-d8-d2.

Case 1.4 White moves the knight b0-c2 and the cannon b2-e2.

Since White can move the cannon e2-e4-e8 in the next turn, Black is in check. Black moves the knight b9-c7, the cannon b7-e7 to block the e2 cannon, and the rook a9-a8. Since Black can move the rook a8-b8-b1-e1 or the rook a8-d8-d1-e1 in the next turn, White is in check. Since White cannot capture the king or the a8 rook in this turn, White should do one of the following five cases.

> **Case 1.4.1** White moves the pawn a3-b3 and another pawn to d3.
> Black moves the pawn a6-b6 and the rook a8-a1-e1.

Case 1.4.2 White moves the pawn a3-b3 and the king, or the pawn c3-b3 and the king.

Black can move the rook a8-d8, move it to the same row with the king, and capture it, or can move the rook i9-i8-f8 and move it to capture the king.

Case 1.4.3 White moves the pawn c3-b3 and the pawn e3-d3, the pawn c3-b3 and the knight c2-d4, the pawn c3-d3 and the knight c2-b4, the pawn c3-d3 and the king to get away, or the pawn e3-d3 and the king to get away.

Black moves the rook a8-d8 and the rook i9-i8-f8 for check. No matter how White moves, Black can move a rook to capture the king within three moves.

Case 1.4.4 White moves the cannon e2-b2.

Thus, White has the other one remaining move. If White moves the king to the column d or f, then Black can capture the king by the a8 rook; otherwise, Black can capture the king by the e7 cannon.

Case 1.4.5 White moves the king to get away from the rook.

It is easy to check that no matter where the White's king go to in the palace, Black can capture the king by a rook.

Case 1.5 White moves the king e1-d2 and then moves the cannon b2-e2 for check.

Black moves the rook a9-a8-d8-d2 to capture the king.

Case 2 White does not make Black in check in the first turn.

Case 2.1 White moves the king to another column in her first turn.

If there is no blocking piece, Black can capture the king with a rook; otherwise, Black can use a cannon.

Case 2.2 The White's king is still in the column e

Black moves the knight b9-c7 and the rook i9-i8-f8 in his first turn. Since Black can capture the king in his second turn by the b7 cannon or the f8 rook, White is in check. Hence, White should protect her king from them from the first turn. To capture each of them, White needs at least three moves (e.g. the

rook i0-i1-f1-f8). Hence, White cannot capture both of them in two turns. Since the king must be in the palace, White should not move only the king. We then consider how to block the two pieces in two turns (four moves). Since White needs at least four moves to block or get away from the f8 rook (e.g. the king e1-e0, the guard d0-e1, the grand f0-f1, and the pawn g3-f3), White does not have enough moves left to block the b7 cannon. Therefore, White is in checkmate.

This completes the proof. $\qquad\square$

For an $(i, 3)$ multimove game on Korean chess with $1 \leq i \leq 2$, Black can easily apply the above strategy. Hence, we get the following lemma.

Lemma 2.4.8. *For an $(i, 3)$ multimove game on Korean chess with $1 \leq i \leq 2$, if White uses the Twin Ma form, Black has a winning strategy.*

Lemma 2.4.9. *For a $(2, 1)$ multimove game on Korean chess, if Black uses the Twin Ma form, White has a winning strategy.*

Proof. White moves the rook a0-a1-d1 to make Black in check. Note that Black cannot capture the White's king or the d1 rook in his first turn. Also, he cannot move the king to get away from the rook. Thus, he should block the d1 rook by moving either the pawn c6-d6 or the pawn e6-d6. In either way, White moves the rook d1-d6-d4 for checkmate. $\qquad\square$

Lemma 2.4.10. *For a $(1, 2)$ multimove game on Korean chess, if White uses the Twin Ma form, Black has a winning strategy.*

Proof. Note that White cannot capture the Black's king or make Black in check in his first turn. If White moves the king to e0, Black moves the knight b8-c7 and then the cannon b7-e7 for checkmate; otherwise, Black can easily apply the White's strategy in Lemma 2.4.9 to capture the White's king. $\qquad\square$

2.4.2 Twin Sang

We use the board in Figure 2.4 to prove the lemmas that there exists a winning strategy of white, and the board in Figure 2.5 for the lemmas with a winning strategy of Black.
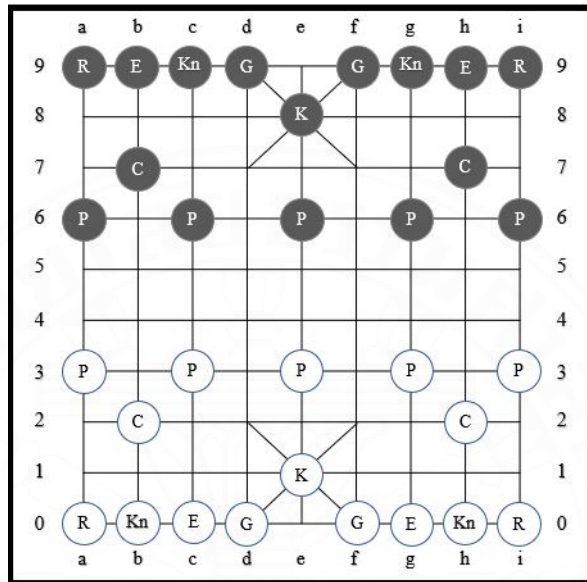
Figure 2.4 The board where White has a winning strategy when Black uses the Twin Sang form
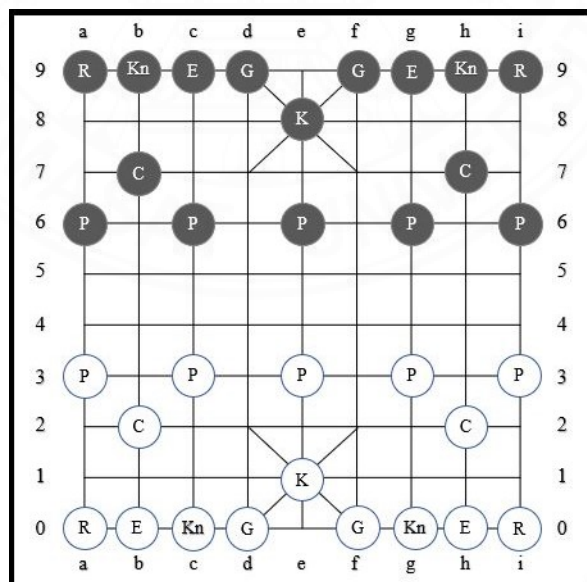
Figure 2.5 The board where Black has a winning strategy when White uses the Twin Sang form

Since White can still use the strategy in Lemma 2.4.2 for $i \geq 4$, we easily get the following lemma.

Lemma 2.4.11. *For an (i, j) multimove game on Korean chess with $i \geq 4$, if Black uses the Twin Sang form, White has a winning strategy.*

Moreover, Black can apply the strategy in Lemma 2.4.4 to get a winning strategy for an (i, j) multimove game on Korean chess with $1 \leq i \leq 3$ and $j \geq 4$. Hence, we get the following lemma.

Lemma 2.4.12. *For an (i, j) multimove game on Korean chess with $1 \leq i \leq 3$ and $j \geq 4$, if White uses the Twin Sang form, Black has a winning strategy.*

Lemma 2.4.13. *For a $(3, 3)$ multimove game on Korean chess, if Black uses the Twin Sang form, White has a winning strategy.*

Proof. White first moves the rook a0-a1-d1 and the knight b0-c2. Then Black is in check by the d1 rook. Note that Black cannot capture the White's king or make the game end with a draw. If Black does not move a knight or an elephant at all, White can use the strategy in Lemma 2.4.5. Otherwise, we consider the following cases in the White's second turn.

Case 1 The Black's king is in the column d.

No matter how Black moved, White can capture the king with the d1 rook.

Case 2 The Black's king is at e7 or e8.

If Black moved a piece to block the d1 rook, White can still capture the king by the d1 rook.

Next, we assume Black moved two pieces to d5 and d6 to block the rook (for example, Black moved the elephant e9-d6 and the pawn c6-c5-d5). Since Black needs at least three moves to do that, the Black's king must be at e8. White can capture the king with the b2 cannon.

Case 3 The Black's king is at e9.

If Black moved two pieces to the column d, White can capture the king by the b2 cannon; otherwise, White can capture the king by the d1 rook.

Case 4 The Black's king is at f7 or f8.

If Black moved two pieces to the column d (for example, Black moves the elephant b9-d6 and the knight c9-d7), White can capture the king by the i0 rook. If this is not the case, Black has at most one blocking piece in the column d, White can capture the king with the d1 rook.

Case 5 The Black's king is at f9.

No matter how Black moved, White can capture the king with the d1 rook.

We are done with the proof. □

For a $(3, j)$ multimove game on Korean chess with $1 \leq j \leq 3$, White can easily apply the above strategy. Hence, we get the following lemma.

Lemma 2.4.14. *For a $(3, j)$ multimove game on Korean chess with $1 \leq j \leq 3$, if Black uses the Twin Sang form, White has a winning strategy.*

Lemma 2.4.15. *For a $(2, 3)$ multimove game on Korean chess, if White uses the Twin Sang form, Black has a winning strategy.*

Proof. Similar to Lemma 2.4.7, White does not have a drawing strategy. Then we give a Black's winning strategy by determining the following two cases.

Case 1 White makes Black in check in the first turn.

If White does not move an elephant or a knight, Black can use the strategy in Lemma 2.4.7. Thus, we consider only when White moves at least one of those. Since the board is symmetric, we consider the following five cases.

Case 1.1 White moves the rook a0-a1-d1.

Black moves the pawn c6-d6 to block the rook and then moves the rook i9-i8-f8 for check. Since White cannot capture the e8 king or the f8 rook in this turn. If White uses at most one piece to block the f8 rook, Black can capture the blocking piece and the king by the f8 rook. If White uses two pieces to block the f8 rook (for example, White moves the knight g0-f2 and the elephant h0-f3), the White's king is still at e1. Black can capture the king by the f8 rook.

Case 1.2 White moves the e3 pawn to another column and moves the knight c0-d2 (to make Black in check by the b2 cannon).

Black moves the knight b9-c7 and the cannon b7-e7-e1.

Case 1.3 White moves the knight c0-d2 and the cannon b2-e2.

Black moves the knight b9-c7, the cannon b7-e7 to block the e2 cannon, and the rook a9-a8. Since Black can move the rook a8-b8-b1-e1 or the rook a8-d8-d2-e1 in the next turn, White is in check. Since White cannot capture the king or the a8 rook in this turn, we consider the following cases in the Black's second turn.

Case 1.3.1 The White's king is at d0 or d2.

Note that White uses at least two moves to do this (for example, White moves the knight d2-b3 and the king e1-d2). If there is a knight at b3, Black moves the rook a8-d8 and then capture the king; otherwise, Black moves the rook a8-b8, moves it again to the same row as the White's king, and captures it.

Case 1.3.2 The White's king is at d1.

Black can capture the king by the rook a8-d8-d2-d1 or the rook a8-b8-b1-d1.

Case 1.3.3 The White's king is at e0.

If White moves the king to another position, Black can capture the king by the f8 rook. If a White's cannon is not in the column e, Black can capture the king with the e7 cannon. If a guard is not at d0, Black moves the rook a8-b8-b0-e0. If a knight is not at d2, Black moves the rook a8-d8-d0-e0. If some piece is at b1 or b3, Black moves the cannon e7-b7-b0-e0. Otherwise, Black moves the rook a8-b8 and the rook i9-i8-f8. Since Black can move the rook b8-b1-e1-e0 or the rook f8-f0-e0, White is in check. To capture a rook, White has to use one more in the second turn and two moves in the third turn; however, Black can still use the remaining rook to capture the king. To block both rooks, White needs at least three moves (for example, White moves the knight g0-f2 in her second turn and moves the elephant h0-f3 and the pawn a3-b3).

Hence, at least one of the following hold: a White's cannon is not in the column e; a guard is not at d0; a knight is not at d2; and some piece is at b1 or b3. Hence, Black can capture the king by the b8 rook, the e7 cannon, and the f8 rook.

Case 1.3.4 The White's king is at e1.

If White moved at least one piece to block the a8 rook in the columns b or d, Black can move the a8 rook to capture the king by using the other column. If White moved two pieces to block the a8 rook in both columns, Black moves the rook a8-d8 and the rook i9-i8-f8 to checkmate.

Case 1.3.5 The White's king is at e2.

Black can capture the king by the a8 rook.

Case 1.3.6 The White's king is in the column f.

Black can capture the king by the i9 rook or the h0 cannon.

Case 1.4 White moves the knight c0-d2-e4.

Note that White can move the knight e4-d6-e8 in her second turn. Black moves the pawn e6-e5 to block the e4 knight, the knight b9-c7, and the rook i9-i8 for check. Since White cannot block the b7 cannon and the i8 rook in two moves, White should move the king to another column. However, Black can capture the king by moving the b7 cannon to the same column as the White's king (and the e6 pawn if needed).

Case 1.5 White moves the knight c0-d2-c4.

Black can easily apply the strategy in Case 1.4.

Case 2 White does not make Black in check in the first turn.

If White moves the king to another column in her first turn, Black can use the strategy in Case 2.1 of Lemma 2.4.7. Otherwise, Black moves the knight b9-c7 and the rook i9-i8-f8 in his first turn. White is now in check by the b7 cannon or the f8 rook. Note that White cannot capture both of them in her first two turns. If the White's king is not at d1 or e0 after her second turn, Black can still capture the king by at least one of them, no matter how White moves in her second turn. If this is not the case, White needs at least three moves to block them, together

with moving the king. To do that, White must move the knight g0-f2 and the h2 cannon to d2 or e2 (for example, White moves the pawn g3-f3 and the knight g0-f2 in her first turn, and the cannon h2-e2 and the king e1-e0 in her second turn). If the White's king is at e0, Black can capture the king by moving the pawn g6-h6 and the cannon h7-h0-e0; otherwise, Black moves the rook a9-a8-d8 and the rook f8-f7 for checkmate.

In summary, the proof is now complete. □

For an $(i, 3)$ multimove game on Korean chess with $1 \leq i \leq 2$, Black can easily apply the above strategy. Hence, we get the following lemma.

Lemma 2.4.16. *For an $(i, 3)$ multimove game on Korean chess with $1 \leq i \leq 2$, if White uses the Twin Sang form, Black has a winning strategy.*

Lemma 2.4.17. *For a $(2, 1)$ multimove game on Korean chess, if Black uses the Twin Sang form, White has a winning strategy.*

Proof. White moves the rook a0-a1-d1 to make Black in check. Note that Black cannot capture the White's king or the d1 rook in one move. Since he cannot move the king to get away from the rook, he should block the d1 rook. Then Black moves either the pawn c6-d6, the pawn e6-d6, or the elephant b9-d6. No matter how Black moves, White moves the d1 rook to capture the d6 piece and then to e6 for checkmate. □

Lemma 2.4.18. *For a $(1, 2)$ multimove game on Korean chess, if White uses the Twin Sang form, Black has a winning strategy.*

Proof. Note that White cannot capture the Black's king or make Black in check in her first turn. On the other hard, Black can check White by the rook a9-a8-d8 or i9-i8-f8 in his first turn. If White does not block one of the rooks in the first his turn, Black can apply the proof of the Lemma 2.4.17. Then White should block one of the rooks in the first turn. No matter how White moves a piece to block, Black moves the rook a9-a8-d8 or i9-i8-f8 to check. White must block that rook with a pawn or an elephant. If White do so, Black moves the rook to capture it and moves the rook again to e3 for checkmate; otherwise, Black can capture the king with the rook. □

2.4.3 Ma-Sang-Ma-Sang

The board shown in Figure 2.6 is used for lemmas that White has a winning strategy in this subsection; otherwise, we use the board in Figure 2.7.
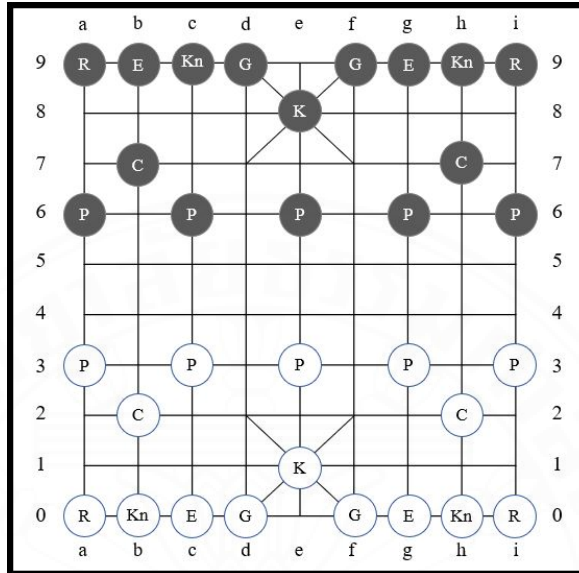

Figure 2.6 The board where White has a winning strategy when Black uses the Ma Sang Ma Sang form
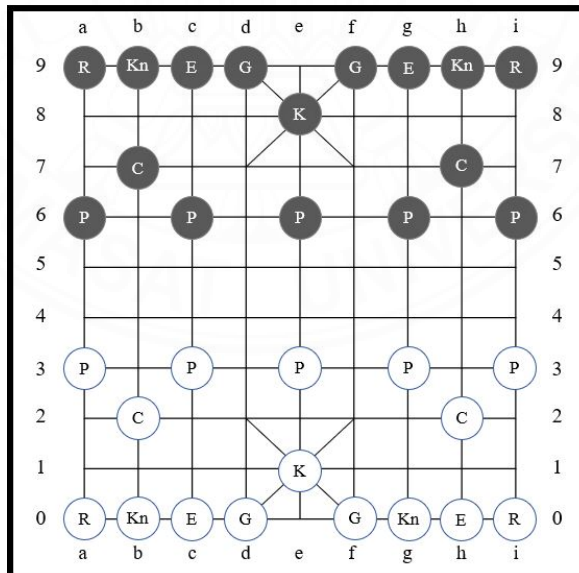

Figure 2.7 The board where Black has a winning strategy when White uses the Ma Sang Ma Sang form

White can use the strategy in Lemma 2.4.2 for an (i, j) multimove game on Korean chess with $i \geq 4$. Hence, we get the following lemma.

Lemma 2.4.19. *For an (i, j) multimove game on Korean chess with $i \geq 4$, if Black uses the Ma-Sang-Ma-Sang form, White has a winning strategy.*

Moreover, Black can apply the strategy in Lemma 2.4.4 to get a winning strategy for an (i, j) multimove game on Korean chess with $1 \leq i \leq 3$ and $j \geq 4$. Hence, we get the following lemma.

Lemma 2.4.20. *For an (i, j) multimove game on Korean chess with $1 \leq i \leq 3$ and $j \geq 4$, if White uses the Ma-Sang-Ma-Sang form, Black has a winning strategy.*

Lemma 2.4.21. *For a $(3, 3)$ multimove game on Korean chess, if Black uses the Ma-Sang-Ma-Sang form, White has a winning strategy.*

Proof. White first moves the rook a0-a1-d1 and the knight b0-c2. Now Black is in check. Note that Black cannot capture the White's king or make the game end in a draw. If Black does not move the b9 elephant or the c9 knight, then White can use the strategy in Lemma 2.4.5. If Black does not move the g9 elephant or the h9 knight, White can use the strategy in Lemma 2.4.13. Suppose Black moves at least one of the b9 elephant and the c9 knight, and at least one of the g9 elephant and the h9 knight. Then White can capture the king by the d1 rook or the cannon.   □

For a $(3, j)$ multimove game on Korean chess with $1 \leq j \leq 3$, White can easily apply the above strategy. The resulting lemma is as follows.

Lemma 2.4.22. *For a $(3, j)$ multimove game on Korean chess with $1 \leq j \leq 3$, if Black uses the Ma-Sang-Ma-Sang form, White has a winning strategy.*

We can apply Lemmas 2.4.8 and 2.4.16 to get the following lemma.

Lemma 2.4.23. *For an $(i, 3)$ multimove game on Korean chess with $1 \leq i \leq 2$, if White uses the Ma-Sang-Ma-Sang form, Black has a winning strategy.*

Lemma 2.4.24. *For a $(2, 1)$ multimove game on Korean chess, if Black uses the Ma-Sang-Ma-Sang form, White has a winning strategy.*

Proof. White moves the rook i0-i1-f1 for check. No matter how Black moves, White can apply the strategy in Lemma 2.4.17 to get a winning strategy.   □

Lemma 2.4.25. *For a* $(1, 2)$ *multimove game on Korean chess, if White uses the Ma-Sang-Ma-Sang form, Black has a winning strategy.*

Proof. Note that White cannot capture the Black's king or make Black in check in her first turn. On the other hard, Black can check White by the rook a9-a8-d8 or i9-i8-f8 in his first turn. If White moves the king to another column, Black can move a rook to the same column as the White's king to checkmate. If White moves the king to e0, Black moves the knight b8-c7 and then the cannon b7-e7 for checkmate. If White does not move a piece to d3, Black moves the rook a9-a8-d8 and applies the strategy in Lemma 2.4.10. If White moves a piece to d3, Black moves the rook i9-i8-f8 to check. Then White must move a piece to f3 to block the f8 rook. After that, Black moves the rook f8-f3-g3 to checkmate. □

2.4.4 Sang-Ma-Sang-Ma

Like the previous subsections, the player who has a winning strategy uses the Twin Ma form. Moreover, the other player uses the Sang-Ma-Sang-Ma form. In other words, if White has a winning strategy, the board is shown in Figure 2.8; otherwise, it is shown in Figure 2.9. We can notice that if we flip the board in Figure 2.8 (respectively, Figure 2.9) with the column e being the symmetry line, we get the board in Figure 2.6 (respectively, Figure 2.7). Hence, The player who has a winning strategy in the previous section also has a winning strategy in this section. For example, White can move the rook a0-a1-d1-d7-e8 to capture the king in a $(4, 2)$ multimove game on Korean chess when Black uses the Ma-Sang-Ma-Sang form. As a result, if Black uses the Sang-Ma-Sang-Ma form, White can move the rook i0-i1-f1-f7-e8 to capture the king (see Figures 2.10 and 2.11).

Therefore, all the results presented in Section 2.4.3 can be obtained analogously in this section.
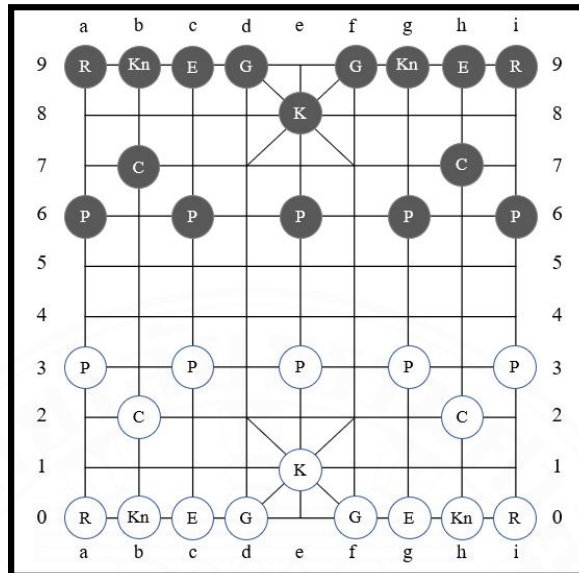
Figure 2.8 The board where White has a winning strategy when Black uses the Sang Ma Sang Ma form
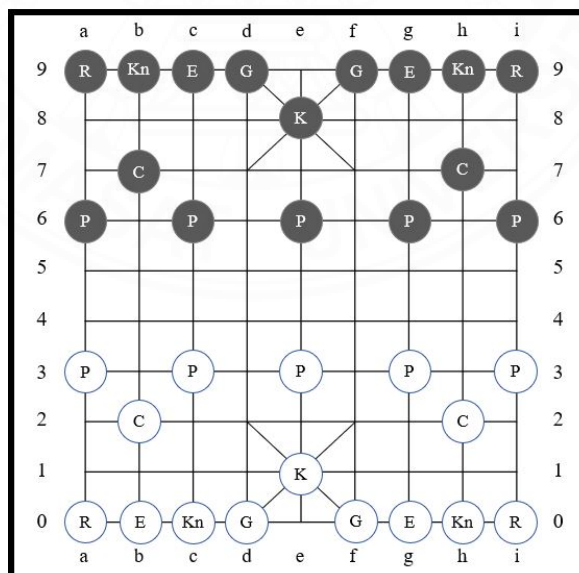

Figure 2.9 The board where Black has a winning strategy when White uses the Sang Ma Sang Ma form
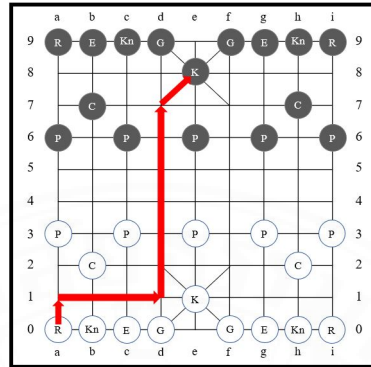
Figure 2.10 The moving of the White's rook when Black uses the Ma-Sang-Ma-Sang form
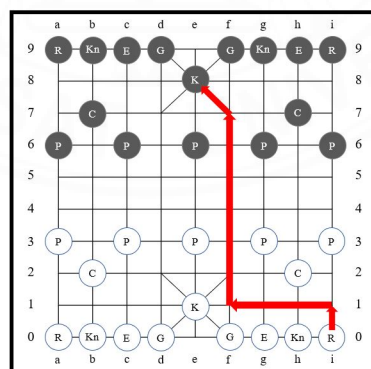

Figure 2.11 The moving of the White's rook when Black uses the Sang-Ma-Sang-Ma form

CHAPTER 3

CONCLUSION

Let i and j be positive integers with $(i, j) \notin \{(1, 1), (2, 2)\}$. The player who has a winning strategy for an (i, j) multimove game on Thai chess and Korean chess is shown in Table 3.1. These results are the same as ones for an (i, j) multimove game on Chess; however, the strategies in the proofs are different. Furthermore, the player who has a winning strategy for an (i, j) multimove game on Japanese chess and Chinese chess is shown in Table 3.2.

White/Black	$j = 1$	$j = 2$	$j = 3$	$j \geq 4$
$i = 1$	?	Black	Black	Black
$i = 2$	White	?	Black	Black
$i = 3$	White	White	White	Black
$i \geq 4$	White	White	White	White

Table 3.1 The player who has a winning strategy for an (i, j) multimove game on Thai chess and Korean chess

White/Black	$j = 1$	$j = 2$	$j \geq 3$
$i = 1$	?	Black	Black
$i = 2$	White	?	Black
$i \geq 3$	White	White	White

Table 3.2 The player who has a winning strategy for an (i, j) multimove game on Japanese chess and Chinese chess

The provided winning strategies attempt to capture the king in the first turn or keep the opponent player in check every turn until the game ends. We can notice that if i (the number of moves per turn for White) is large enough, White can win the game no matter how large j is. Let i_0 be the minimum number of moves such that White can capture the king in the first turn. Thus, $\left\lceil \frac{i_0}{2} \right\rceil$ is the minimum number of moves such that White can check Black in the first turn, so we get the following lemma.

Lemma 3.1. *Let i_0 be the minimum number of moves such that White can capture the king in the first turn. For an (i, j) multimove game, White can check Black in the first turn if and only if $i \geq \left\lceil \frac{i_0}{2} \right\rceil$.*

Some proofs in this dissertation can be shorten by using this lemma. For example, in Lemma 2.1.8, we have $i_0 = 3$. Since $i = 1 < 2 = \lceil \frac{i_0}{2} \rceil$, Lemma 3.1 guarantees that White cannot check Black in the first turn, so we do not need to show the first paragraph in the proof. However, we have chosen to retain the proofs in those lemmas because we aim to provide readers with the same techniques we use in the proofs for each of them.

In some games, White can "pass" in her first turn so that she can win by stealing Black's winning strategy. For example, in a (3,2) multimove game on Thai chess, White can move the king d1-c2-d2-d1 in the first turn, and she can use Black's strategy in Lemma 2.1.6 to win. Unfortunately, there are some other games in which White cannot do that. For example, in an $(3, 2)$ multimove game on Chess, since the arrangements of pieces of White and Black are different when the game starts, White then cannot use Black's winning strategy. This technique gives the following theorem.

Theorem 3.2. *For an (i, j) multimove game with identical piece arrangements for White and Black, White has a winning strategy if she can pass in her first turn, and Black has a winning strategy for a (j, i) multimove game.*

First, we note that if $i > 1$, White can always pass in her first turn for an (i, j) multimove game on Thai chess, Japanese chess, Chinese chess, and Korean chess. Next, the arrangements of pieces of White and Black are the same in an (i, j) multimove game on Thai chess, Japanese chess, and Chinese chess. Moreover, for an (i, j) multimove game on Korean chess if Black has a winning strategy for a (j, i) multimove game on Korean chess in some starting form, White can just choose that form in a (i, j) multimove game on Korean chess, so we immediately get Corollary 3.3.

Corollary 3.3. *For an (i, j) multimove game with $i > 1$ on Thai chess, Japanese chess, Chinese chess, and Korean chess, White has a winning strategy if Black has a winning strategy in a (j, i) multimove game on the same kind of chess.*

For an (i, i) multimove game with identical piece arrangements for White and Black, where White can pass in her first turn, if Black has a winning strategy, we can use Theorem 3.2 to show that White would also have a winning strategy. This is a contradiction, so we get the following theorem.

Theorem 3.4. *For an* (i, i) *multimove game with identical piece arrangements for White and Black, if White can pass in her first turn, Black does not have a winning strategy.*

       In a (2,2) multimove game on Thai chess, Japanese chess, and Chinese chess, White can always pass in her first turn, so Theorem 3.4 implies Black does not have a winning strategy. On the other hand, for a (2,2) multimove game on Korean chess, if Black has a winning strategy for some form, White would also have a winning strategy by choosing that form, which is a contradiction. Therefore, we get the following corollary.

Corollary 3.5. *For an* $(2, 2)$ *multimove game on Thai chess, Japanese chess, Chinese chess, and Korean chess, Black does not have a winning strategy.*

BIBLIOGRAPHY

[1] Averbakh, I., & Averbakh, Y. (2012). *A history of chess: From Chaturanga to the Present Day*. Germany: Russell Enterprises.

[2] Berger, E. R., & Dubbs, A. (2015). Winning strategies in multimove chess (i, j). *Journal of Information Processing, 23*(3), 272–275. https://doi.org/10.2197/ipsjjip.23.272

[3] Boros, E., Elbassioni, K.M., Gurvich, V.A., & Makino, K. (2012). On Nash equilibria and improvement cycles in pure positional strategies for Chess-like and Backgammon-like n-person games. *Discrete Mathematics, 312*(4), 772-788. https://doi.org/10.1016/j.disc.2011.11.011

[4] Cazaux, J., & Knowlton, R. (2017). *A world of chess: its development and variations through centuries and civilizations*. North Carolina: McFarland & Company, Inc.

[5] Chia, G. L., & Ong, S. (2005). Generalized knight's tours on rectangular chessboards. *Discrete Applied Mathematics, 150*(1–3), 80–98. https://doi.org/10.1016/j.dam.2004.11.008

[6] Donovan, L. (2015). *Chess: the complete guide to chess - master: chess tactics, chess openings, and chess strategies (3rd ed.)*. South Carolina: CreateSpace Independent Publishing Platform.

[7] Ionascu, E. J., Pritikin, D., & Wright, S. E. (2008). k-dependence and domination in kings graphs. *The American Mathematical Monthly, 115*(9), 820–836. https://doi.org/10.1080/00029890.2008.11920596

[8] Laisin, M., Okoli, O. C., Chukwuma, E. I., & Okaa-Onwuogu, C. A. (2020). Construction of a three-dimensional chess board for Bishop movement within the Forbidden Area with vector directives. *World Journal of Innovative Research, 9*(1), 101–105. https://doi.org/10.31871/wjir.9.1.27

[9] Laskar, R., & Wallis, C. (1999). Chessboard graphs, related designs, and domination parameters. *Journal of Statistical Planning and Inference, 76*(1-2), 285–294. https://doi.org/10.1016/s0378-3758(98)00132-3

[10] Nhat, V. Q., & Lee, G. (2013). Chessboard and pieces detection for Janggi chess playing robot. *International Journal of Contents, 9*(4), 16–21. https://doi.org/10.5392/ijoc.2013.9.4.016

[11] Ma, L. (2020). Xiangqi vs chess—the cultural differences reflected in Chinese and Western games. *Open Journal of Social Sciences, 08*(03), 52–61. https://doi.org/10.4236/jss.2020.83006

[12] Matsubara, H., Iida, H., & Grimbergen, R. (1996). News, information, tournament and reports from chess to Shogi to Go. *ICGA Journal, 19*(2), 103–112. https://doi.org/10.3233/icg-1996-19208

[13] Reek, J. van, Uiterwijk, J. W. H. M., & Herik, H. J. van den. (1998). Planning a strategy in chess. *ICGA Journal, 21*(3), 183–192. https://doi.org/10.3233/icg-1998-21305

[14] Spoerer, K., Sirivichayakul, T., & Iida, H. (2013). Homogeneous group performance in chess. *Procedia Technology, 11*, 1272-1276. https://doi.org/10.1016/j.protcy.2013.12.324

[15] Wagon, S. (2014). Graph theory problems from hexagonal and traditional chess. *The College Mathematics Journal, 45*(4), 278–287. https://doi.org/10.4169/college.math.j.45.4.278

[16] Wu, R., & Beal, D. (2001). Solving Chinese chess endgames by database construction. *Information Sciences, 135*, 207-228. https://doi.org/10.1016/s0020-0255(01)00137-2

[17] Yen, S. J., Chen, J. C., Yang, T. N., & Hsu, S. C. (2004). Computer Chinese chess. *ICGA Journal, 27*(1), 3–18. https://doi.org/10.3233/icg-2004-27102

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